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ANISOTROPIC MODEL OF DAMAGE FOR GEOMATERIALS AND CONCRETE

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Abstract. A damage model for geomaterials and concrete is proposed. This model takes into account the anisotropic character induced by the degradation of material. The law of behavior obtained by differentiation of the free energy expresses the unilateral effect observed in traction-compression as well as the residual strains caused by the damage.

The approach suggested requires the identification of a reduced number of parameters having a clear physical significance. An application to the cases of the uniaxial traction-compression loading shows a good adequacy with the experimental observations.

1 Introduction

Geomaterials and concretes are regarded as isotropic and heterogeneous materials before any mechanical loading at a mesoscopic scale. The application of a mechanical loading allows onset of defects whose direction of propagation depends on the local stress field. Before any loading, these materials are characterized by a significant density of microscopic cracks that propagate in a direction normal to the tension but tend to close in the case of compression [1,2] with possibility of frictional slip on the lips of discontinuities.

The complete crack closure causes a recovering of the material rigidity. This phenomenon is called unilateral effect [3]. In addition one observes during a simple mechanical test that the cancellation of the loading leads to a more or less important irreversible strain. These effects are caused by frictions, which appear at the crack closure.

A mechanical modeling approach of these materials must describe the behaviors and associated phenomena described below. The damage mechanics constitutes a theoretical tool adapted to describe the complex mechanisms associated with damage and rupture under mechanical loading. Thus the formulation of the model must take account of the principal characteristics described which are summarized below:

(1) degradation of the properties of material by creation of defects in the structure
(2) an anisotropic behavior as a consequence of damage
(3) unilateral effect at the crack closure in the case of the application of compressive stresses.
(4) dissymmetry in behavior between tension and compression
(5) occurrence of irreversible deformation after total unloading.

Many research treats damage phenomenon in concretes and geomaterials. The production of several models is mainly imposed by the complexity and the variety of the behaviors observed. According to the type and the level of imposed loading, the response can evolve from a linear elastic behavior to a nonlinear behavior with development of a damage depending on the structure and mechanical properties as well as loading field. Certain approaches privilege the simplicity of the formulation by using a scalar parameter [4] to describe the density of the defects induced by the loading in place of a more realistic approach that would use tensorial formulation to describe a system of defects strongly influenced by the local stress fields. Some models take into account the unilateral effect (3) [1, 2, 5, 6, 7] and the dissymmetry between tension and compression (4). Irreversible deformation (5) is rarely included in the formulation of the behavior laws [1, 2, 3].

Badel *et al.* [1] make a thorough analysis of the various models and note many inaccuracies in the formulations suggested. In addition some models [2] are applicable only for low mechanical loading and cannot reproduce high damage values. Their implementation in a numerical code leads to some problems in convergence.

The present model takes accounts of the various characteristics of behavior quoted above. Our approach is based on the formulation of a potential of free energy as a scalar function of the strain and damage tensors. This choice avoids singularity in the expression of the stress tensor obtained by derivation of the potential of free energy [1]. State variables are reduced to two rank symmetric tensorial variables $\underline{\underline{\varepsilon}}$ (strain variable) and $\boldsymbol{D}$ (damage variable).

The model takes also account of dissymmetry between traction and compression and contains a term related to the residual effects.

In last part we propose an application to the case of a tensile and then of a compression tests. A comparison with experimental results will be carried out to test the acuity of the model.

2 - Formulation

2.1 Micromechanical description of the damage

We consider a volume of sufficiently large size compared to heterogeneities. The material is elastic linear and isotropic before loading. Let us note S_n the cross-section of elementary volume, and $\boldsymbol{n}$ its normal unit vector. Due to the existence of cracks surfaces and stress concentration, the net cross-sectional area of the element (effective resistant area) is S_n^{eff} instead of S_n $(S_n^{eff} \leq S_n)$. The surface density of discontinuities on the quoted cross-area is given by:

$$D_n = \frac{S_n - S_n^{eff}}{S_n} \qquad (1)$$

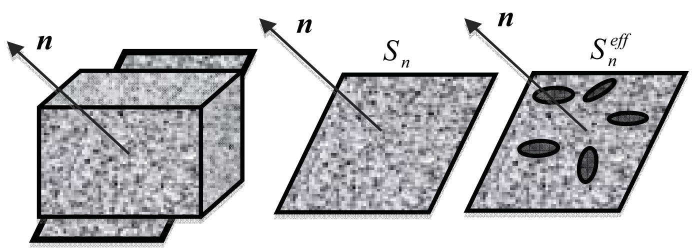

Figure 1. Representative volume element

Defects of random orientations and size are described by several systems of parallels microcracks. A two rank symmetric tensorial variable $\boldsymbol{D}$ is thus chosen indicating orientation of microcracks sets as well as generation and growth of decohesion surfaces:

$$\boldsymbol{D} = \sum_i D_{ni} \boldsymbol{n}_i \otimes \boldsymbol{n}_i \qquad (2)$$

where $\boldsymbol{n}_i$ describes the orientation of the *i-th* set of parallel crack-like defects.

