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THEORETICAL ISSUES FOR CAD SUPPORT OF FOOTWEAR MANUFACTURING

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Abstract. To a great extent, the design and comfort of shoes are defined by the shape of the shoe-last. The proposed mathematical model of a shoe-last consisting of the basic part in the range of 0.0D – 0.9D and the forepart 0.9D - [1D+(δ+Δ)], where D is the foot length and δ and Δ are the standard and the decorative allowances. The shape of the forepart performing the design function and dependent on fashion tendencies is determined by a number of parameters and is designed in a reference coordinate system in a real-time mode with due regard for the client's requirements. The reference coordinate system utilizes the concepts of a mass centre and three points being anthropometric constants on the surface of a foot (a shoe-last) that are identical in any foot/last and permit one to orient it.

1 Introduction

The principal purpose of this article is to develop a mathematical model of a shoe-last, consisting of a basic part in the range of 0.0D - 0.9D and a forepart 0.9D - [1D+(δ+Δ)], performing the design function and depending on the fashion tendencies in the reference coordinate system. The principal task of footgear designing and production is to satisfy the demand for comfort; at the same time, the shoes must meet the requirements concerning their endurance, wearability, etc. The shoes comfort is ensured by the fact that the shape and sizes of a human foot are taken into account. Direct reproduction of a foot as the principal unit of shoes production, the shoe-last, does not ensure a comfortable shape of the shoe interior. In order to obtain the carcass of a foot surface that might be used in forming the carcass of the last surface, it is necessary to take the predetermined conditions into account, namely the possible transformation of the tri-dimensional shape of the average foot (averaging in a specific sex-age group within the borders of a specific territory [10]) in the process of walking, running, etc., so that the distortions of the foot shape in the process of using the shoes would not bring about a sense of pain and disrupt natural processes of human vital functions. As we pass from the virtual average foot to the surface of a last, besides the factors analyzed above we must take into account the properties of the set of materials from which shoe uppers are made, the purpose of the shoe in question, the conditions of its wearing, etc. [8]. A last is the principal link in shoes production, and the dominating role in shoe design belongs to the mathematical model of the last. The whole set of design and production preparation tasks (obtaining unfolded uppers, designing the details of shoe bottoms, designing the machine attachments) is solved in reference to the surface of a last. Data about the shape of the shoe-last surface may source from a sample formed by a designer manually and measured by a contact or contactless method or from a model of the surface constructed on the basis of transfer from some average foot to a last formed with regard for a large number of factors [3, 8]. In any case, one has to choose a coordinate system for analyzing the last surface and carrying out further design work.

The prerequisites for creating such a coordinate system are the presentation of a shoe-last as a rotation body [9] being self-evident for a number of years, as well as a number of applied problems requiring a concerted application of accumulated statistical, medical and other data in the field of footgear designing and production. In particular, the style of a last is determined by the shape of its forepart as far as the footprint outline and the longitudinal profile are concerned, and therefore the lasts of various styles intended for the same position of the foot (the same heel height) must differ only by their foreparts. Thereby, it becomes possible to divide the last surface into two parts: the basic zone being stable and the forepart subjected to the influence of fashion. If we accept section 0.9D as the reference point for the longitudinal coordinate of the toe, it is possible to form the last foreparts of various lengths and shapes without altering the shape of the last body and without any negative consequences in respect of the shoe interior shape alteration. The concept and location of cross-section 0.9D is intuitively understandable on the basis of production of lasts as rotation bodies on duplicating machines needs adjusting and determining in case of digital measurements with the help of 3D scanners and other devices in the coordinate systems of these measuring devices.

For the purpose of coordinating the definitions pertaining to the already known and newly discovered peculiarities of a foot/last surface, one has to construct a coordinate system connected with a shoe-last that can be used for the purpose of passing from a local coordinate system of a measuring device to a reference coordinate system.

There are two methods of constructing such a system: determining a set of invariant characteristics connected with the internal shape of a surface or defining a set of singular points (geometrical entities) that are always identical and their number is sufficient to orient a foot (or a last). We use the latter method in our work.

