Bond Graph Approach to Build Reduced Order Observers in Linear Time Invariant Systems

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Abstract. This paper proposes a method to build reduced order observers for linear time invariant systems modelled by bond graph. The method is based on the Luenberger's algebraic method for the design of reduced order observers. The calculation of the observer gain is based on the pole placement technique for linear systems modelled by bond graph. One example with two outputs is developed where one of the measurements is dependent on all state variables.

I. Introduction

Effective control and monitoring of a process requires reliable and continuous real-time information on the state variables of the process. In practice, however, continuous, on-line state measurements are rarely available. State estimators (observers), which are deterministic or stochastic, and static or dynamic, have as purpose to reconstruct the inaccessible but important state variables from other easily available measurements. The Kalman filter and the Luenberger observer were the first ones to be introduced in the 1970s. The present work concerns deterministic linear time invariant systems modelled by bond graphs and because of that, the observers are built based on the Luenberger's method.

The principal advantage of implementing a reduced order observer is that it will estimate only those state variables which cannot be directly measured, thus, the order of the model of the observer will be lower than the order of a complete order observer, and therefore the computational cost to estimate these variables is also lower. Karnopp (1979) proposed a method for bond graph models to build both complete order observers and reduced order observers for linear systems. In his work, the method used bond graph basically for the architecture of the observer. The calculus of the gain of the observer is made with the classical algebraic methods.

In this paper, the method proposed is a procedure to build the reduced order observers directly from the bond graph model. It can be used in both cases, when the output depends on all state variables and when the output is related to one state variable; in the second case the procedure is simpler.

The use of bond graphs allows employing techniques of structural analysis to determine the observability of a model, which is a necessary condition for the construction of Luenberger's observers. In this paper, the verification of structural observability of the bond graph model is made with the technique proposed by Sueur and Dauphin-Tanguy (1991).

The Luenberger's method includes matrix manipulations which can imply the calculation of matrix inverses. The main objective of this paper is to give a graphical method that can be applied on a bond graph model directly to build the observers without the generation and manipulation of the state and output equations of the system. The calculation of the matrices will be made directly from the bond graph model and using the graphical advantages of the bond graphs, the determination of the invertibility of some matrices will be made from a structural point of view.

Rahmani et al.(1994) proposed a method to calculate the gain of a controller gain by pole placement directly from the bond graph model. In the present work an extension of this method is used to calculate the observer's gain. Because of that, it is possible to calculate the gain with the knowledge of the causal cycles families in the bond graph of the observer. In this case the characteristic polynomial of the reduced observer ($P_{(\bar{A}_{bb} - \bar{K}\bar{A}_{ab})}(s)$)

is selected and then the calculus of K is based on the polynomial coefficients.

As an application, one example with two outputs is studied here, where one of the measurements depends on all state variables.

The paper is organized as follows: section II shows the bond graph implementation of reduced order observers. Section III shows the principles to design the observer starting from a bond graph model. Section IV shows one example. Finally Section V shows the conclusions of this work.

II. Bond Graph Approach

Given a linear time invariant system, and considering that its bond graph model is equivalent to:

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{1}$$

with $A \in \Re^{n \times n}, B \in \Re^{n \times p}, C \in \Re^{m \times n}$, y regroups the outputs of the system.

That means that in the bond graph model there are nor causal loops between R-elements neither derivative causalities that could generate an implicit state equation. From this bond graph model, the proposed procedure to build the reduced order observers is shown in the next subsection.

II. 1 Proposition of a procedure to build the reduced order observers

Step 1. General conditions to build the reduced order observer
There exist two conditions related to the system to be verified before the construction of the Luenberger's observer:

      (a) Only the information related to the non-redundant outputs should be considered. This means that:

$$m = \text{number of non-redundant outputs of the system.}$$

      (b) The system has to be observable.

To verify the condition (a) in the bond graph model, it is possible to use the following property about the struct-rank[C] shown by Sueur and Dauphin-Tanguy (1991):

Property 1. The struct-rank[C] is equal to the number of detectors in a bond graph model that can be dualized without creating causality conflicts and accepting the change of causality of the dynamical elements in integral causality.

With this property, it is possible to identify the redundant outputs that can exists, then these outputs can be neglected and by consequence, the number of outputs that remain is equal to the struct-rank[C]:

$$\text{struct-rank}[C] = m$$

To verify the condition (b) the concept of structural observability can be properly used.   The structural observability of the bond graph model can be verified with the technique proposed by Sueur and Dauphin-Tanguy (1991). They have shown that if all the dynamical elements in integral causality are causally connected with a detector and all the I-C elements in integral causality are in derivative causality when a derivative assignment is performed over the initial bond graph, then the system is structurally state observable by the detectors.

