ZONAL MODEL OF CLIMATIC CONDITIONS IN A LECTURE ROOM

I. Bratuž, M. Anatasijević Kunc
Faculty of Electrical Engineering, Tržaška 25, 1000 Ljubljana, Slovenia

Corresponding Author M. Atanasijević-Kunc, University of Ljubljana, Faculty of Electrical Engineering
Tržaška 25, 1000 Ljubljana, Slovenia; maja.atanasijevic@fe.uni-lj.si,
Tel: +386 (0)1 4768-314, Fax: +386 (0)1 4264-631

Abstract. The aim of this study is to develop a model of climatic conditions in a lecture room. Various types of mathematical models can be applied to the description of climatic conditions in order to obtain the desired model accuracy. The type of approach applied is usually a compromise between accuracy and availability of data of a realistic example and the purpose of model usage. One of these approaches is zonal modelling as a link between nodal and CFD (*Computer Fluid Dynamics*) modelling. In the paper, a zonal model is presented and evaluated in order to describe the important parameters that represent indoor air quality. The paper will show that, by using zonal modelling and with a low number of zones employed, a good approximation of a realistic situation may be achieved.
The model was developed with a view to evaluating the regulation of three parameters which indicate the indoor air quality, i.e. the temperature, CO_2 concentration and relative humidity. Due to the correlation between these measured parameters, we included in the model all the necessary physical interactions in order to maintain nonlinearity of the system and coupling between the parameters.

1 INTRODUCTION

The developed model is based on zonal modelling. Zonal modelling may be represented as an example of a CFD model with rather coarse grid. This type of modelling offers the possibility of describing air conditions in a room with desired accuracy. It also gives a good insight into the relations between modelled parameters.

The zonal modelling approach starts with dividing a room into sub-zones where each sub-zone represents its own control volume, where the distribution of heat and air pollutants is assumed to be uniform. Each sub-zone is described with its own set of heat and mass balance equations. By dividing the room in to smaller sub-zones and by determining their size and shape, we can also predict the paths of air movement in the room.

The model was developed for a room equipped with a HVAC (*Heating, Ventilation and Air Conditioning*) system. This system has the ability to heat or cool the room by controlling the air temperature at the inlet. The room was also provided with a sensor for carbon dioxide concentration. In addition to measuring the fresh air temperature, we were also able to measure the air temperature at the air outlet as well as the external temperature. Because of a large window that represented a significant degree of solar heat as compared to the heat within the room, we also measured the illumination of the room. This enabled us to determine the solar heat emitted through the window.

Notwithstanding that we were only able to measure the temperature and carbon dioxide concentration in the room, we also decided to model the presence of water vapour in the air, the main source of water vapour being the presence of a human person and the flow of fresh air coming from outside. In this way, we will be able to detect the air vapour saturation and condensation in the room leading to the previously mentioned health problems. Moreover, the air vapour model will also improve the heat model by adding the water heat capacity to heat balance equations.

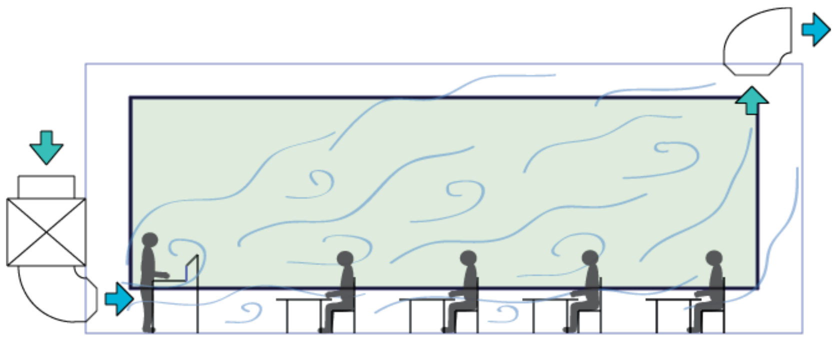

Figure 1 Modelled lecture room

In this paper, we will demonstrate that rather a coarse grid of zones may suffice for evaluating a HVAC system regulator. A good approximation of climatic conditions in the room was achieved by dividing the room into eight sub-zones. Additionally, the room model includes a model of a building envelope. Convection is calculated for the natural and forced ventilation differently, where the speed of the air inside and outside the building is included in the calculation. Furthermore, we have included in the model dynamics the presence of persons and appliances in the room, the solar heat and heat transfer as a term of thermal radiation.

