ROBUST CONTROL APPROACH FOR INPUT-OUTPUT LINEARIZABLE NONLINEAR SYSTEMS WITH MODELING ERRORS BASED ON HIGH-GAIN PI-OBSERVER

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Abstract. This paper presents a robust control approach for the class of MIMO input-output linearizable nonlinear systems with modeling errors. As known, the exact feedback linearization method can be applied to control input-output linearizable nonlinear systems, if all the states are available for control and the system model is precise. The mentioned two aspects are exactly the key problems of this classical nonlinear control method. To the first point, the information of the system states can be obtained by measurements or estimations from an observer design. Secondly, modeling errors can be allowed in general. The solution approach developed in this contribution is to reduce/compensate the effects of modeling errors by a feedback of the estimated modeling error calculated by a specific high-gain disturbance observer. At the same time, the non-measured states of the transformed system can also be estimated. Comparing with other possible robust control methods for nonlinear system, the proposed approach offers not only a robust control design, but also the estimation of disturbances. This idea was for SISO cases partially proposed in [1]. In this contribution, the approach is extended and proved for general input-output linearizable nonlinear systems. A detailed instruction of the approach for applications on mechanical systems is given. An example of nonlinear MIMO systems is given to illustrate the application and the success of the approach.

1 Introduction

The lack of robustness of the classical nonlinear control method, the exact feedback linearization method, strongly limits its application. Many researches (e.g. in [2, 3]) have developed different methods to solve this problem. However, in most of the methods the modeling errors or the disturbance are considered with known bounds and dynamics. The approach discussed in this paper takes no information or assumptions from the dynamics of the modeling errors and disturbances, because the modeling errors and disturbances can be estimated together with the system states by a high-gain PI-Observer.

The high-gain PI-Observer is a disturbance observer, that estimates both the system states like a Luenberger observer and additionally the disturbances as external inputs acting to the system. It takes additionally the integration of the estimation errors as input to the observer and therefore generates some extended states [4]. The PI-Observer concept has been developed by many authors [5, 6, 7] for different purposes, in [5, 6] mainly to improve the robustness of the state estimations. In [7], the goal is to use the PI-Observer approach to estimate both the original states and the extended states with high gains, which is exactly the function of the PI-Observer used here.

The key points in this contribution are the combination of the advantages of the exact feedback linearization method and the PI-Observer and the application of the approach to mechanical systems.

The paper is organized as follows: in the second chapter, the considered group of nonlinear systems with disturbances/uncertainties and the robustness problem are stated. In the third chapter, the PI-Observer design is briefly introduced. Then, the proposed robust control approach is detailed in the fourth chapter. The instruction of its application on mechanical systems is given in the fifth chapter with an example. The last chapter concludes this paper.

2 Problem Statement

The nominal model of the considered nonlinear systems can be described by

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

$$y = h(x), \tag{2}$$

where $x \in R^n$ denotes the state vector, $u \in R^r$ the input vector, $y \in R^m$ the output to be controlled. It is assumed that the vector fields $f(\cdot)$ on R^n, $g(\cdot)$ on $R^{n\times r}$, and $h(\cdot)$ on R^m are smooth. The nominal model of the system is assumed as input-output linearizable and the remaining zero dynamics is assumed to be stable. Note that to realize the input-output linearization in MIMO case the number of the inputs should equal to the number of the outputs, namely $m = r$.

If the modeling errors, disturbances, and other unknown effects acting on the nonlinear system are considered as external unknown inputs of the system (1) denoted by $Nn(x,t)$, an 'exact' system model can be set up by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + Nn(x,t), && (3)\\ y &= h(x), && (4) \end{aligned}$$

where the matrix $N = \begin{bmatrix} N_{col_1} & \cdots & N_{col_p} \end{bmatrix} = \begin{bmatrix} N_{row_1} \\ \vdots \\ N_{row_n} \end{bmatrix} \in R^{n\times p}$ that locates the external unknown inputs $n(x,t) \in R^p$ is assumed as known. The dimension of the unknown inputs is assumed as p, $0 < p \leq n$. For unknown matrix N some strategies are already proposed [8] and will not be detailed in this paper.