As a symmetric two rank tensor, $\boldsymbol{D}$ can be represented in its principal axes and corresponding principal directions $\boldsymbol{n}_k$, $k = 1, 2, 3$ as follows:

$$\boldsymbol{D} = \sum_{k=1}^{3} D_k \boldsymbol{n}_k \otimes \boldsymbol{n}_k \qquad (3)$$

So, any system of cracks-sets is equivalent to three orthogonal sets of parallel cracks [2].

2.2 Thermodynamic free energy

The microstructural change in materials of inelastic strain and damage is induced by the development of microscopic cavities and other defects [8]. We postulate that the Helmotz free energy ψ is a function of strain $\boldsymbol{\varepsilon}$ and damage $\boldsymbol{D}$:

$$\psi = \psi(\boldsymbol{\varepsilon}, \boldsymbol{D}) \qquad (4)$$

The function $\psi(\boldsymbol{\varepsilon}, \boldsymbol{D})$ is restrained to depend linearly on $\boldsymbol{D}$, corresponding to the hypothesis of non-interacting cracks and is at most quadratic in $\underline{\underline{\varepsilon}}$ assuming thus linear elasticity for fixed $\boldsymbol{D}$. According to these assumptions, a general form of the function $\psi(\boldsymbol{\varepsilon}, \boldsymbol{D})$ is proposed as a Taylor series [3, 8, 9]:

$$\psi(\boldsymbol{\varepsilon},\boldsymbol{D})=\psi(0,\boldsymbol{D})+\frac{\partial\psi(0,\boldsymbol{D})}{\partial\boldsymbol{\varepsilon}}:\boldsymbol{\varepsilon}+\frac{\partial^2\psi(0,\boldsymbol{D})}{\partial\boldsymbol{\varepsilon}^2}:\boldsymbol{\varepsilon}:\boldsymbol{\varepsilon} \tag{5}$$

This expression can be written in the form:

$$\psi(\boldsymbol{\varepsilon},\boldsymbol{D})=\psi^0+\psi^r+\psi^e \tag{6}$$

The terms ψ^0, ψe, ψ^r respectively, represents the initial, residual and elastic recoverable free energy of the material. The first term ψ^0 has no influence in the law of state and appears only in the dissipation.

The material is considered isotropic in the absence of damage and the anisotropy appears when the mechanical loading generates oriented defects. Thus for undamaged material the elastic recoverable free energy can be expressed as follows:

$$\psi^e(\boldsymbol{\varepsilon},\boldsymbol{D}=\boldsymbol{0})=G_0\boldsymbol{\varepsilon}^D:\boldsymbol{\varepsilon}^D+\frac{3}{2}K_0\left[Tr\boldsymbol{\varepsilon}\right]^2 \tag{7}$$

where G_0 and K_0 are the shear and bulk modulus of the undamaged material related to Lame coefficients by the following relation:

$$G_0=\mu_0 \text{ and } K_0=\frac{3\lambda_0+2\mu_0}{3}$$

$\boldsymbol{\varepsilon}^D$ is the deviatoric strain tensor defined by:

$$\boldsymbol{\varepsilon}^D=\boldsymbol{\varepsilon}-\frac{1}{3}Tr(\boldsymbol{\varepsilon})\boldsymbol{1} \tag{8}$$

We assume that the occurrence of damage affects the volume change and the deviatoric part of the strain tensor. Defects in material are supposed to be accompanied by reduction in effective volume of damaged unit element compared with a virgin one and with reduction in effective surface area that is associated with the elastic deviatoric strain.

According to these assumptions Papa *et al.* [10] proposed the following expression for the potential of free energy:

$$\psi^e(\boldsymbol{\varepsilon},\boldsymbol{D}=\boldsymbol{0})=G_0\left[(\boldsymbol{1}-\boldsymbol{D})^{\frac{1}{2}}:\boldsymbol{\varepsilon}^D\right]:\left[(\boldsymbol{1}-\boldsymbol{D})^{\frac{1}{2}}:\boldsymbol{\varepsilon}^D\right]+\frac{3}{2}K_0\left[\mathrm{Tr}\,\boldsymbol{\varepsilon}\right]^2(1-d) \tag{9}$$

where d is the volumetric damage expressed as the average of the eigenvalues of damage tensor:

$$d=\beta\,\mathrm{Tr}(\boldsymbol{D}) \tag{10}$$

The residual term ψ^r of the expression (6) is written as a linear function of strain and damage tensors [9]:

$$\psi^r=-\eta\,\mathrm{Tr}(\boldsymbol{D}.\boldsymbol{\varepsilon}) \tag{11}$$

Onset and propagation of defects in material is assumed to be associated with positive strain field [11,12, 13, 14]. We take into account this last assumption by affecting the positive part of volumetric strain by a coefficient function of the state damage. We assume that the negative part is not concerned by the damage evolution.