Step 2. Division of the state vector
The Luenberger's method for building reduced order observers consists in dividing the state variables of the model into measurable variables and non-measurable variables. The measurable variables are the state variables that can be directly measured from a sensor or can be calculated directly from the measurement of the sensor. With this classification, the state equation of the system can be written as a function of the measurable ($x_a \in \Re^m$) and non-measurable ($x_b \in \Re^{n-m}$) variables:

$$\begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} u$$

$$y = \begin{bmatrix} C_a & C_b \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} \tag{2}$$

Afterwards, with a linear transformation, T:

$$T = \begin{bmatrix} C_a & C_b \\ 0 & I^{(n-m)\times(n-m)} \end{bmatrix} \tag{3}$$

the state equation is written as a function of the output and the non-measurable state:

$$\begin{bmatrix} \dot{y} \\ \dot{x}_b \end{bmatrix} = \begin{bmatrix} \overline{A}_{aa} & \overline{A}_{ab} \\ \overline{A}_{ba} & \overline{A}_{bb} \end{bmatrix} \begin{bmatrix} y \\ x_b \end{bmatrix} + \begin{bmatrix} \overline{B}_a \\ \overline{B}_b \end{bmatrix} u \tag{4}$$

where:

$$\overline{A}_{aa} = \left(C_a A_{aa} + C_b A_{ba} \right) C_a^{-1}$$
$$\overline{A}_{ab} = \left(C_a A_{ab} + C_b A_{bb} - \overline{A}_{aa} C_b \right)$$
$$\overline{A}_{ba} = A_{ba} C_a^{-1} \tag{5}$$
$$\overline{A}_{bb} = A_{bb} - A_{ba} C_a^{-1} C_b$$
$$\overline{B}_a = C_a B_a + C_b B_b$$
$$\overline{B}_b = B_b$$

With this representation the inverse of the C_a matrix is needed. Because of that, after the selection of x_a, the following condition: rank(C_a) = m must be verified to guarantee the existence of C_a^{-1}.

The rank of the C matrix can give information about the inversibility of C_a. Taking only the outputs in the detectors that can be dualized without any causality conflict, Property 1 implies that there exists an invertible submatrix of C with dimensions m x m, called C_a, corresponding to a selection of components x_a. Using this reasoning, the dimension of vector x_a always has to be equal to the struct-rank[C]:

$$\dim(x_a) = \text{struct-rank}[C] = m$$

The bicausality concept can be appropriately used to select the x_a vector guaranteeing the existence of C_a^{-1}. The bicausality allows fixing or imposing at the same time a variable and its conjugate as bicausal bonds decouple the effort and flow causalities. In the context of the inversion problem, imposing the output variable without modifying the energy structure (or constraint equations) of the system can be carried out with an SS element having a flow source /effort source causality (Ngwonpo et al., 1996).

By Property 1 it is possible to determine struct-rank[C] by means of the dualization of detectors. When this procedure is made, the conjugate variable is equal to zero, because the detector is supposed to be ideal (no power dissipated or stored). Then, to obtain information about C_a^{-1}, the detectors can be substituted by SS elements leading to a null power flow on that bond. From this analysis, Theorem 1 can be introduced.

Theorem 1. The inverse of the C_a submatrix exists (or the struct-rank[C_a] = m), if the following operations can be made in the bond graph model without introducing any causality conflict:

   (i)   All detectors are substituted by SS elements as shown in Figure 1.
   (ii)   The dynamical elements associated with x_a change its integral causality into a bicausality as it is shown in Figure 2.
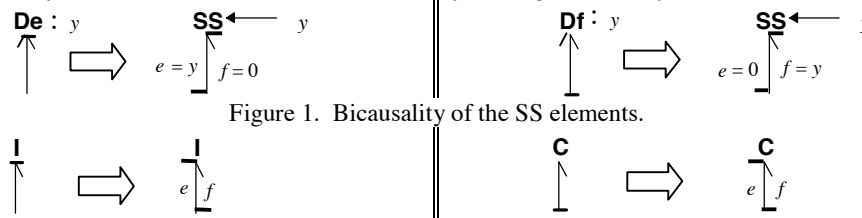   (iii) The dynamical elements associated with x_b stay in integral causality.