Human person presence was included in modelling as a heat, pollutant and water vapour source. Due to the fact that the zonal model divides the room in to smaller volumes, we were also able to include the information about person location inside the room into the model.

For modelling purposes, we created the Matlab/Simulink toolbox. The toolbox was designed so that the functionality blocks provided the ability of modular building [2, 3, 11, 15].

2 ROOM MODEL

Figure 2 shows the layout of the lecture room that was divided into eight sub-zones. A horizontal line that divides room in two parts limits the heat and pollutant dynamics because of the presence of human person in the lower part of the room. The height of the lower zone is 2m. This zone also delimits the air jet coming from the air diffuser. We expect that air jet will not fully develop before entering into the second half of the room [8, 14].

The partitioning of the room by additional three vertical divisions offers the possibility to determine the number of persons present in each part of the room. These divisions also give us the possibility to adjust the air jet dynamics and its development throughout the room. The air jet development is presented as a part of the air jet entering the upper portion of the room (Figure 3).

For each sub-zone in which the air jet is divided, the mass and energy conservation equations are solved and it is also assumed that it has a uniform air mass and heat distribution.

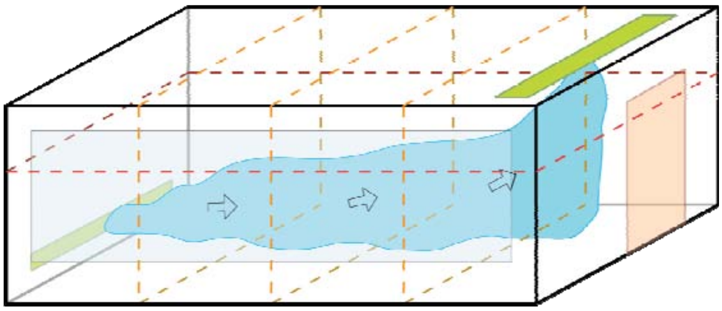

Figure 2 Zonal partitioning of the lecture room

Each sub-zone has its own set of equations describing the mass flow through the volume. As it has already been mentioned, we are going to simulate the air, water vapour and carbon dioxide flows. For this purpose, the model must contain a balance equation for each observed parameter [9, 10, 12]. To this end, the air mass has to be divided into three elements the air is composed of. These are dry air, water vapour and carbon dioxide. Due to the fact that the carbon dioxide concentration is very small compared to the mass of the dry air (m_{da}) and water vapour (m_{wv}), the carbon dioxide concentration may be excluded from the mass and heat flow equations and is calculated separately.

$$\sum_i \dot{m}_{da} = 0 \tag{2.1}$$

$$V \frac{d\rho_{wv}}{dt} = \sum_i \dot{m}_{wv,i} + \dot{m}_{wv,source} \tag{2.2}$$

$$V \frac{d\rho_{CO_2}}{dt} = \sum_i \dot{m}_{CO_2,i} + \dot{m}_{CO_2,source} \tag{2.3}$$

The heat balance equation includes heat from humans, building envelope transfer characteristics, heat flow caused by the mass flow trough sub-zones, heat from the sun and appliances in the lecture room and long wave radiation.