Badel *et al.* [1] pointed out the difficulty to represent the response of the material in compression loading with a potential of free energy given by (9). The axial stress increases with axial strain and no maximum value can be reached. This observation leads us to affect the damage energy due to the deviatoric part of the strain field by a coefficient α $(1\le\alpha\le10)$. This allows to accelerate the decrease of this part of free energy.

The final form of the Helmotz strain energy we propose is given by the expression:

$$\psi(\boldsymbol{\varepsilon},\boldsymbol{D}) = G_0\left[(\boldsymbol{1}-\alpha\boldsymbol{D})^{\frac{1}{2}}:\boldsymbol{\varepsilon}^D\right]:\left[(\boldsymbol{1}-\alpha\boldsymbol{D})^{\frac{1}{2}}:\boldsymbol{\varepsilon}^D\right]+ \\ \frac{3}{2}K_0\left\{\left[(\mathrm{Tr}\,\boldsymbol{\varepsilon})_+\right]^2(1-\beta\,\mathrm{Tr}\,\boldsymbol{D})-\left[(-\mathrm{Tr}\,\boldsymbol{\varepsilon})_+\right]\right\}-\eta\,\mathrm{Tr}(\boldsymbol{D}.\boldsymbol{\varepsilon}) \tag{12}$$

where $(\boldsymbol{x})_+$ is formed from positive eigenvalues of tensor $(\boldsymbol{x})$:

$$(\boldsymbol{x})_+ = (x_i)_+\boldsymbol{e}_i\otimes\boldsymbol{e}_i \text{ and } (x_i)_+ = Sup(x_i,0).$$

α , β and η are material coefficients.

This form of potential of free energy is equivalent to that classically proposed by several theoretical models [3, 8]:

$$\psi(\boldsymbol{\varepsilon},\boldsymbol{D}) = G_0\,\mathrm{Tr}(\boldsymbol{\varepsilon}^D.\boldsymbol{\varepsilon}^D)+\frac{K_0}{2}\frac{(\mathrm{Tr}\,\boldsymbol{\varepsilon})^2}{3}+\alpha_1\,\mathrm{Tr}\,\boldsymbol{D}(\mathrm{Tr}\,\boldsymbol{\varepsilon})^2 \\ +\alpha_2\,\mathrm{Tr}\,\boldsymbol{D}\,\mathrm{Tr}(\boldsymbol{\varepsilon})^2+\alpha_3(\mathrm{Tr}\,\boldsymbol{\varepsilon})(\mathrm{Tr}\,\boldsymbol{\varepsilon}.\boldsymbol{D})+\alpha_4\,\mathrm{Tr}((\boldsymbol{\varepsilon})^2.\boldsymbol{D}) \tag{13}$$

Derivation of equation (12) with respect to $\boldsymbol{\varepsilon}$ gives stress-strain relationship:

$$\boldsymbol{\sigma} = \frac{\partial\psi(\boldsymbol{\varepsilon},\boldsymbol{D})}{\partial\boldsymbol{\varepsilon}} = 2G_0(\boldsymbol{1}-\alpha\boldsymbol{D})^{1/2}.\boldsymbol{\varepsilon}.(\boldsymbol{1}-\alpha\boldsymbol{D})^{1/2}+ \\ \frac{K_0}{3}[(\mathrm{Tr}\,\boldsymbol{\varepsilon})_+(1-\beta\,\mathrm{Tr}\,\boldsymbol{D})-(-\mathrm{Tr}\,\boldsymbol{\varepsilon})_+]\boldsymbol{1}-\eta\boldsymbol{D} \tag{14}$$

We check that when $\boldsymbol{\varepsilon}=\boldsymbol{0}$ (unloading stage) then $\boldsymbol{\sigma}=-\eta\boldsymbol{D}$. This term reflects the irreversibility due to the damage occurred during loading.