2 Constructing a Reference Coordinate System

A singular point in a foot (a last) whose position is stable is the extreme point of the heel curvature [5], the other such point is that of the mass centre of the whole last. The mass centre of the last (a point cloud) is unique; it does not depend on the principles of approximating and measuring its surface (the density of point measurement on the surface is of some significance [6]). If we draw a straight line through these two points – the heel curvature point and the mass centre, we get an unambiguous and invariable axis for constructing a coordinate system in the last. The above axis is sufficient to solve the problems of lasts production on duplicating machines and designing the parts of shoes bottoms, as well as production tools. If contactless measuring devices are used, the measuring device coordinate system is located outside the foot/last body and at some distance from it. As the reference point of the coordinate system, we choose the mass centre of the measured point cloud and transfer the measuring device coordinate system to that point. To change over to a new coordinate system, it is necessary to determine the location of the last mass centre in the old coordinate system $O_0X_0Y_0Z_0$ (Figure1), and then we define the rotation angles of axes of one coordinate system in relation to the other, calculate the coordinates conversion matrix and convert the coordinates of the last surface points to the new coordinate system $OXYZ$ connected with the last mass centre.

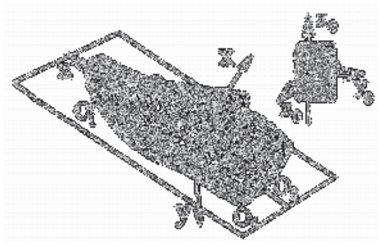

Figure 1. $O_0X_0Y_0Z_0$ – the coordinate system of the measuring device ; O - the centre of mass of the foot/last surface point cloud; O_1, O_2– the characteristic (spatial) points of the foot/last edge; O_3 – the extreme point of the heel curvature; OXYZ – the reference coordinate system

It becomes necessary to define the plane in relation to which the foot/last must be oriented. The designing of articles with the help of cross-sections [8] is based on another invariant as well: the space curve of the last (foot) print edge. Measuring a foot differs from measuring a last because the perimeter points of the footprint edge are to be defined separately with the help of a special algorithm [7]. After forming an invariant space curve of the foot/last print edge, we use additional anthropometric information about two additional base points O_1 and O_2 located on the outside and inside of the edge outline; these points are located in the same plane. Various sources name and define these points differently; in particular, the Russian GOST [4] standard defines them as the points located on the last print edge in sections 0.73D and 0.62D on the inside and outside of the print, respectively.

The points must be defined invariably, namely by the distance from a certain highlighted reference point along the edge curve of the last print. In addition, one should define a highlighted reference point on the edge curve of the foot/last print. To a large extent, its choice depends on the researcher's free will; however it makes sense to propose the following simple algorithm for defining a reference (highlighted) point on the print edge curve. Let us define the reference point of the print edge as a point located at the end of the longest segment wholly situated inside the print; whereupon one should choose from two points the one located at a greater distance from the centre of mass on the print edge curve. As a matter of fact, the case in hand is a strict determination of the location of the point whose analogue is sometimes determined (not quite correctly) as the extreme point of the heel curvature of the footprint edge. In Figure1, the abovementioned point is marked as O_4.

The presence of singular points of the print edge located at the same distance from the reference plane [4, 5] permits us to orient the foot/last in relation to this plane by introducing the second coordinate axis as follows: let us define the second axis as a straight line passing through the mass centre point and being perpendicular to the segment connecting the singular points of the foot/last print edge. As a result of such a construction, a new axis will run perpendicular to both the foot/last rotation axis Z and the plane in relation to which the foot/last must be oriented. For the time being, the coordinate system in which the measured foot/last is located is that of the measuring device transferred to the foot mass centre and having one common axis with the coordinate system being constructed. As a matter of fact, the case at hand at this stage is that we must rotate the existing coordinate system laterally around the axis Z by an angle equal to the difference between the current direction of any of the remaining axes of the device and that of the perpendicular to the reference plane.

Since we use the coordinate system of the measuring device, presume it to be orthogonal and rotate it at definite angles or move it as a single whole without disturbing its structure, it turns out that constructing a really reference coordinate system of a foot/last depends on how precisely an orthogonal coordinate system is reproduced by a measuring device (instrument).