$$\rho_a \cdot V \cdot c_{p,a} \cdot \dot{T}_a = P_{w,c} + P_{da,m} + P_{wv,m} + P_h + P_{apl} \tag{2.4}$$

where

$$P_{w,c} = \sum_i h_{conv,i} \cdot A \cdot (T_{ws,i} - T_{a,z}) \tag{2.5}$$

$$P_{da,m} = \sum_i \dot{m}_{da,i} \cdot c_{p,da} \cdot (T_{a,i} - T_{a,z}) \tag{2.6}$$

$$P_{wv,m} = \sum_i \dot{m}_{wv,i} \cdot c_{p,wv} \cdot (T_{a,i} - T_{a,z}) \tag{2.7}$$

$$c_{p,a} = c_{p,da} \cdot \frac{x_{da} \cdot M_{da}}{M_a} + c_{p,wv} \cdot \frac{x_{wv} \cdot M_{wv}}{M_a} \tag{2.8}$$

$$\rho_a = \rho_{da} + \rho_{wv} \tag{2.9}$$

Relative humidity and carbon dioxide were calculated from equations from (2.10) to (2.13).

$$p_z = \left(\rho_{da} \cdot \mathcal{R}_{da} + \rho_{wv} \cdot \mathcal{R}_{wv}\right) \cdot T_{a,z} \tag{2.10}$$

$$RH = \frac{\rho_{wv} \cdot \mathcal{R}_{wv} \cdot T_{a,z}}{P_{wv,sat}} \tag{2.11}$$

$$x_{CO_2,z} = \frac{m_{CO_2,z}}{m_{ha,z}} \tag{2.12}$$

$$m_{ha,z} = \left(\rho_{da,z} + \rho_{ha,z}\right) \cdot V \tag{2.13}$$

The lecture room is surrounded by two external and two internal walls. The external temperature of the building was measured. The temperatures in the surrounding rooms were estimated with regard to acclimatization settings (the temperature variation in these rooms was neglected).

External walls are 0.45 m thick and are composed of 0.15m of insulation and 0.30m of concrete. On the larger external side of the lecture room there is a window that represents a major part of the area of that side. The ceiling and the floor are made of concrete 0.20m thick concrete and face the upper and lower spaces of the building with a temperature estimated to equal that in the laboratory (Figure 4).

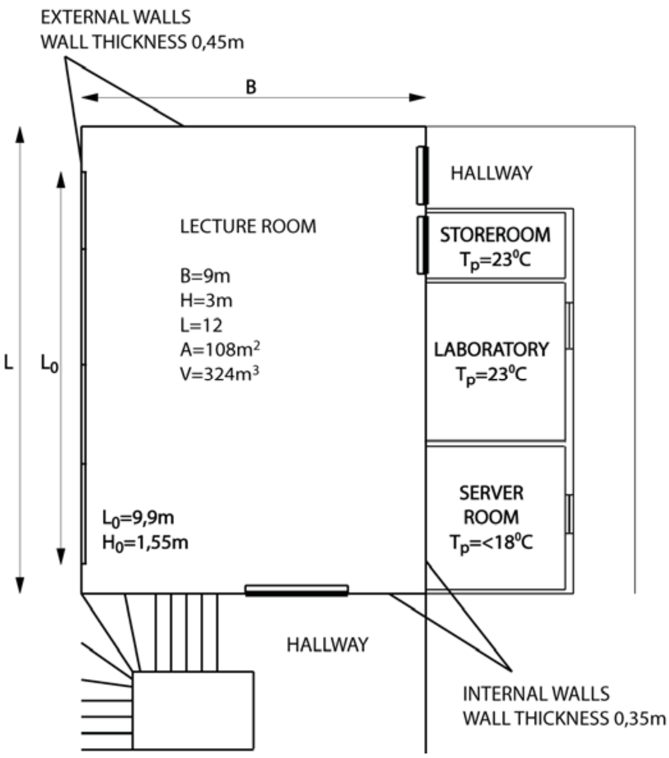

Figure 3 Lecture room layout [6]

For each building envelope surface a heat transfer balance equation (2.14) is calculated as a sum of convection heat transfer, long wave radiation, heat flow through the wall and, on some surfaces, also the sun heat [7].