Derivation of equation (13) with respect to $\boldsymbol{D}$ yields to the damage driving forces:

$$\boldsymbol{Y} = -\frac{\partial\psi(\boldsymbol{\varepsilon},\boldsymbol{D})}{\partial\boldsymbol{D}} = \alpha G_0\boldsymbol{\varepsilon}^D.\boldsymbol{\varepsilon}^D+\frac{3}{2}K_0\beta(\mathrm{Tr}\,\boldsymbol{\varepsilon})_+^2\boldsymbol{1}+\eta\boldsymbol{\varepsilon} \tag{15}$$

We can distinguish in (15) the damage driving forces linked to the residual stress term of the strain energy:

$$\boldsymbol{Y}_1 = \eta\boldsymbol{\varepsilon} = \eta[(\boldsymbol{\varepsilon})_+ + (\boldsymbol{\varepsilon})_-] = \boldsymbol{Y}_1^+ + \boldsymbol{Y}_1^- \tag{16}$$

and the damage driving forces linked to the reversible part of free energy:

$$\boldsymbol{Y}_2 = \alpha G_0\boldsymbol{\varepsilon}^D.\boldsymbol{\varepsilon}^D+\frac{3}{2}K_0\beta(\mathrm{Tr}\,\boldsymbol{\varepsilon})_+^2\boldsymbol{1} \tag{17}$$

It is assumed that the evolution of the damage $\boldsymbol{D}$ is mainly governed by the positive part of the damage driving force $\boldsymbol{Y}_1^+$, which depends on the tensile (positive) strain tensor $(\boldsymbol{\varepsilon})_+$ [1, 2, 12].

The elastic domain is defined by a yield function $f\le 0$ written in the $\boldsymbol{Y}$-space. Concerning the form of the damage threshold for our class of material, the positive part of damage force $\boldsymbol{Y}_1^+$ is assumed to play a determining role. Then we assume that the threshold function is defined as follows [2, 13]:

$$f(\boldsymbol{Y}_1^+,\boldsymbol{D}) = \sqrt{\frac{1}{2}\mathrm{Tr}[(\boldsymbol{\varepsilon})_+^2]}-K(\boldsymbol{D})\le 0 \tag{18}$$

$K(\boldsymbol{D})$ is the damage threshold depending on the damage tensor $\boldsymbol{D}$:

$$K(\boldsymbol{D}) = a\,\mathrm{Tr}[(\boldsymbol{1}-\boldsymbol{D})^{-\gamma}]+b \tag{19}$$

This function depends on three additional parameters *a, b* and *γ*. Threshold function is not built on physical argument but allows obtaining realistic results regarding to concrete and geomaterials behavior observed in some experimental tests [14, 15].

The damage evolution is expressed using the normality rule:

$$\dot{\boldsymbol{D}} = \dot{\Lambda}\frac{\partial f(\boldsymbol{Y}_1^+, \boldsymbol{D})}{\partial \boldsymbol{Y}_1^+} \tag{20}$$

$\dot{\Lambda}$ is the damage multiplier which can be determined from consistency equation $\dot{f}(\boldsymbol{Y}_1^+, \boldsymbol{D}) = 0$:

$$\dot{\Lambda} = \frac{\dfrac{\partial f}{\partial \boldsymbol{Y}_1^+} : \dot{\boldsymbol{Y}}_1^+}{\dfrac{\partial f}{\partial \boldsymbol{D}} : \dfrac{\partial f}{\partial \boldsymbol{Y}_1^+}} \tag{21}$$

The loading-unloading conditions are the so-called Kuhn-Tucker conditions:

$$\dot{\Lambda} \geq 0, f(\boldsymbol{Y}) \leq 0, \dot{\Lambda} f(\boldsymbol{Y}) = 0 \tag{22}$$

3 Model prediction in uniaxial loading of tension and compression.

3.1 Uniaxial simple tension test

The model proposed can be developed analytically and explicitly in the uniaxial loading like tension test. Let be $\boldsymbol{e}_1$ the direction of the tension loading. The stress, strain and damage tensor are given below:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix}, \boldsymbol{D} = \begin{pmatrix} D_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{23}$$

It is assumed here that the damage evolves only in the direction of positive strain associated to ε_1. In this case $\boldsymbol{\varepsilon}$ and $\boldsymbol{D}$ have the same principal directions.

The damage elastic behavior is obtained from (14):

$$\begin{cases} \sigma_1 = [(\lambda_0 + 2\mu_0)[1 - \dfrac{(4\alpha + 2\beta)\mu_0 + 3\beta\lambda_0}{3(\lambda_0 + 2\mu_0)} D_1]\varepsilon_1 + \\ \quad 2\lambda_0[1 - \dfrac{2(\alpha - \beta)\mu_0 - 3\beta\lambda_0}{3\lambda_0} D_1]\varepsilon_2 - \eta D_1 \\ \sigma_2 = \dfrac{2}{3}\mu_0(\varepsilon_2 - \varepsilon_1) + \dfrac{3\lambda_0 + 2\mu_0}{3}(\varepsilon_1 + 2\varepsilon_2)(1 - \beta D_1) = 0 \end{cases} \tag{24}$$

The second expression of (24) gives the relation between axial and transversal strain:

$$\varepsilon_2 = -\frac{3\nu_0 - \beta(1 + \nu_0)D_1}{3 - \beta(1 + \nu_0)D_1}\varepsilon_1 = -\nu(D_1)\varepsilon_1 \tag{25}$$

Analyze of expressions (25) shows that $\nu(D_1)$ decreases as damage evolves (Fig. 2). This reflects the fact that the axial tensile strain evolves faster than the transverse one. This property of the model is consistent with experimental results in tension tests.