With the classical input-optput linearization method [9], the model (3) can be transformed into the following form

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} + \tilde{N}(x)n(x(t),t), \tag{5}$$

by choosing the inputs as

$$u = -E^{-1}(x) \begin{bmatrix} L_f^{r_1} h_1(x) \\ \vdots \\ L_f^{r_m} h_m(x) \end{bmatrix} + E^{-1}(x) \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}, \tag{6}$$

where

$$E(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}, \tag{7}$$

$$\tilde{N}(x) = \begin{bmatrix} L_{N_{col_1}} L_f^{r_1-1} h_1(x) & \cdots & L_{N_{col_p}} L_f^{r_1-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{N_{col_1}} L_f^{r_m-1} h_m(x) & \cdots & L_{N_{col_p}} L_f^{r_m-1} h_m(x) \end{bmatrix}. \tag{8}$$

From Eq.(5), it can be seen clearly that with the classical input-output linearization the transformed system equation is sensitive to the modeling errors, which is the key problem of the classical nonlinear method. For real applications of this control approach it has to be assumed that i) the states are measurable and ii) the model has to be known so that the transformation can be defined as known.

3 PI-Observer Design

In this chapter, the PI-Observer design developed in [10] is introduced briefly.

For systems described by

$$\dot{z} = Az + Bu + Nn(z,t),\ y = Cz, \tag{9}$$

with the state vector z of order n, the input vector u of order l, the measurement vector y of order m, and the time variant and unknown external input vector $n(t)$ of order r. The states z and the unknown inputs $n(t)$ can be estimated by a PI-Observer design

$$\begin{aligned} \begin{bmatrix} \dot{\hat{z}} \\ \dot{\hat{n}} \end{bmatrix} &= \underbrace{\begin{bmatrix} A & N \\ 0 & 0 \end{bmatrix}}_{A_e} \begin{bmatrix} \hat{z} \\ \hat{n} \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u + \underbrace{\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}}_{L} (y - \hat{y}), \\ \hat{y} &= \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} \hat{z} \\ \hat{n} \end{bmatrix}, \end{aligned} \tag{10}$$

if the extended system (A_e, C_e) is fully observable. This includes the condition

$$rank \begin{bmatrix} \lambda I_n - A & -N \\ 0 & \lambda I_r \\ C & 0 \end{bmatrix} = n + r$$

for all λ of $det[\lambda I - A] = 0$. This condition includes that the dimension of the unknown input vector $n(t)$ has to be less than or equal to the number of independent measurements, namely $r \leq m$ (proofs refer to [10, 11]).

Based on Eq. (15), the estimation errors as $e = \hat{z} - z$ and $f_e = \hat{n} - n$, the error dynamics of the extended system are

$$\begin{bmatrix} \dot{e} \\ \dot{f}_e \end{bmatrix} = \underbrace{\begin{bmatrix} A - L_1 C & N \\ -L_2 C & 0 \end{bmatrix}}_{A_{e,obs}} \begin{bmatrix} e \\ f_e \end{bmatrix} - \begin{bmatrix} 0 \\ \dot{n} \end{bmatrix}. \quad (11)$$

With suitable observer gain matrix L, the PI-Observer can estimate at the same time both the system states and the unknown external inputs.