$$\sum_{l} \frac{(T_{w,l} - T_{w,i})}{R_{l-i,rad}} + \frac{(T_{s,i} - T_{a,z})}{R_{i,conv}} + q_{sun} + q_l = 0 \tag{2.14}$$

$$R_{l-i,rad} = \frac{1}{\varepsilon_l \cdot A_i \cdot G_{l-i} \cdot 4 \cdot \sigma \cdot \overline{T}^3} \tag{2.15}$$

$$R_{i,conv} = \frac{1}{h_{conv,i} \cdot A_i} \tag{2.16}$$

The heat transfer through the building envelope from the surroundings to the internal air consists of heat flow in terms of conduction through the wall and convection on both sides of the wall. In some situations, convection or conduction can be neglected due to the ratio between convection and conduction. This can be determined by calculating the Biot's number (Bi). If the Biot's number is large, the convection factor in heat flow may be neglected.

$$Bi = \frac{h \cdot L}{k} \tag{2.17}$$

The heat convection flow in case of forced ventilation must be calculated differently than in the case of natural convection. Natural convection is expected on those walls where the air jet has no influence on the air mass movement. Forced convection depends on the Reynolds number (Re) which characterizes the relative influences of inertial and viscous forces in a fluid problem. The Reynolds number is directly related to the effect of ar jet turbulence near the surface.

$$\mathrm{Re}_x = \frac{\rho \cdot u_\infty \cdot x}{\mu} \tag{2.18}$$

$$\overline{Nu_L} = \frac{\overline{h} \cdot L}{k_f} = 0.664 \cdot \mathrm{Re}_L^{\frac{1}{2}} \cdot \mathrm{Pr}^{\frac{1}{3}} \tag{2.19}$$

In the equation (2.19), Pr represents a dimensionless number that approximates the ratio between dynamic viscosity thermal diffusivity, Nu represents the Nusselt's number, h and k are the heat transfer coefficient and thermal conductivity, and L is the characteristic dimension of the surface. Pr is determined by the gas structure. Since the Earth's atmosphere mostly consists of diatomic gases such as N_2 and O_2, the Prandtl's (Pr) number can be set as $\mathrm{Pr} = 5/7$.

For natural convection, the forces that drive the air mass upwards or downwards are present because of buoyancy. When the air mass moves towards the floor, the buoyancy is negative and positive when the air moves upwards. In real life, the situation in case of natural convection is more complicated than in case of forced convection. In case of forced convection, the boundary layer, in which the air is influenced by viscous forces of the surface, manly consists of the air moving in a single direction. In case of natural convection, the boundary layer consists of condensation film and moving air. Moreover, the movement directions of condensed water film and plume are not necessarily the same.

$$Gr_L = \frac{g \cdot \beta \cdot \Delta T \cdot L^3}{\nu^2} \tag{2.20}$$

$$Ra_L = Gr_L \cdot \mathrm{Pr} \tag{2.21}$$

$$\overline{Nu_L} = 0.678 \cdot Ra_L^{\frac{1}{4}} \cdot \left(\frac{\mathrm{Pr}}{0.952 + \mathrm{Pr}} \right)^{\frac{1}{4}} \tag{2.22}$$

In spite of the complexity of the natural convection, the heat transfer calculation is simplified and the condensation film is neglected. Despite the fact that equations between the natural and forced convection are different, an analogy among some numbers may be found. Thus, the significance of Prandtl's number for forced convection is the same as the significance of Grashof's number (Gr) for natural convection. The product between numbers Gr and Pr represents Rayleigh's number (Ra).

The transient building envelope models are three. The first one is the model for insulated wall, the second one is the wall without insulation, and the third one is the window model. A three-point model is used for a wall, although a five-point model would be more appropriate [5], but this leads to a higher degree model. This means that the wall temperature is calculated at three points: at both surfaces and in the middle of the wall (in case of insulated wall, the temperature inside the wall is calculated at the juncture between insulation and concrete).