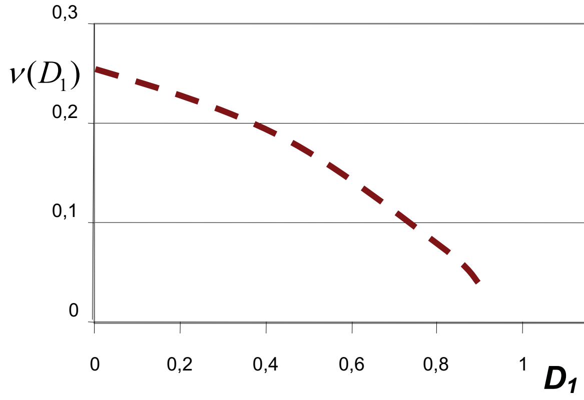

Figure 2. Evolution of Poisson ratio ν*(D) vs* damage D_1.

Concerning threshold function in tension, equations (18) and (19) leads to the following equation:

$$\varepsilon_1 - \sqrt{2}[a(2+\frac{1}{(1-D_1)^\gamma})+b] \le 0 \tag{26}$$

At the onset of damage ($f(\boldsymbol{Y}_1^+, \boldsymbol{D}) = 0, \boldsymbol{D} = \boldsymbol{0}$), the strain threshold ε_{10} is given by:

$$\varepsilon_{10} = \sqrt{2}(3a+b) \tag{27}$$

Beyond of the value ε_{10} , the damage parameter is positive ($D_1 > 0$) and the expression (26) yields to:

$$D_1 = 1 - \left[\frac{\sqrt{2}a}{\varepsilon_1 - \varepsilon_{10} + \sqrt{2}a}\right]^{1/\gamma} \tag{28}$$

We can easily verify that if $\varepsilon_1 \to \infty \Rightarrow D_1 \to 1$, which is the limit value of damage parameter. As a synthesis of the uniaxial loading simulation we can conclude that the damage model depends on 6 parameters $\alpha, a, b, \beta, \eta, \gamma$. The parameters *a* and *b* are linked by the relation (27).

