# Engine Modeling and Control System Design Considering the Twist of the Crankshaft

Hiroo Hirahara[1], Keita Yoshida[1],Masami Iwase[1],Shoshiro Hatakeyama[1], Teruyoshi Sadahiro[1], Nobuharu MIYATAKE[2]
[1]Tokyo Denki University, Japan, [2]MITSUBISHI RESEARCH INSTITUTE , Inc. Japan

Corresponding author: Hiroo Hirahara, Department of Computers and System Engineering,Tokyo Denki University – Hatoyama-cho Ishizaka, Hiki, Saitama 350-0394 – Japan, hirahara@hatalab.k.dendai.ac.jp

**Abstract.** The exhaust emission regulations and improving safety of cars become strict year by year, and better fuel efficiency and more comfortable driving response are also desired strongly. The present engine works are based on the crankshaft angle, i.e. the timing of the spark angle and fuel injection depend on the crankshaft angle. However, the twist of the crankshaft cannot be neglected in the measurement of the crankshaft angle and it influences the engine control. In this paper, the influence of the twist of the crankshaft is studied by using a model of an engine and its simulators. The engine model is derived by the Projection Method.

## 1 Introduction

The engine torque control is important for a lot of reasons, e.g. automobile emissions, safety, fuel efficiency, comfortable driving response and so on. And, in the recent year, high speed and high performance semiconductors that are able to execute more complex control method are cheaper and cheaper. On these backgrounds, in order to improve the engine torque control drastically, it is imperative to control and to estimate engines based on the nonlinear model of the engine. The current engine is controlled by the crankshaft angle, i.e. the spark timing depends on the crankshaft angle. In the current engine, in generally, the crankshaft angle is measured from a sensor mounted in the front of the engine. However, the twist of the crankshaft cannot be neglected in the measurement of the crankshaft angle and it affects the engine control. This problem is pointed out by McKelvey [1] and Kallenberge [3]. The gap of the combustion timing between the front side and the rear side is occurred because the twist is generated in the crankshaft of an in-line multi-cylinder engine and it causes undesirable variation of the torques. In this case, irregularity of rotational speeds causes vibrations. This problem can be solved to measure the crankshaft angle of every piston. But it is difficult to attach sensors on the every piston, because of the space of sensors, costs of the engine and so on. So the crankshaft angle is measured on either the first-cylinder or the end cylinder, and it is important to estimate the engine torques by it. In our recent research, the piston crank mechanism has been expressed by a nonlinear model[2]. In this study, considering the twist of the crankshaft, the engine model is enhanced more realistic, and is applied to the high precision engine torque estimation.

In this paper, at first, a nonlinar model of the engine considering the twist of the crankshaft is proposed using the Projection Method [4]. Then the way to estimate the engine torque is also proposed. The proposed method is verified by numerical simulations.

## 2 Modeling

In case of the piston-crank engine, it consists of three parts that are a piston, a connecting rod and a crankshaft. To model the nonelinar engine model that has the effect of the twist of the crankshaft, the Projection Method which is proposed by Blajer [4] is used. At first, the modeling of the each engine parts are done, then the single-cylinder engine model is derived connecting these models by some simple positional constraint conditions. the nonlinear model of strictly-structured engine is derived using the modeling result of the single cylinder engine is connected using the crankshaft in considering the twist. The piston crank model and parameters are shown in Figure 1 and Table 1.

### 2.1 Dynamical system without any constraints

In the PJ method, the dynamical system without any constraints must be derived at first. The COG position of the crank part is inside the crank radius. Therefore, we assume that $r_G$ = 0.4 r. The generalized coordinate $x$ and generalized velocity $v$ are defined as follows:

$$x = \begin{bmatrix} x_p & z_p & x_c & z_c & x_r & z_r & \phi & \theta_a \end{bmatrix}^T, \quad (1)$$
$$v = \begin{bmatrix} \dot{x_p} & \dot{z_p} & \dot{x_c} & \dot{z_c} & \dot{x_r} & \dot{z_r} & \dot{\phi} & \dot{\theta}_a \end{bmatrix}^T, \quad (2)$$

where $(x_p, z_p), (x_c, z_c)$, and $(x_r, z_r)$ are the coordinate of the COG of the piston, of the connecting rod, and of the crank, respectively, $\phi$[rad] is an angle between the connecting rod and the piston and $\theta_a$[rad] is an angle of the