For the window, a two-point transient model is used. The window model is simplified in such a way that the heat conduction of the glass is neglected as it is the heat capacity of the gas between two glasses [4].

$$c_{wg,\text{int}} \cdot \frac{dT_{s,\text{int}}}{dt} = q_{s,\text{int}} + h_{conv,\text{int}} \cdot (T_{a,z} - T_{s,\text{int}}) + \frac{1}{r_g} \cdot \left(T_{s,\text{int}} - T_{s,ext}\right) \tag{2.23}$$

$$c_{wg,ext} \cdot \frac{dT_{s,ext}}{dt} = q_{s,ext} + h_{conv,ext} \cdot (T_{a,ext} - T_{s,\text{int}}) + \frac{1}{r_g} \cdot \left(T_{a,ext} - T_{s,ext}\right) \tag{2.24}$$

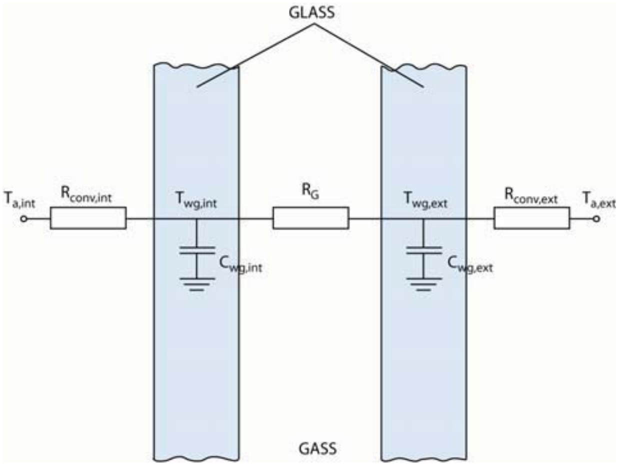

Figure 4 Two-point window transient model

Although an optimum model for the building envelope would be a five-point transient model [5], the decision to choose a three-point transient model was based on the fact that the three-point model results in a lower degree model with sufficient accuracy.

3 Simulation results

The temperature response of the lecture room was based on the experiment that started by heating the room and continued by its cooling. Since the system is multivariable, we added an additional elicitation to the system such as the presence of persons in the room. The presence of persons in the room varied during the experiment.

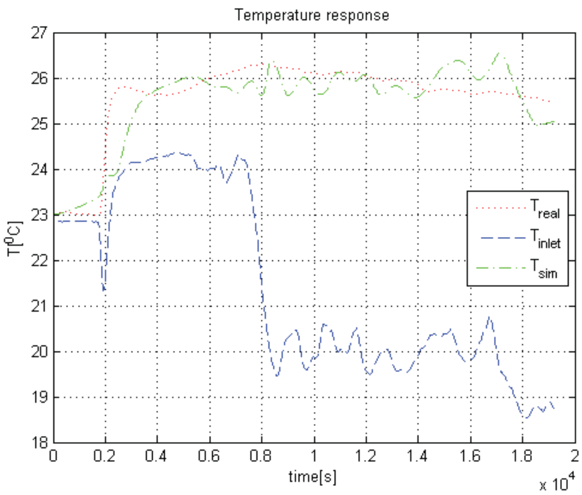

Figure 5 Real and simulated temperature response (T_{real} - real temperature response; T_{sim} - simulated temperature response; T_{inlet} - air temperature measured at the air inlet)

By comparing the results of the real and simulated temperature response (Figure 5), the difference of the both temperature responses may be noticed. The first important difference may be observed at the beginning of the both responses. The simulated temperature response shows the dynamics as early as at the beginning in spite of the fact that there is no air ventilation. This can be explained on the grounds that the simulated temperature response is actually the temperature of the last sub-zone which is coupled to the rest of the system over the radiation and conduction heat. On the contrary, the real response temperature is measured at the air outlet which may be regarded to be thermally insulated from the rest of the room when there is no air flow. At the outset of room ventilation (Figure 8), the measured real system response temperature drops, although the air temperature at the inlet is actually higher. This may be regarded as a HVAC response time.