4 Proposed Approach

From Eq.(5), m decoupled motions can be found and a uniform equation (12) can be used to describe the motions,

$$y_i^{(r_i)} = v_i + \sum_{j=1}^{p} L_{N_{col_j}} L_f^{(r_i-1)} h_i(x) n_j(x,t), \quad i = 1, \cdots, m, \quad (12)$$

which can be written in a state space form

$$\dot{x} = Ax + bu + \bar{N}\bar{n}, \quad (13)$$
$$y = cx, \quad (14)$$

with state vector $x = \begin{bmatrix} y_i \\ \dot{y}_i \\ \vdots \\ y_i^{(r_i)} \end{bmatrix}$, system matrix $A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{r_i \times r_i}$, input matrix $b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_i \times 1}$, matrix $\bar{N} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_i \times 1}$, output matrix $c = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1 \times r_i}$, input $u = v_i$, disturbance $\bar{n} = \sum_{j=1}^{p} L_{N_{col_j}} L_f^{(r_i-1)} h_i(x) n_j(x,t)$. In this case, it is obvious that the system is fully controllable and fully observable according to (A, b) and (A, c).

Based on the information in the last chapter, the decoupled motions has exactly an appropriate structure for a PI-Observer design. Therefore, m PI-Observers can be constructed for the m motions separately, for example

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\bar{n}}} \end{bmatrix} = \begin{bmatrix} A & \bar{N} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\bar{n}} \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y - \hat{y}),$$
$$\hat{y} = \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\bar{n}} \end{bmatrix}. \quad (15)$$

With properly chosen observer gain matrices L_1 and L_2 [10], which have the corresponding dimension $r_i \times 1$ and 1×1 and fulfill the conditions mentioned in chapter 3, the transformed states and the transformed disturbances can be estimated with the PI-Observer.

As a robust control, the state feedback control $u = K\hat{x} - \bar{N}\hat{\bar{n}}$ can be taken to stabilize the transformed system dynamics, because the estimations $\hat{x}$ and $\hat{\bar{n}}$ are available from the PI-Observer and the transformed system is fully controllable. With the help of Eq.(6) the inputs to the original system can be calculated. At the same time from the m PI-Observers m external inputs/disturbances $\bar{n}$ can be estimated in the transformed coordination and maximum m independent disturbances in the original coordinates can be therefore regenerated/calculated. Of course all the states and outputs in the original coordinates should be available to realize the input-output linearization as usual and this will be explained specifically for mechanical systems.

The whole control loop will be stable, while the transformed system dynamics is stabilized and the remaining zero dynamics of the input-output linearization is assumed stable.

5 Application on mechanical systems

Without loss of generality nonlinear MIMO mechanical systems can be described by n second order differential equations

$$\begin{bmatrix} \ddot{x}_1 \\ \vdots \\ \ddot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n) \\ \vdots \\ f_n(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n) \end{bmatrix} + \begin{bmatrix} g_{11}(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n) & \cdots & g_{1r}(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n) \\ \vdots & \ddots & \vdots \\ g_{n1}(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n) & \cdots & g_{nr}(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n) \end{bmatrix} u + \begin{bmatrix} d_1(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n,t) \\ \vdots \\ d_n(x_1,\dot{x}_1,\cdots,x_n,\dot{x}_n,t) \end{bmatrix}, \quad (16)$$

$$y_{meas} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad (17)$$

$$y_{contr} = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}. \quad (18)$$

Note that $2n$ states are required to describe the system dynamics from the n second order differential equations. Usually the $2n$ states are chosen as n displacements and the related n velocities. Here the n displacements are assumed as measurements y_{meas} and the number of the inputs u and the number of the outputs to be controlled y_{contr} are the same. Some of the displacements are assumed to be controlled. The system (16) can be written by $2n$ first order differential equations in a general form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + Nd(x,t), \\ y_{contr} &= h(x) = Cx, \end{aligned} \quad (19)$$

with $2n$ states, if the state vector is taken as $x = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ \vdots \\ x_n \\ \dot{x}_n \end{bmatrix}$ and corresponding $f(x)$, $g(x)$, N, $d(x,t)$, and C are considered.

If the system described in (19) is input-output linearizable and the number of independent disturbances in $d(x,t)$ is less than m, a robust control design with the proposed method can be applied.

