PARAMETER STUDIES OF MODE INTERACTIONS IN NOSE LANDING GEAR SHIMMY

Phanikrishna Thota[1], Bernd Krauskopf[1] and Mark Lowenberg[2]
[1]Dept. of Engineering Mathematics, [2]Dept. of Aerospace Engineering
University of Bristol, United Kingdom.

Corresponding author: Phanikrishna Thota
Dept. of Engineering Mathematics
University of Bristol
Queen's Building, University Walk, Bristol BS8 1TR, UK.
Email: Phani.Thota@bris.ac.uk

Abstract. In this paper we study a mathematical model of an aircraft nose landing gear with torsional and lateral-bending degrees of freedom. The model also includes the geometric effects of a shock absorber (oleo) and a non-zero rake angle. Specifically, we investigate how the changes in the height of the nose landing gear due to the presence of an oleo affect shimmy oscillations. The nonlinear effects of both the elastic tyre force and the geometry of the gear are incorporated into the model. We conclude from the bifurcation analysis that a reduction in the overall height of the nose landing gear increases the area in the plane of velocity and vertical force that corresponds to lateral shimmy oscillations, but has no significant effect on torsional shimmy dynamics.

1 Introduction

Shimmy oscillations are a classical vibration problem in wheeled vehicles caused by a variety of structural and tyre flexibilities [1, 2, 5]. These oscillations are undesirable because of their adverse effects on vehicles and their passengers. Shimmy oscillations are observed in cars, motorcycles, and indeed aircraft. In this work we study their nature in an aircraft nose landing gear. Namely, we consider the interaction between different vibrational modes of a nose landing gear that contribute to shimmy oscillations under variations in system parameters.

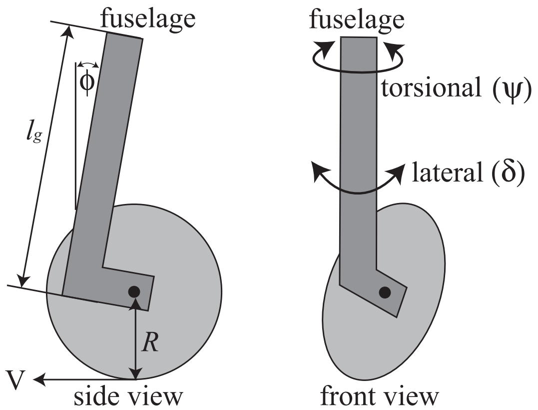

Figure 1: Schematic side and front views of an aircraft nose landing gear.

Figure 1 shows a schematic of the nose landing gear of a typical midsize passenger aircraft. It consists of a wheel with an elastic tyre that is attached to a strut via a mechanical trail (or caster) of length e. The strut is inclined to the vertical at a rake angle ϕ and rotates about its own axis with a torsion angle ψ. It also experiences a sidewards lateral bending motion δ about an axis passing through the fuselage centreline. The strut houses various mechanisms that control the dynamics of a landing gear. Mainly, it consists of a shock absorber, also called an oleo, that dampens any vertical motion in the landing gear. Pneumatic oleos use a combination of gas and oil to achieve the required damping characteristics. The compression and expansion of the gas in the oleo with changes in the vertical load are directly associated with the changes in the overall height of the landing gear. We consider a linear relationship between the stroke l_o of the oleo around its operating point and the vertical force F_z. In this paper we study the effects of this geometric change in landing gear height (we neglect the oleo dynamics) on shimmy oscillations.

From a dynamical systems point of view, the onset of shimmy oscillations is via the transition through a Hopf bifurcation when a parameter is changed. This leads to the onset of periodic dynamics of the wheel away from the straight-line motion. In order to study shimmy dynamics with changes in the gear height, we investigate a mathematical model of a nose landing gear that includes torsional and lateral bending vibrational modes. This model also includes the relevant effects of a non-zero rake angle. We perform a bifurcation analysis of the model equations with the software package AUTO [3], where the parameter values used are typical for a midsize passenger aircraft. Specifically, we present two-parameter bifurcation diagrams in the plane of forward velocity V and vertical force F_z on the gear. They contain different regions that indicate the dominance of different vibrational modes. The bifurcation diagrams also suggest that these bifurcation curves intersect, which may lead to a complicated mixed-mode response involving frequencies of more than one vibrational mode; see [7] for more details.