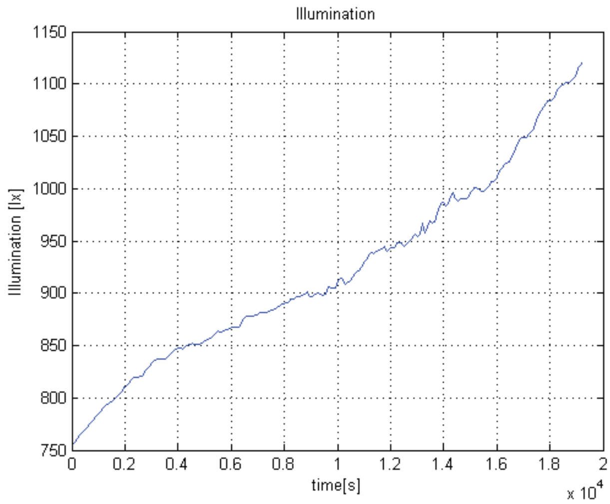

Figure 6 Illumination of the lecture room during the experiment

During the experiment, the number of persons present rise from zero to eighteen (Figure 8), and the illumination of the room increased likewise (Figure 6). At the same time as the persons entered the room, the temperature at the air inlet was decreased. Although the temperature at the air inlet was lower, the temperature at the air outlet remained almost unchanged. It may be observed that, in this region, the modelled system follows the real system. This means that the summation of the energies in the model is correct.

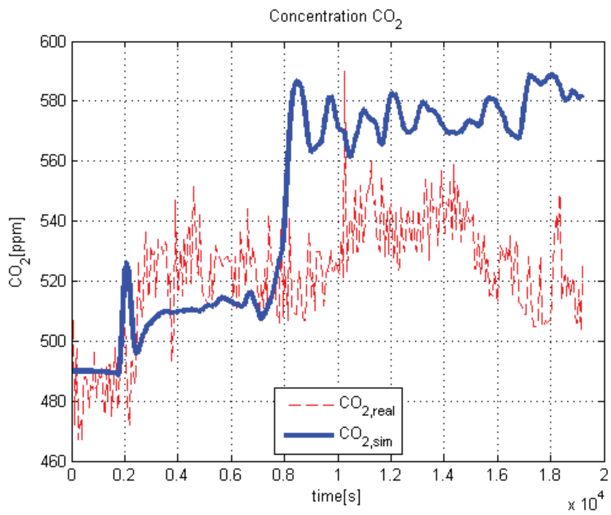

Figure 7 CO_2 concentration ($CO_{2,real}$ - measured concentration of the carbon dioxide; $CO_{2,real}$ - simulated concentration of the carbon dioxide)

If we compare the measured and the simulated results of CO_2 concentration, the initial matching of both responses may be noticed. As the persons entered the room, the difference between both responses significantly increased. This difference appear as a result of various facts: The first one is the air flow through the room, which is at that time at its maximum. This may cause the elimination of carbon dioxide from the air before it mixes with the air and reaches the sensor. Additional difference of concentration responses can be explained due to the fact that, in this model, we expect a complete mixing of the air, which in reality is not true. The last possible explanation of the difference is the actual position of the persons inside the zone. If the persons are far from the sensor, the measurement of the concentration within the zone is poorly conditioned.

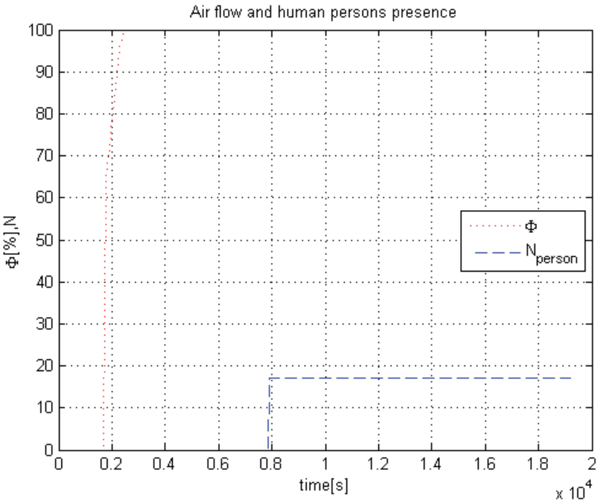

Figure 8 Air flow (Φ) and presence of the persons (N_{person})