Figure 1. Bicausality of the SS elements.

Figure 2. Bicausality of the elements associated with x_a.

Proof. Consider the junction structure equation of the initial bond graph model yields:

$$\begin{bmatrix} \dot{x}_a \\ \dot{x}_b \\ D_{in} \\ y \end{bmatrix} = \begin{bmatrix} S_{11}^{aa} & S_{11}^{ab} & S_{13}^{a} & S_{14}^{a} \\ S_{11}^{ba} & S_{11}^{bb} & S_{13}^{b} & S_{14}^{b} \\ S_{21}^{a} & S_{21}^{b} & S_{23} & S_{24} \\ S_{31}^{a} & S_{31}^{b} & S_{33} & 0 \end{bmatrix} \begin{bmatrix} Z_a \\ Z_b \\ D_{out} \\ u \end{bmatrix}$$

(6)

To perform a bicausality assignment in the storage elements and to change the detectors by SS elements is equivalent to inverse some terms in (6); then a new output structure vector $\begin{bmatrix} Z_a^t & \dot{x}_b^t & D_{in}^{*t} \end{bmatrix}$ is built as it is shown in equation (7):

$$\begin{bmatrix} Z_a \\ \dot{x}_b \\ D_{in}^{*} \end{bmatrix} = \begin{bmatrix} M_1 & M_2 & M_3 & 0 \\ M_4 & M_5 & M_6 & M_7 \\ M_8 & M_9 & M_{10} & M_{11} \end{bmatrix} \begin{bmatrix} y \\ Z_b \\ D_{out}^{*} \\ u \end{bmatrix}$$

(7)

If (7) is solvable, that is if $(I - M_{10} L^{*})$ is invertible, where I is an identity matrix of appropriate dimensions and L^{*} is such that $D_{out}^{*} = L^{*} D_{in}^{*}$, it follows, with $Z_a = F_a x_a$ and $Z_b = F_b x_b$,

$$x_a = \overline{M}_1 y + \overline{M}_2 x_b$$

(8)

On the other hand, supposing the output equation in equation (2):

$$y = \begin{bmatrix} C_a & C_b \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix}$$

(9)

To calculate x_a from equation (2) the condition struct-rank[C_a] = m has to be satisfied, then

$$x_a = C_a^{-1} y - C_a^{-1} C_b x_b$$

(10)

Obviously, there are two possible ways to invert the relation (16), directly with the output submatrix C_a, or using the bicausality assignment. The same equation have to be obtained, so equation (8) has to be equal to equation (10), meaning that the struct-rank[C_a] = m and the inverse of C_a exists. ∎

Once the selection of x_a is made such as the condition of $\text{rank}(C_a) = m$ is verified, then the calculation of C_a and C_b is directly derived from the initial bond graph model by calculating the gain of the causal path of length one (1) from the I or C-elements associated with the time derivative of x_a and x_b to the output y.

Step 3. Calculation of C_a^{-1} directly from a bond graph model

From Theorem 1, it is possible to deduce the following theorem that allows calculating directly C_a^{-1} from a bond graph model.

Theorem 2. After applying the three conditions of Theorem 1, it is possible to calculate C_a^{-1} directly from the new bond graph with bicausalities as:

$$C_a^{-1} = F_a^{-1} M_a \tag{11}$$

where M_a is the gain matrix between the output y and the complementary state variables Z_a, calculated through the calculation of the gain of the causal path of length one (1) from y to Z_a and F_{ia}^{-1} is a diagonal positive definite matrix as:

$$F_a^{-1} = \begin{bmatrix} C_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & I_j \end{bmatrix}_a \tag{12}$$

Proof. If the three conditions of Theorem 1 are satisfied, then from the bond graph with bicausality assignment, the complementary variables Z_a can be calculated knowing y. This means that is possible to calculate the gain matrix (M_a) between the output y and the complementary state variables through the calculation of the gain of the causal path of length one from y to Z_a.

By definition:
$$Z_a = F_a x_a$$

where F_a is a diagonal positive definite matrix (each element, I, C, is a one port element):

$$F_a = \begin{bmatrix} 1/C_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/I_j \end{bmatrix}_a$$

Then as $x_a = F_a^{-1} Z_a$, finally C_a^{-1} can be calculated as:
$$C_a^{-1} = F_a^{-1} M_a \qquad\qquad \blacksquare$$

Before explaining steps 4, 5 and 6 of the method, the following two definitions are required.