2 Mathematical Model

The torsional and lateral modes, including their coupling through the tyre, can be modeled by the equations

$$I_t\ \ddot{\psi} + M_{K_\psi} + M_{D_\psi} + M_{K_\alpha} + e_{\text{eff}}\, F_{K_\lambda} + \frac{c_\lambda\ \dot{\psi}\ \cos(\phi)}{\text{V}} - F_z\ \sin(\phi)\ e_{\text{eff}}\sin(\theta) \quad = \quad 0, \tag{1}$$

$$I_l\ \ddot{\delta} + M_{K_\delta} + M_{D_\delta} + (l_g - l_o)\ F_{K_\lambda}\ \cos(\theta)\ \cos(\phi) - F_z\ e_{\text{eff}}\ \sin(\theta) \quad = \quad 0, \tag{2}$$

$$\dot{\lambda} + \frac{V}{L}\lambda - V\ \sin(\theta) - (l_g - l_o)\ \dot{\delta}\ \cos(\delta) - \big(e_{\text{eff}} - h\big)\ \cos(\theta)\ \dot{\psi}\ \cos(\phi) \quad = \quad 0. \tag{3}$$

Equations (1), (2) and (3) govern the torsional, lateral and tyre dynamics respectively. The stroke l_o of the oleo enters as a correction to the landing gear height l_g. Here, I_t and I_l are the mass moments of inertia corresponding to the torsional and lateral bending modes of the landing gear. Equation (3), which describes the dynamics of the lateral deformation λ of the tyre, is a modified version of the von Schlippe's stretched string model [6]. Here, the original tyre equation is modified to include the effects of the lateral bending mode δ.

The second and third terms in Eqs. (1) and (2) correspond to stiffness and damping moments in the vibrational modes, and are given by $M_{K_{(\psi,\delta)}} = k_{(\psi,\delta)}\ (\psi,\delta)$ and $M_{D_{\psi,\delta}} = c_{(\psi,\delta)}\ (\dot{\psi},\dot{\delta})$, where $k_{(\psi,\delta)}$ and $c_{(\psi,\delta)}$ are the stiffness and damping coefficients in either torsional or lateral bending mode, depending on the index. The following terms in both Eqs. (1) and (2) represent the influence of the tyre on the landing gear dynamics. In the above equations, c_λ is the damping coefficient corresponding to the tyre's thread width and $\theta = \psi\ \cos(\phi)$. A detailed explanation of these terms can be found in Thota et al. [7].

The self-aligning moment M_{K_α} is given by the piecewise continuous function

$$M_{K_\alpha} \quad = \quad \begin{cases} k_\alpha \frac{\alpha_m}{\pi} \sin\left(\alpha \frac{\pi}{\alpha_m}\right)\ F_z & \text{if } |\alpha| \le \alpha_m, \\ 0 & \text{if } |\alpha| > \alpha_m, \end{cases} \tag{4}$$

and the lateral restoring force F_{K_λ} due to tyre deformation is given by

$$F_{K_\lambda} = k_\lambda\ \tan^{-1}(7.0\ \tan(\alpha))\ \cos\big(0.95\ \tan^{-1}(7.0\ \tan(\alpha))\big)\ F_z. \tag{5}$$

Here, k_α and k_λ are the torsional and lateral stiffnesses of the tyre. The slip angle α is related to the lateral deformation λ by $\alpha = \tan^{-1}(\lambda/L)$, where L is the *relaxation length* of the tyre. The constant α_m is the limit on the slip angle α beyond which the self-aligning moment is taken to be zero. The rake angle ϕ enters into the model via the effective caster length as given by $e_{\text{eff}} = e\cos\phi + (R + e\sin\phi)\tan\phi$.

3 Bifurcation analysis

We now perform a bifurcation analysis of the nose landing gear model Eqs. (1)–(3) with the continuation software AUTO [3]. Here, we consider the dependence of the dynamics on the forward velocity V and the vertical force F_z, where we fix all other system parameters at realistic values as used and given in [7]. Specifically, we compare the dynamics of the model with a constant height l_g ($l_o = 0$) of the landing gear with the case where the stroke l_o varies due to the presence of an oleo. Here, the stroke l_o is given by $l_o = \frac{F_z}{460}$ m, which is a realistic approximation for the nose landing gear of a midsize passenger aircraft.