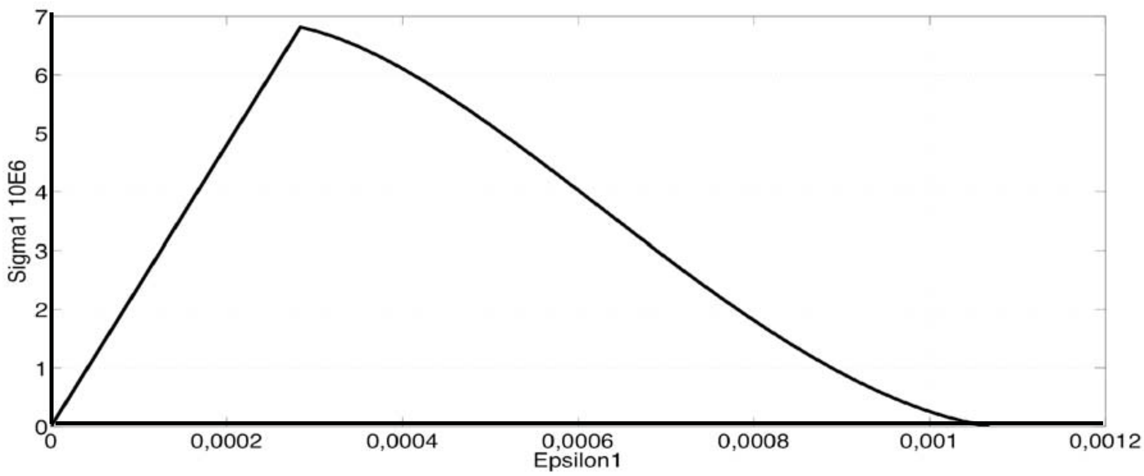

Figure 3. Simulation of stress-strain test in tension loading

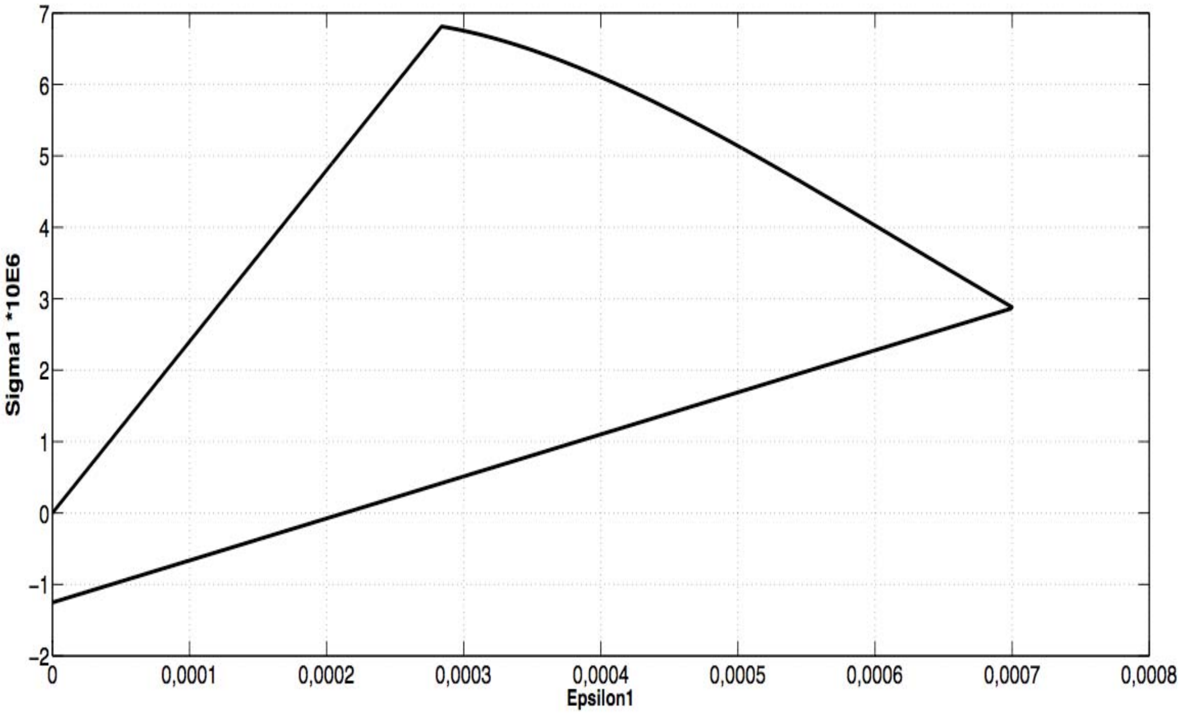

Figure 4 . Numerical test of unloading phase during tension

3.2 Uniaxial compression test

We choose $\boldsymbol{e}_1$ as the direction of the compression loading, then strain, stress and damage are now of the form:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \boldsymbol{e} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix}, \; \boldsymbol{D} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{pmatrix}$$

The damage elastic behavior is obtained from (14):

$$\begin{cases} \sigma_1 = (\lambda_0 + 2\mu_0)\varepsilon_1 + 2\lambda_0\varepsilon_2(D) \\ \sigma_2 = [2(\lambda_0 + \mu_0) - 2\dfrac{\mu_0}{3}\alpha D]\varepsilon_2 + (\lambda_0 + 2\dfrac{\mu_0}{3}\alpha D)\varepsilon_1 - \eta D = 0 \end{cases} \tag{29}$$

The second expression of (29) gives the relation between axial and transversal strain:

$$\varepsilon_2 = -\frac{3\lambda_0 + 2\mu_0\alpha D}{6(\lambda_0 + \mu_0) - 2\mu_0\alpha D}\varepsilon_1 + \frac{3\eta D}{6(\lambda_0 + \mu_0) - 2\mu_0\alpha D} \tag{30}$$

Equations (18) and (19) leads to the expression of the threshold function:

$$\varepsilon_2 - [a(1 + \frac{2}{(1-D)^\gamma}) + b] \leq 0 \tag{31}$$

The initial value of the strain at the beginning of the damage process is then obtained for $f(\boldsymbol{Y}_1^+, \boldsymbol{D}) = 0$ and $\boldsymbol{D} = \boldsymbol{0}$. In the transverse direction $\boldsymbol{e}_2$ the strain ε_{20} is given by:

$$\varepsilon_{20} = 3a + b \tag{32}$$

and

$$\varepsilon_2 = \varepsilon_{20} + 2a[\frac{1}{(1-D_2)^\gamma} - 1] \quad (33)$$

Then the expression of the damage D_2 :

$$D_2 = 1 - [\frac{2a}{\varepsilon_2 - \varepsilon_{20} + 2a}]^{1/\gamma} \quad (34)$$

We can obviously verify that if $D_2 = 0 \Rightarrow \varepsilon_2 = \varepsilon_{20}$ and if $D_2 \to 1,\ \varepsilon_2 \to \infty$.