For the nonlinear feedback in the input-output linearization process in (6)-(8), usually all the system states, the outputs, and the derivatives of the outputs in the original coordinated are required. In the application on mechanical systems it can be seen that the outputs to be controlled are usually the displacements which are assumed as measurements and the derivatives of the outputs can be obtained directly from the PI-Observers. The velocities, which are also states in the original coordinates, can be estimated by PI-Observers as the states in the transformed coordinates when the corresponding displacements are to be controlled. Otherwise additional PI-Observers can be designed to estimate the velocity based on the measured displacement. This will be shown with the example in next chapter.

Another important point is that the estimation of modeling errors/disturbances in the original coordinates is available based on the estimations of the PI-Observers.

6 Application Example

An example of nonlinear MIMO mechanical systems, shown in Fig. 1 [12], is given to illustrate the proposed method. The system can be modeled by

$$\begin{aligned} m\ddot{x}_1 &= k(-2x_1+x_2) + k_p[-x_1^3 + (x_2-x_1)^3] + u_1 + d_1, \quad (20) \\ m\ddot{x}_2 &= k(x_1-2x_2+x_3) + k_p[(x_3-x_2)^3 - (x_2-x_1)^3] + u_2 + d_2, \\ m\ddot{x}_3 &= k(x_2-x_3) + k_p(x_2-x_3)^3 + d_3, \\ y_{meas} &= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \\ y_{contr} &= \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = \begin{bmatrix} x_1 & x_3 \end{bmatrix}^T, \end{aligned}$$

where the dynamics of the disturbances d_1, d_2, d_3 are unknown to the control design and taken in the simulation as $d_1 = 0$, $d_2 = 100sin(10t) + 500$, and $d_3 = 0.2d_2$.

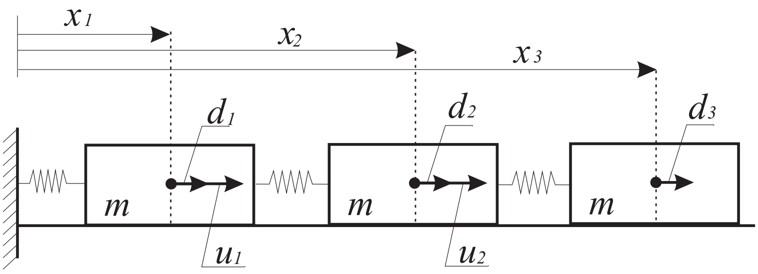

Figure 1: Nonlinear MIMO mechanical system example

The input-output linearized form of the system can be written in

$$\ddot{y}_1 = v_1 + \frac{d_1}{m} = v_1 + \bar{n}_1, \tag{21}$$

$$y_2{}^{(4)} = v_2 + [\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2](\frac{d_2}{m} - \frac{d_3}{m}) = v_2 + \bar{n}_2, \tag{22}$$

if the inputs are chosen as

$$u_1 = m\left[v_1 - \frac{k}{m}(-2x_1 + x_2) - \frac{k_p}{m}\left[-x_1^3 + (x_2 - x_1)^3\right]\right], \tag{23}$$

$$\begin{aligned} u_2 = & \frac{m}{\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2} \\ & \{v_2 - \left[\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2\right]\left[\frac{k}{m}(x_1 - 3x_2 + 2x_3) + \frac{k_p}{m}\left[2(x_3 - x_2)^3 - (x_2 - x_1)^3\right]\right] \\ & -6\frac{k_p}{m}(x_2 - x_3)(\dot{x}_2 - \dot{x}_3)^2\}. \end{aligned} \tag{24}$$

The remaining zero dynamics $\ddot{x}_2 = \frac{k}{m}\left[(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2 + d_2\right]$ is stable, if the disturbance d_2 is bounded.