Figure 2 shows the two-parameter bifurcation diagrams of Eqs. (1)–(3) in the (V, F_z)-plane for constant (a) and variable (b) heights of the landing gear. The figure highlights a curve H_t (grey) of Hopf bifurcations of the torsional

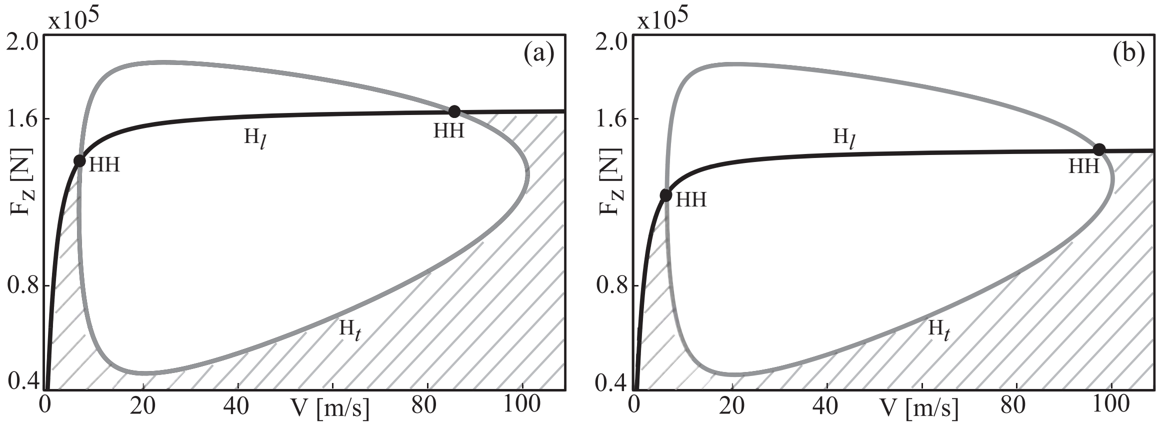

Figure 2: Two-parameter bifurcation diagrams of Eqs. (1)–(3) in the (V, F_z)-plane for constant (a) and variable (b) gear heights, consisting curves of Hopf bifurcations. In both the cases, the Hopf bifurcation curve H_t (grey) of the torsional mode forms an isola and interacts with the Hopf bifurcation curve H_l (black) of the lateral mode in two double Hopf points HH. In the shaded region, the zero equilibrium solution associated with the straight-line rolling of the wheel is stable.

mode and a curve H_l (black) of Hopf bifurcations of the lateral bending mode. While the torsional Hopf bifurcation curve H_t forms an isola (a closed curve), the lateral Hopf bifurcation curve H_l crosses the bifurcation diagram from left to right intersecting the torsional Hopf bifurcation curve H_t in two Hopf-Hopf bifurcation points HH. Such codimension-two bifurcation points organize the parameter space into the regions corresponding to different types of shimmy [4, 7]. The shaded region represents a stable zero equilibrium solution associated with the straight-line rolling of the wheel.

Figures 2(a) and (b) represent qualitatively similar dynamics, which means that we find the same bifurcations and regions for both constant and variable heights of the landing gear. However, there are differences of a quantitative nature. Specifically, the lateral Hopf curve H_l in Fig. 2(a) levels off at ≈ 160 kN, while the same occurs at ≈ 140 kN in Fig. 2(b). This difference (≈ 20 kN) of vertical force F_z in the H_l curves suggests, that in the presence of an oleo the landing gear is susceptible to lateral shimmy oscillations for a wider range of vertical force values. This is due to the increased stroke length l_o for higher vertical force values, which decreases the stabilizing moment associated with the term $\left((l_g - l_o)\, F_{K_\lambda}\; \cos(\theta)\; \cos(\phi)\right)$ in Eq. (2). On the contrary, an oleo has little effect on the torsional Hopf curves H_t, that is, the torsional shimmy oscillations are not affected by the changes in the stroke of the nose landing gear.

4 Conclusion

In this work we investigated a mathematical model of a nose landing gear with the inclusion of the torsional and lateral bending modes. The model also features the geometric effects of a non-zero rake angle and a shock absorber that dampens the vertical motion. The torsional and lateral bending modes are coupled through the tyre force generated due to the interaction of the elastic tyre with the ground. The main focus of our study was the effect of an oleo on the two-parameter bifurcation diagram in the (V, F_z)-plane that divides the parameter space into regions of different types of shimmy oscillations. Specifically, we conclude that the presence of oleos increases the area in the parameter space where lateral shimmy oscillations occur, while it has little effect on torsional shimmy oscillations. Namely, for a large vertical force the gear height decreases, and this results in a decrease in the stability in the lateral motion by a reduction in a stabilizing moment. This type of sensitivity study may help enhance the design and evaluation of a nose landing gear over the entire relevant range of operating parameters.

5 Acknowledgments

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