The compression stress-strain diagram resulting from a simulated test compression is shown in figure 5. We note in figure 6 that the Poisson's ratio remains constant in the undamaged elastic phase and increases when damage evolves in the second phase.

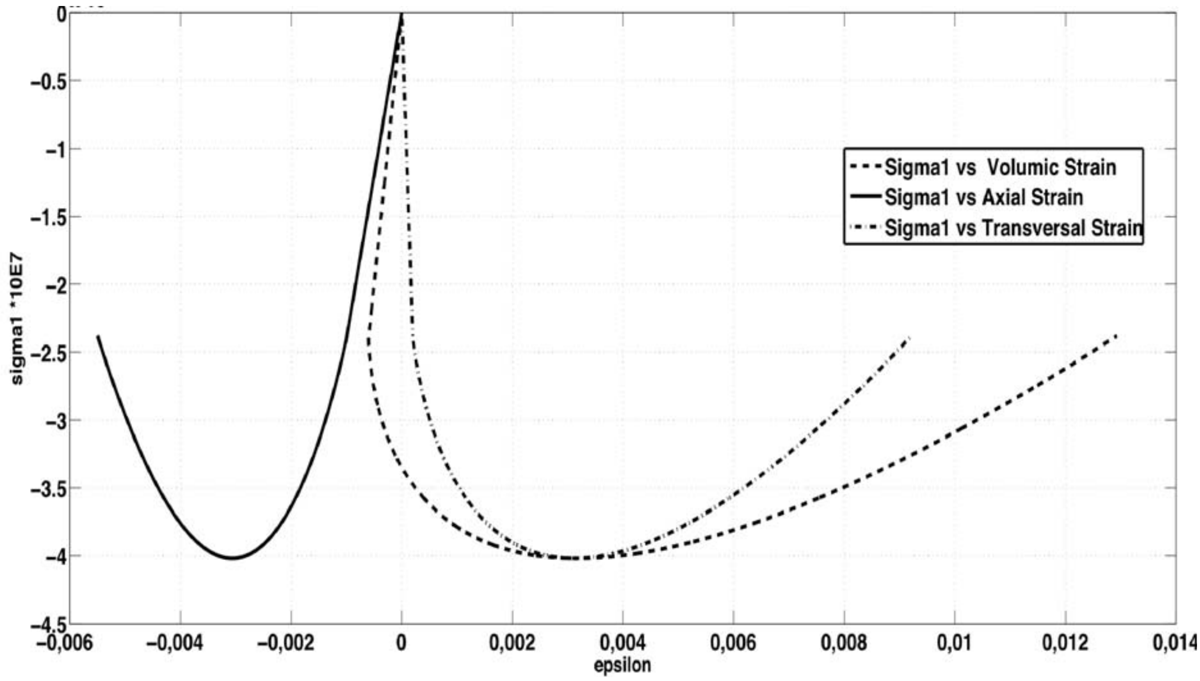

Figure 5 Stress-strain diagram simulation in compression

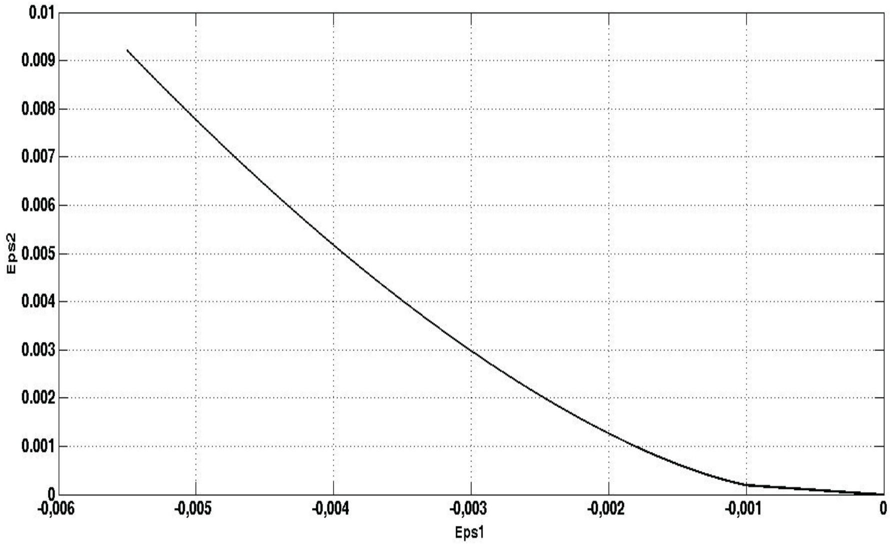

Figure 6 Poisson's ratio evolution

The parameters of the model used in this simulation are obtained from experimental tests on repair mortar [16]:

$E = 24$ Gpa , $\nu = 0.2$, $\eta = 7.25$ Mpa , $\gamma = 3.7$, $a = 2.9\times10^{-4}$, $b = -6.7\times10^{-4}$, $\alpha = 5$, $\beta = 1.76$

3.3 Dissymmetry in tension and compression

In the elastic phase we can demonstrate analytically that the model reflects the asymmetry between tension and compression by comparison between $(\varepsilon_{10})_t$ the elastic tension strain threshold and $(\varepsilon_{10})_c$ the elastic compression strain threshold.