Two PI-Observers can be designed for the transformed decoupled dynamics (21) and (22)

$$\dot{z}_a = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{\bar{n}}_1 + L_{1a}(y_1 - \hat{y}_1), \tag{25}$$

$$\dot{\hat{\bar{n}}}_1 = L_{2a}(y_1 - \hat{y}_1), \tag{26}$$

$$\hat{y}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} z_a, \tag{27}$$

and

$$\dot{z}_b = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} z_b + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \hat{\bar{n}}_2 + L_{1b}(y_2 - \hat{y}_2), \tag{28}$$

$$\dot{\hat{\bar{n}}}_2 = L_{2b}(y_2 - \hat{y}_2), \tag{29}$$

$$\hat{y}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} z_b, \tag{30}$$

with the state vectors $z_a = \begin{bmatrix} \hat{y}_1 \\ \dot{\hat{y}}_1 \end{bmatrix}$ and $z_b = \begin{bmatrix} \hat{y}_2 \\ \dot{\hat{y}}_2 \\ \ddot{\hat{y}}_2 \\ \hat{y}_2^{(3)} \end{bmatrix}$, to estimate the transformed states and disturbances, namely $\hat{x}_1$, $\dot{\hat{x}}_1$, $\hat{\bar{n}}_1$, $\hat{x}_3$, $\dot{\hat{x}}_3$, $\ddot{\hat{x}}_3$, $\hat{x}_3^{(3)}$, and $\hat{\bar{n}}_2$. To construct the inputs in (23) and (24), besides the displacements x_1, x_2, and

x_3 the velocities $\dot{x}_2$ and $\dot{x}_3$ are also required. As a transformed coordinate, the velocity $\dot{x}_3$ can be estimated by the PI-Observer (28)-(30). To estimate the velocity $\dot{x}_2$, another PI-Observer can be designed by

$$\dot{z}_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_3 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{\bar{n}}_3 + L_{1c}(x_2 - \hat{x}_2), \quad (31)$$

$$\dot{\hat{\bar{n}}}_3 = L_{2c}(x_2 - \hat{x}_2), \quad (32)$$

$$(33)$$

with state vector $z_c = \begin{bmatrix} \hat{x}_2 \\ \hat{\dot{x}}_2 \end{bmatrix}$, $v_3 = \frac{k}{m}\left[(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2\right]$, and $\bar{n}_3 = \frac{d_2}{m}$.

With the estimations from the three PI-Observers mentioned above, the system (16) can be transformed into an input-output linearized form with nonlinear feedback (23) and (24). To realize the robust control, linear control method can be applied to the linearized model (21) and (22), for example with linear state feedback control

$$v_1 = -20\hat{\dot{x}}_1 - 100(x_1 - x_{1ref}) - \hat{\bar{n}}_1, \quad (34)$$

$$v_2 = -200\hat{x}_3^{(3)} - 15000\hat{\ddot{x}}_3 - 500000\hat{\dot{x}}_3 - 6250000(x_3 - x_{3ref}) - \hat{\bar{n}}_2. \quad (35)$$

The desired values taken in the simulation are $x_{1ref} = 0.25$ and $x_{3ref} = 0.3$. The dynamics of the disturbances d_1, d_2, and d_3 are calculated from the estimations $\hat{\bar{n}}_1$, $\hat{\bar{n}}_2$, and $\hat{\bar{n}}_3$.

The simulation results in comparison with classical input-output linearization and Luenberger observer design are given in Fig. 2.