Definition 1. The modulated flow source associated with an I-element, denoted **MSf$_I$**, is a flow source that imposes a flow on the connected junction equal to:

$$f = \frac{1}{I}\beta \tag{13}$$

where β is the module in the source and I is the inductance value of the I-element associated.
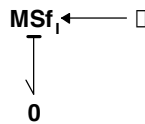
Figure 3. Modulated flow source associated with an I-element

Definition 2. The modulated effort source associated with a C-element, denoted **MSe$_C$**, is an effort source imposing an effort on the connected junction equal to:

$$e = \frac{1}{C}\beta \tag{14}$$

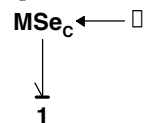where β is the module in the source and C is the capacitance value of the associated C-element.

Figure 4. Modulated effort source associated with a C-element

The observer's bond graph model is the same than the system model with the pertinent changes described in the steps 4, 5 and 6.

Step 4. Addition of the term Ky.

In the dynamical elements associated with $\hat{x}_b$, the term Ky is added to calculate the state:

$$\hat{x}_b = \hat{z} + Ky \tag{15}$$

In the case where $\hat{x}_b$ is a momentum of an I-element, this operation is equivalent to change in the observer the I-element of the system's model, by the bond graph shown in Figure 5.
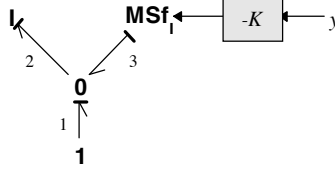

Figure 5. Bond Graph model to add Ky

Then,

$$\hat{p}_2 = \hat{z} = \int \dot{\hat{z}}\, dt \Rightarrow \hat{f}_1 = \frac{1}{I}\hat{p}_2 + \frac{1}{I}Ky \tag{16}$$

In the case where $\hat{x}_b$ is a displacement in a C-element, this operation is equivalent to change in the observer the C-element of the system's model, by the bond graph shown in Figure 6.
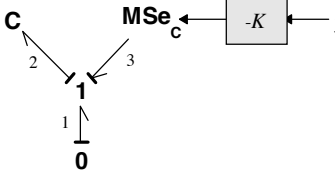
Figure 6. Bond Graph model to add Ky

Then,

$$\hat{q}_2 = \hat{z} = \int \dot{\hat{z}}\, dt \Rightarrow \hat{e}_1 = \frac{1}{C}\hat{q}_2 + \frac{1}{C}Ky \tag{17}$$

Step 5. Change of the dynamical elements associated with x_a.

The dynamical elements in the observer associated with x_a (I-elements or C-elements) change their dynamical equation as follows:

$$\hat{x}_a = C_a^{-1}(y - C_b\,\hat{x}_b) \tag{18}$$

thus: $\hat{e}_C = \frac{1}{C}\hat{x}_a$ or $\hat{f}_I = \frac{1}{I}\hat{x}_a$

Then they are not dynamical elements, because:

$$\hat{x}_a \neq \int \dot{\hat{x}}_a\, dt \tag{19}$$

They are algebraic elements, therefore, in the observer's bond graph, C-elements change to modulated effort sources **MSe$_C$** and I-elements change to modulated flow sources **MSf$_I$**. By means of this change of elements the order reduction occurs.

Step 6. Addition of the term ϕ.

In this step, the calculation of a term ϕ is required. This term ϕ is equal to:

$$\phi = -K\left[C_a\,\hat{\phi}_a + C_b\,\hat{\phi}_b\right] \tag{20}$$

where, $\hat{\phi}_a$ and $\hat{\phi}_b$ are flows when the state variables x_a and x_b are associated with a C-element (or efforts when x_a and x_b are associated with an I-element).

Then, the term ϕ is also a flow when the state variable x_b is associated with a C-element (or an effort when x_b is associated with an I-element). Afterwards the addition of ϕ is made with a modulated source as it is shown in Figure 7.

Figure 7. Addition of the term ϕ

The $\hat{\phi}_a$ variable is the flow in the MSe_C associated with x_a (if x_a is a displacement in a C-element) or the effort in the MSf_I associated with x_a (if x_a is a momentum in an I-element) as it is shown in Figure 7.

The $\hat{\phi}_b$ variable is the effort in the observer's bond graph before the addition of ϕ (if x_b is a momentum in an I-element) or the flow in the observer's bond graph before the addition of ϕ (if x_b is a displacement in a C-element) as it is shown in Figure 7.