In case of measuring an existing specimen of a last, the measured specimen can be placed in a reference coordinate system both as it is and after separating the fashion-dependent forepart starting with section 0.9D, depending on the task that has been set, e.g. a local modification of the model or its comparison with an existing standard basic part of a sample last. In the latter case, we need to recalculate coordinates of the centre of mass after the forepart is separated and to repeat the procedure of the coordinate system construction.

3 The Mathematical Model of a Last Composed of the Basic (Permanent) Part and a Variable Forepart

The constructed coordinate system allows one to take advantage of additional anthropometric information and to present the model of a last as two parts.

3.1 Analysis of the Shape of the Basic Part of a Last

Since the basic part of a last is made permanent, this permits us to analyze the properties of the basic part surface: to construct the first and the second quadratic forms of the surface, to determine the Gaussian curvature of the surface in every point and to highlight surface sections consisting of the same type points; which is important, e.g., when constructing the unfolded pattern of a surface section , to compare the peculiarities of form change of feet/lasts of different sex-age groups of the population whose form change is so different that the idea of comparing cross-sections of different kinds being analogous in length fractions that seems natural at first sight turns out to be groundless [6, 2], etc. The abovementioned explorations remain significant until the next period of mass measuring of the population's feet and determining a new averaged foot in every territorial sex-age group. The discussion of the results of the explorations described in this Clause is beyond the scope of this publication.

3.2 Designing the Last Forepart

The necessity to consider the matter of designing the forepart (starting with section 0.9D) becomes evident as well, if we consider in retrospect some shapes of foreparts of the lasts used in mass production of shoes (Figure 2).

Figure 2. Typical shapes of the foreparts of lasts used during the recent years

Like other components taking part in the shaping of a man's outward appearance, the design of shoes is subjected to the influence of fashionable tendencies to a great extent. Variable as the fashion is, shoes must always meet the requirements of size matching, comfort and hygiene. That means that the possibility of altering the basic part of a last is very restricted and regimented by the so-called "indifference threshold". The forepart is the only free-form area [1] which may be designed freely with due regard for the requirements concerning the forepart matching with the principal carcass of the last.

Although the length, axial line curvature and the taper factor are not formally restricted, there are aesthetic restrictions; the scope of their alteration is defined by a database recording the peculiarities of shoe toes form change during the last fashion periods. The data analysis permits one to obtain the maximum and the minimum values of the parameters and to find a correlation matrix establishing the scope of their mutual changeability.

While designing the forepart of a last, one can adopt the line passing through the aggregate of the forepart cross-sections' mass centers as the axial line. The axial line is characterized by three invariant parameters: its length, twist and curvature. If we add the section profile taper and form change factors to the above three parameters, the forepart of the last surface will be completely determined.

One can single out two principal shapes of a shoe toe cap: with and without a pronounced quadrangularity. The latter shape of the toe, whose upper section resembles a parabola to a certain degree, has become more widespread, although silhouettes of intermediate shapes whose profile is close to a rectangular one are more widely used in the modern fashion. Therefore it is convenient to use a superelliptic function $x^n / a^n + y^n / b^n = 1$ for the shape of the upper part of the forepart section, where n is the value of the exponent governing the shape of the curve being the forepart form change parameter at the same time. Altering parameters a and b along the forepart sets the toe taper factor. Studies of actual specimens of serially-produced lasts have shown that the twists of foreparts axial lines are insignificant and the surface of the last toe cap can be described by means of a set of two orthogonal families of parabolas.

Figure 3. An example of constructing new last models on the basis of a permanent basic part: a – the basic part of the last; b, c, d – foreparts having different form change and taper parameters

4 Conclusion

The reference coordinate system (RCS) is mainly intended for the purposes of standardization, coordination and accrued sectoral data exchange. An especially convenient feature of the proposed system is the standardization of the set of basic parts of the lasts on the basis of the moulds of average typical feet of various territorial sex-age groups for mass-produced shoes. The basic part of a last is not subject to the influence of fashion, unlike its fore-part starting with the section 0.9D; this part is designed by means of altering parameters (Figure 3) or modified separately by hand.

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