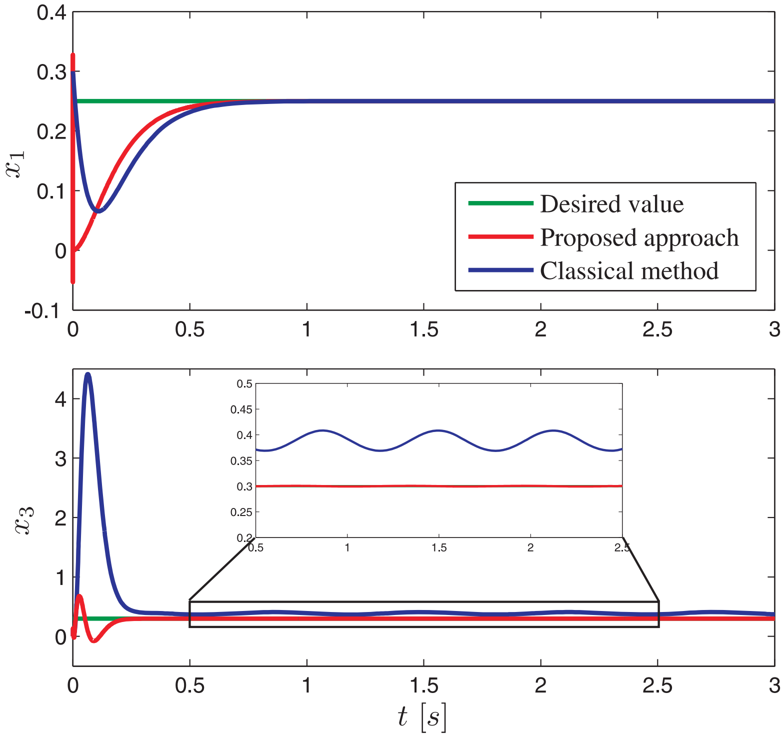

Figure 2: Comparison of the control results

According to the results, the proposed approach shows firstly robustness against external disturbances, which can also be understood as modeling errors and parameter uncertainties. In comparison, the classical input-output linearization method with estimated states from Luenberger observer and the same state feedback control as in the proposed approach is strongly influenced by the disturbances and leads to large control error, especially in controlling x_3.

Secondly, the proposed approach estimates the velocities based on the measurements of the displacements and therefore avoids numerical differentiation of the measured signals to get the information of all the states.

At last, the proposed approach not only realizes a robust control for the considered class of nonlinear MIMO systems, but also generates an estimation of the unknown disturbances/modeling error is a breakthrough in both application and nonlinear robust control. The estimated disturbance from the example is shown in Fig. 3.

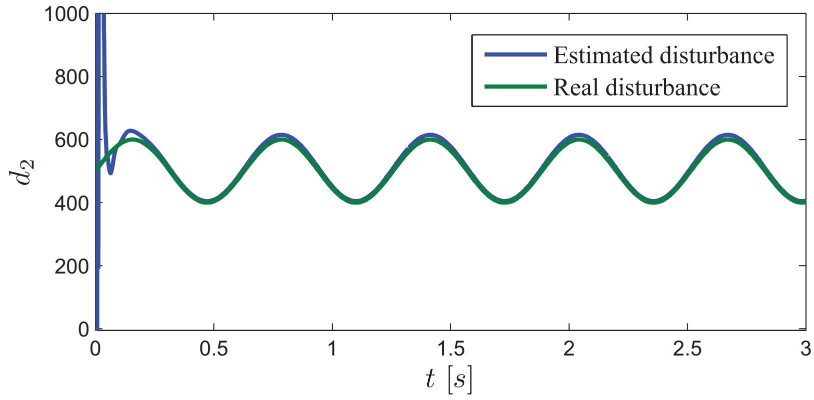

Figure 3: Estimation of the disturbance d_2

7 Conclusions

In this contribution, a robust control design approach is proposed for input-output linearizable nonlinear MIMO systems with modeling errors and therefore extends the previously developed approach by the authors for SISO systems. The presented approach solves the robustness problem and the requirements of all the states information in the classical nonlinear control method (the input-output linearization method) and provides the estimation of the disturbances/modeling errors at the same time. The application of the approach on mechanical systems is detailed with an example. The results illustrate clearly the effects and advantages of the proposed robust control method with the use of PI-Observer technique.

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