From an algebraic point of view, the graphical operations shown in steps 4, 5 and 6 are equivalent to these of Luenberger's method for reduced order observers in linear system (Luenberger, 1966). The state equation for the estimated of x_b is:

$$\dot{\hat{x}}_b = \overline{A}_{ba}\, y + \overline{A}_{bb}\, \hat{x}_b + \overline{B}_b\, u + K\left(\overline{A}_{ab}\, x_b - \overline{A}_{ab}\, \hat{x}_b\right) \tag{21}$$

with $K \in \Re^{(n-m) \times m}$.

Thus, the dimension of the observer is equal to $(n - m)$, where n is the total number of state variables and m is the number of non-redundant outputs of the system.

As x_b is non-measurable, in equation (21) the term $\overline{A}_{ab} x_b$ must be substituted by an expression derived from equation (4), which is a function of y and u as it is shown in equation (22):

$$\overline{A}_{ab} x_b = \dot{y} - \overline{A}_{aa}\, y - \overline{B}_a u \tag{22}$$

Afterwards, to avoid the time derivation of the output, the estimated state is calculated by means of an auxiliary variable, $\hat{z}$:

$$\hat{z} = \hat{x}_b - Ky \tag{23}$$

With this variable and an algebraic manipulation, the state equation for the estimated states is:

$$\dot{\hat{z}} = \overline{A}_{ba}\, y + \overline{A}_{bb}(\hat{z} + Ky) + \overline{B}_b\, u - K(\overline{A}_{aa}\, y + \overline{A}_{ab}\,(\hat{z} + Ky) + \overline{B}_a\, u) \tag{24}$$

Equation (24) can be written as:

$$\dot{\hat{z}} = \overline{A}_{ba}\, y + \overline{A}_{bb}(\hat{z} + Ky) + \overline{B}_b\, u + \phi \tag{25}$$

where: $\phi = -K(\overline{A}_{aa}\, y + \overline{A}_{ab}\,(\hat{z} + Ky) + \overline{B}_a\, u) = -K\left[C_a\, \hat{\phi}_a + C_b\, \hat{\phi}_b\right]$ (26)

and the terms $\hat{\phi}_a$ and $\hat{\phi}_b$ correspond to:

$$\hat{\phi}_a = A_{aa}\, \hat{x}_a + A_{ab}\,(\hat{z} + Ky) + B_a\, u \tag{27}$$

$$\hat{\phi}_b = A_{ba}\, \hat{x}_a + A_{ba}\,(\hat{z} + Ky) + B_b\, u \tag{28}$$

Finally, the proposed procedure to build the reduced order observers from a bond graph model can be summarized as follows:

Step 1. Selection of the non-redundant outputs and verification of the structural observability of the model.
Step 2. Selection of x_a, verification of rank(C_a) = m and calculation of C_a and C_b.
Step 3. Calculation of C_a^{-1}.
Step 4. Addition of the term Ky (as in equation (15)).
Step 5. Change of the dynamical elements associated with x_a.
Step 6. Addition of the term ϕ (as in equation (26)).

The observer obtained with this procedure gives the following dynamics for the estimation error, $e = x_b - \hat{x}_b$:

$$\dot{e} = (\overline{A}_{bb} - K\overline{A}_{ab}\,)e \tag{29}$$

Luenberger (1966) has shown that if the system in equation (1) is observable, the pair $(\overline{A}_{bb}, \overline{A}_{ab}\,)$ is also observable, therefore, the eigenvalues of $(\overline{A}_{bb} - K\overline{A}_{ab}\,)$ can be arbitrary selected to define the error dynamics. The next section explains how to use the pole placement technique for linear systems modelled by bond graph to calculate the observer's gain.

III. Observer design

The observer design is based on the pole placement techniques proposed by Rahmani et al. (1994). In this case the characteristic polynomial of the reduced observer ($P_{(\overline{A}bb-\overline{KA}ab)}(s)$) is selected and then the calculation of K is based on the polynomial coefficients. This calculus is possible considering the information signals associated with K and applying the proposed Theorem 3:

Theorem 3. The value of each coefficient of the characteristic polynomial $P_{(\overline{A}bb-\overline{KA}ab)}(s)$, is equal to the total gain of the ith-order families of causal cycles in the bond graph model:

$$P_{(\overline{A}bb-\overline{KA}ab)}(s) = s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1} s + \alpha_n$$

The gain of each involved family of causal cycles must be multiplied by $(-1)^d$ if the family is constituted by d disjoint causal cycles.