Equations (26) and (36) allow to write:

$$(\varepsilon_{10})_t = \sqrt{2}(\varepsilon_{20})_c = -\nu\sqrt{2}(\varepsilon_{10})_c \quad (35)$$

The ratio between strain in tension $(\varepsilon_{10})_t$ and compression strain $(\varepsilon_{10})_c$ is about:

$$\frac{(\varepsilon_{10})_t}{(\varepsilon_{10})_c} = -\nu\sqrt{2} = -0.28$$

For $\nu = 0.2$ we have: $\left|\frac{(\varepsilon_{10})_t}{(\varepsilon_{10})_c}\right| = 0.28$

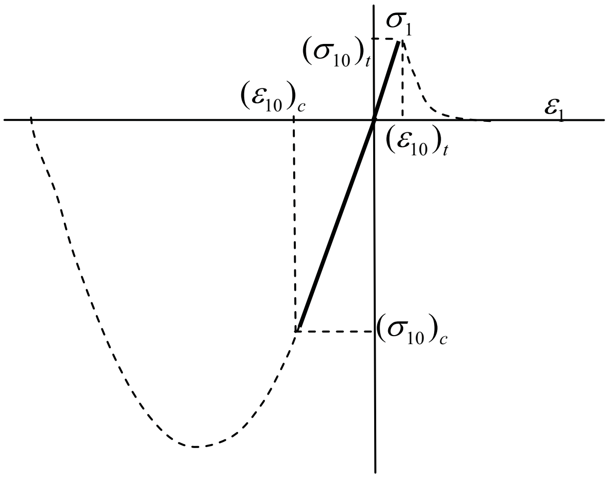

Figure 7. Asymmetric response in tension and compression

The asymmetry in tension-compression is clearly observable on figure 3 and 5. The ratio between compression and tension maximum stress is about 6.2. Experimental tests in tension and compression give a ratio value between 8 and 10 for repair mortar material simulated here. Therefore, our approach is acceptable at this stage of model development.

3.4 Initial elastic domain

The threshold function defined in (18) leads to the following general form:

$$(\boldsymbol{\varepsilon})_+ : (\boldsymbol{\varepsilon})_+ \leq (a+b)^2 \text{ for } \boldsymbol{D} = \boldsymbol{0} \quad (36)$$

Using the eigenvalues of the strain tensor $\boldsymbol{\varepsilon}$ in (36) one obtains in the plane strain $(\varepsilon_3 = 0)$ the following relation:

$$\varepsilon_1^2 + \varepsilon_2^2 \leq (a+b)^2 \quad (37)$$

The representation of the relation (37) is illustrated in figure 8.

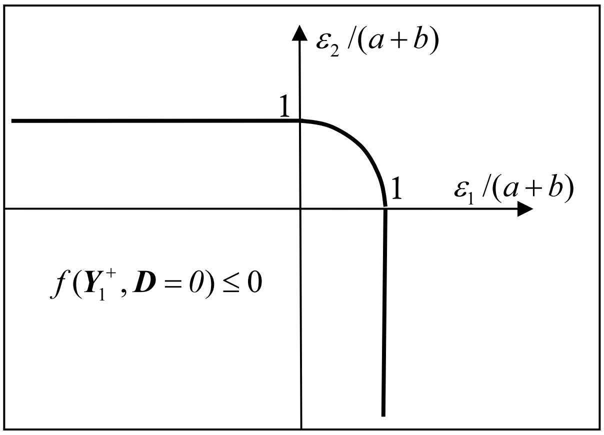

Figure 8. Representation of undamaged domain in plane strain $\varepsilon_3 = 0$

The undamaged domain is an open domain in the negative strain direction.

4 Conclusions

A model based on damage theory has been presented in this paper. The proposed model uses a tensorial variable to describe anisotropic damage. The model has been proved to be effective under several respects in describing the mechanical behavior of concrete and geomaterials under static loading, i.e.:

- occurrence of irreversible strain after total unloading;
- decreasing of young modulus with development of damage;
- dissymmetry in behavior between tension and compression
- unilateral effect as consequences of crack closure

This model requires the identification of a set of seven parameters easily extracted from tension-compression test.

A general good agreement is obtained between the experimental tension compression tests and theoretical simulations. More Developments are needed concerning the response of this model in the case of general loading paths.

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