4 Conclusion

By comparing the real and simulated responses it may be established that the model shows a similar temperature response as the real system. This means that the model has an appropriate energy summation that may be observed from the correct temperature response in spite of the fact that the heat source has changed (solar heat and the number of persons in the room).

The CO_2 responses are due to a number of factors involved that are difficult to compare. Better results in CO_2 would require additional fragmentation of the room, resulting in a coarser grid of zones. Smaller zones would enable a better local determination of carbon dioxide concentration.

The future development of the model should follow the implementation of the air jet model in the room model. This would improve the air speed determination at the walls, which is related to the forced convection and the air. This air model would also improve the CO_2 concentration response and provide a more accurate determination of air flow through the room. As it has already been mentioned, the future model should have a room fragmentation, although the justification of such a coarser grid should be studied.

5 References

[1] Maroni M., Seifer B., Lindval T.: *Indoor Air Quality, A comprehensive reference book, Vol. 3.* ELSEVIER, 1995

[2] Rasmus L. J., Grau K., and Heiselberg P. K.: *Integration of a multizone airflow model into a thermal simulation.* Proceedings: Building Simulation 2007

[3] Lorenzetti D. M.: *Assessing multizone airflow software.* Environmental Energy, Technologies Division, Indoor Environment Department, Lawrence Berkeley National Laboratory, Berkeley CA, USA, December 2001

[4] Furlan B.: *Izdelava simulacijskega modela toplotnega odziva stavbe s spremenljivimi lastnostmi ovoja in aplikacija mehkega vodenja, Magistrska naloga.* Fakulteta za elektrotehniko, Univerza v Ljubljani, 1999

[5] Seem J.E.: *Modeling of heat in buildings, Ph.D thesis.* University of Wisconsin, Madison, USA, 1987

[6] Lampe T.: *Vodenje klimatske naprave, Diplomsko delo visokošolskega strokovnega študija.* Fakulteta za elektrotehniko, Univerza v Ljubljani, 2007

[7] Incropera F. P.: *Fundamentals of heat and mass transfer, 6th edition.* Wiley, 2006

[8] Riederer P.: *Thermal room modelling adapted to the test of HVAC control systems, Ph.D thesis.* Ecole des Mines de Paris, 2002

[9] Zhang Y., Hanby V. I.: *Model-based control of renewable energy systems in buildings.* Ninth International IBPSA Conference, Montréal, Canada, August 15-18, 2005

[10] Ren Z.: *Enhanced modelling of indoor air flows, temperatures, pollutant emission and dispersion by nesting sub-zones within a multizone model.* The Queen's University of Belfast, Faculty of Engineering, School of Computer Science, September 2002

[11] El Khoury Z., Riederer P., Couillaud N., Simon J., Raguin M.: *A multizone building model for Matlab/Simulink enviroment.* Ninth International IBPSA Conference, Montreal, Canada, August 15 - 18, 2005

[12] Cordeiro Mendonça1 K., Wurtz E., Inard C.: *Predicting temperature and moisture distributions in conditioned spaces using the zonal approach.* Ninth International IBPSA Conference, Montréal, Canada, 2005

[13] TRNSYS 16 MANUAL
[14] Edae J. T., Fariborz H.: *A new generation of zonal models.* ASHRAE Transactions, Jul 2006
[15] Mendes N., Oliveira G. H. C., Araújo H. X.: *Building thermal performance analysis by using Matlab/Simulink.* Seventh International IBPSA Conference, Rio de Janeiro, Brazil, August 13-15, 2001