Thus, the causal analysis to calculate K is made only with the family of causal cycles in the observer's bond graph.

IV. Example

Model with two outputs. In this case an electrical network is studied. The outputs are the current in the capacitor and the current in the resistance R_2.
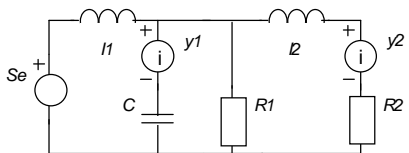
Figure 8. Example 2. Electrical network

The bond graph model is:
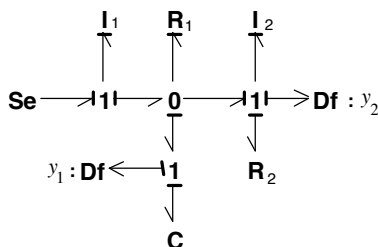
Figure 9. Bond graph model of the example.

Then applying the procedure II.1 for building the reduced order observer:

Step 1. After applying Property 1, it is possible to determine that y_1 and y_2 are non-redundant outputs. The structural analysis (Sueur and Dauphin-Tanguy, 1991) for this model reveals that the system is observable by the detectors Df's.

Step 2. For this system, the momentum in the I-elements are selected as measurable state variables, then by Theorem 1 the new bond graph shown in Figure 10 is generated:
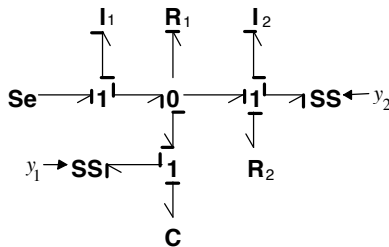

Figure 10. Bond graph with bicausality to calculate C_a^{-1}

Figure 10 shows that there are no causality conflicts, then: struct-rank$[C_a] = 2 = m$

Finally, the values of C_a and C_b derived from Figure 9 are:

$$C_a = \begin{bmatrix} 1/I_1 & -1/I_2 \\ 0 & 1/I_2 \end{bmatrix}$$

$$C_b = \begin{bmatrix} -1/R_1C \\ 0 \end{bmatrix}$$

Step 3. This calculus is made directly from the bond graph model with bicausality (Figure 10).

From Figure 10, the gain matrix between y and Z_a is: $M_a = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Then the inverse of C_a matrix is:

$$\Rightarrow C_a^{-1} = F_a^{-1} M_a = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I_1 & I_1 \\ 0 & I_2 \end{bmatrix}$$

Step 4. Taking the bond graph model of the system in Figure 9, for the observer the addition of the term Ky ($Ky = K_1 y_1 + K_2 y_2$) is made as it is shown in Figure 11.
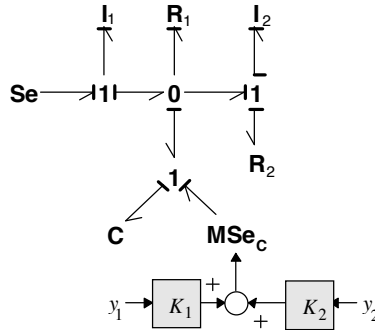

Figure 11. Addition of the term Ky in the observer's model.

Step 5. Figure 12 shows the substitution of the dynamical elements associated with x_a.
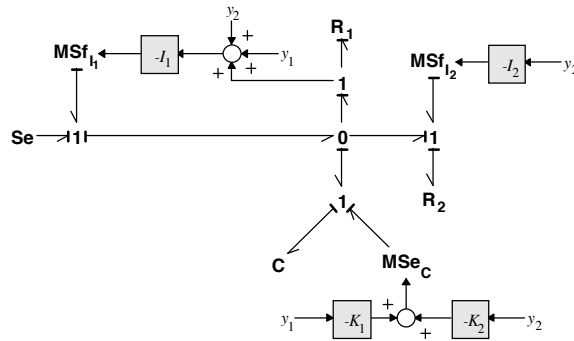
Figure 12. Change of the dynamical elements associated with x_a

Step 6. Addition of the term ϕ:

With this operation, the observer's bond graph is complete as it is shown in Figure 13.

Observer design

Applying the method of Rahmani et al. (1994) in the observer model (Figure 13), the families of causal cycles shown in Table 1 are found.

Table 1. Families of causal cycles in the observer

Family of causal cycle	Gain ($d = 1$)
(a) (C$\leftarrow\rightarrow$R$_1$) : 10-9-8-7-5-12-5-7-8-9-10	$G_{(a)} = (-1)^1 (-1/R_1 C)$
(b) (C$\leftarrow\rightarrow$R$_1$) : 10-9-8-19-20-23-18-11-9-10	$G_{(b)} = (-1)^1 \left(-K_1 /(R_1 C)^2\right)$
(c) (C$\leftarrow\rightarrow$MSf) :10-9-8-7-6-16-17-22-23-18-11-9-10	$G_{(c)} = (-1)^1 (K_1 / I_1 C)$
(d) (C$\leftarrow\rightarrow$R$_1$) : 10-9-8-7-5-12-13-14-15-3-2-4-7-8-9-10	$G_{(d)} = (-1)^1 (1/R_1 C)$
(e) (C$\leftarrow\rightarrow$MSf) :10-9-8-7-4-2-21-23-18-11-9-10	$G_{(e)} = (-1)^1 (K_1 / I_2 C)$
(f) (C$\leftarrow\rightarrow$MSf) :10-9-8-7-6-16-17-18-11-9-10	$G_{(f)} = (-1)^1 (-K_2 / I_2 C)$
(g) (C$\leftarrow\rightarrow$MSf) :10-9-8-7-5-12-13-14-15-3-2-4-7-19-20-23-18-11-9-10	$G_{(g)} = (-1)^1 \left(K_1 /(R_1 C)^2\right)$

Selecting α_1 as the desired coefficient in the characteristic polynomial, $P_{(\bar{A}_{bb} - K\bar{A}_{ab})}(s)$:

$$P_{(\bar{A}_{bb} - K\bar{A}_{ab})}(s) = s + \alpha_1$$

The calculus of K is directly derived from α_1, because:

$$\alpha_1 = G_{(a)} + G_{(b)} + G_{(c)} + G_{(d)} + G_{(e)} + G_{(f)} + G_{(g)}$$

$$\Rightarrow \alpha_1 = \frac{1}{R_1 C} + \frac{K_1}{(R_1 C)^2} - \frac{K_1}{I_1 C} - \frac{1}{R_1 C} - \frac{K_1}{I_2 C} + \frac{K_2}{I_2 C} - \frac{K_1}{(R_1 C)^2} \qquad (30)$$

Selecting an arbitrary value for K_1 ($\overline{K}_1$), K_2 is calculated directly from equation (30) as follows:

$$\Rightarrow K_2 = \left[\alpha_1 + \overline{K}_1\left(\frac{1}{I_1 C} + \frac{1}{I_2 C}\right)\right] I_2 C \tag{31}$$

Simulations

Simulations have been made in 20-Sim software version 2.3 (Controllab, 1998). The simulations show the state dynamics (Figure 14) and the estimation error dynamic (Figure 15). The values of the parameters in this case are: $Se = 10$ V, $I_1 = 2$ H, $I_2 = 2$ H, $C = 3$ mF, $R_1 = 1.5$ Ω, $R_2 = 1.5$ Ω, $\alpha_1 = 30$, $K_1 = 1$ and the initial conditions are: $x_{a1}(0) = x_{a2}(0) = 1$, $x_b(0) = 1$ and $\hat{z}(0) = 0.4$. Then, using equation (31), K_2 is equal to 2.18.

For the implementation, the values of the parameters of the observer with respect to the system have a variation of −10%.
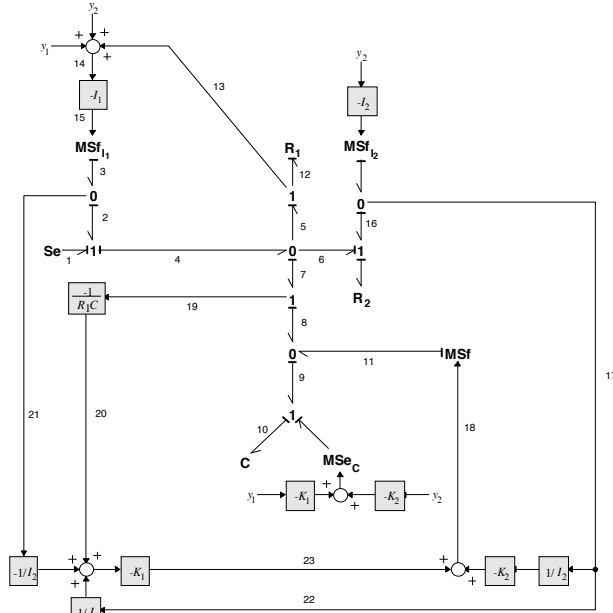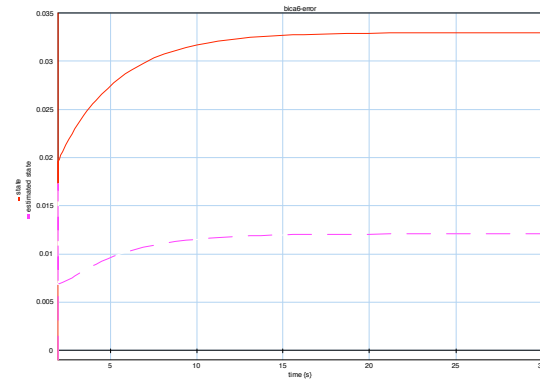

Figure 13. Bond graph model of the observer

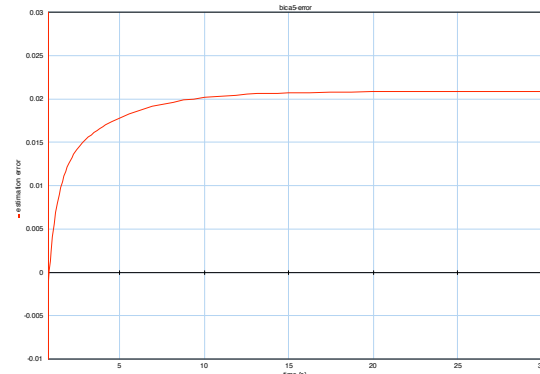Figure 14. Non-measurable state (x_b) and estimated state $(\hat{x}_b)$ vs. time

Figure 15. Estimation error $(x_b - \hat{x}_b)$

The steady state estimation error in Figure 15 has a value of 0.02 approximately, which represents 60% of the real value of x_b. This result shows that although the Luenberger observers (both complete and reduced order observers) are relatively simple in their implementation and design, this type of observers, however, has a drawback that can make it impractical for a particular system: they depend heavily on the precise setting of the parameters and the precise measurement of the output vector. Any disturbance (noise) in the measurement, parameter differences, or internal noises can make the observer unusable (Morgan, 2001). Additionally, Schereir (1997) has shown that the state estimation error does not always converge to zero if a Luenberger observer is applied to reconstruct the states in linear uncertain systems.

V. Conclusions

A bond graph method to design reduced order observers has been shown. As in the algebraic methods, the building of the observers depends on the output matrix C and the inverse of the C_a submatrix. In this paper has been shown how to determine the inversibility of the C_a submatrix (from a structural point of view) and how to calculate the inverse of the C_a submatrix directly from the bond graph model using the bicausality concept. This result leads to a new method to build reduced order observers for LTI systems modelled by bond graphs based on the bicausality concept, which will not needs the calculation of C_a^{-1}, as it is shown in Pichardo-Almarza et al. (2003). The work presented in this paper represents a control tool for people who develop bond graph models, since is possible to build a reduced order observer directly from a bond graph model where the matrix calculation can be directly derived from the observer's graph (including the observer's gain) avoiding the algebraic manipulation of matrices.

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Nomenclature

A	:	state matrix	u	:	input
B	:	input matrix	x	:	state variable
C	:	output matrix , capacitance	$\hat{x}$	:	estimated state variable
e	:	effort	y	:	output
$\hat{e}$	:	estimated effort	$\hat{z}$	:	estimated auxiliary state variable
f	:	flow	Z	:	complementary state variable
$\hat{f}$	:	estimated flow	*Subscripts*		
G	:	gain of causal cycle	a	:	measurable
I	:	inductance, identity matrix	b	:	non-measurable
K	:	observer's gain	C	:	associated with C-element
m	:	number of system's outputs	I	:	associated with I-element
M	:	gain matrix	Se	:	associated with an effort source
n	:	number of system's state variables	*Greek characters*		
p	:	momentum	α	:	desired coefficient in the characteristic polynomial
$\hat{p}$	:	estimated momentum	β	:	module of the modulated sources
q	:	displacement	φ	:	flow or effort associated with calculus of $\dot{\hat{z}}$
$\hat{q}$	:	estimated displacement	ϕ	:	flow or effort associated with calculus of $\dot{\hat{z}}$
R	:	resistance			