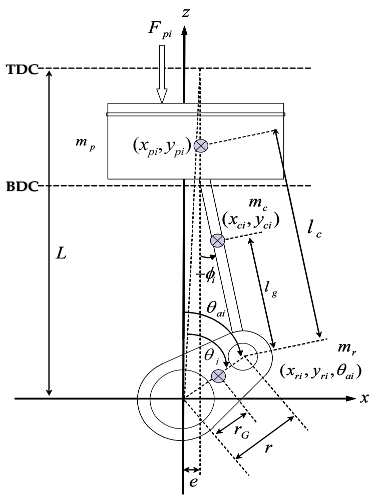


**Figure 1:** Piston crank model

**Table 1:** Model parameters

| | | |
|---|---|---|
| $l_c$ | the length of the connecting rod | 0.14665[m] |
| $l_g$ | the length of the center of gravity (COG) of the connecting rod | 0.1121[m] |
| $r$ | the crank radius | 0.043[m] |
| $e$ | offset of the position of the COG of the piston | 0.0095[m] |
| $L$ | distance between top dead center (TDC) and center of crank | 0.1972[m] |
| $r_G$ | the length of the COG of the crank | 0.0215[m] |
| $m_p$ | the mass of the piston | 0.5[kg] |
| $m_c$ | the mass of the connecting rod | 0.52[kg] |
| $m_r$ | the mass of the crankshaft and the crank | 9.5[kg] |
| $A$ | the cross-section area of the piston | 0.0062[m$^2$] |
| $g$ | acceleration due to gravity | 9.81[m/s$^2$] |

crank(see Fig.1).

The generalized mass matrix M is defined as follows:

$$M = \mathrm{diag}\left[m_p \quad m_p \quad m_c \quad m_c \quad \tfrac{m_r}{2} \quad \tfrac{m_r}{2} \quad J_c \quad J_r\right], \tag{3}$$

where $J_c$ and $J_r$ are the inertia moment of the connecting rod and the crankshaft, respectively.

The generalized forces matrix h is defined as follows:

$$h = \left[0 \quad -m_p g - P\cdot A - c_p \tfrac{dz_p}{dt} \quad 0 \quad -m_c g \quad 0 \quad -\tfrac{m_r g}{2} \quad 0 \quad -c_r\dot{\theta}\right]^T, \tag{4}$$

where $P$ is pressure on the piston, $c_p$ and $c_r$ are the viscous friction coefficient of the piston and the crank, respectively. Therefore, the dynamical system without any constraint is derived as follows:

$$M\ddot{x} = M\dot{v} = h. \tag{5}$$

### 2.2 The constraint matrix and the orthogonal complement matrix to the constraint matrix

To model the dynamical system with constraints, the constraint matrix which is the Jacobian of the restricted matrix and its orthogonal complement matrix is needed. Considering constraints, they are derived as follows. The positional constraints of the elements of the generalized coordinate $x$ are

$$\theta_a = \left(\frac{\pi}{180}\theta\right) + \sin^{-1}\left(\frac{e}{l_c + r}\right), \tag{6}$$

$$\sin\phi = \frac{r\sin\theta_a - e}{l_c}, \tag{7}$$

$$\cos\phi = \frac{\sqrt{l_c^2 - (r\sin\theta_a - e)^2}}{l_c}, \tag{8}$$

$$\phi = \sin^{-1}\left(\frac{r\sin\theta_a - e}{l_c}\right), \tag{9}$$

$$x_p = e, \tag{10}$$

$$z_p = r\cos\theta_a + \sqrt{l_c^2 - (r\sin\theta_a - e)^2}, \tag{11}$$

$$\dot{z_p} = -r\sin\theta_a\dot{\theta}_a - \frac{r\cos\theta_a\,(r\sin\theta_a - e)}{\sqrt{l_c^2 - (r\sin\theta_a - e)^2}}\dot{\theta}_a, \tag{12}$$

$$x_c = e + (l_c - l_g)\sin\phi = e + \left(1 - \frac{l_g}{l_c}\right)(r\sin\theta_a - e), \tag{13}$$

$$z_c = r\cos\theta_a + \frac{l_g}{l_c}\sqrt{l_c^2 - (r\sin\theta_a - e)^2}, \tag{14}$$

$$x_r = r_g\sin\theta_a, \tag{15}$$

$$z_r = r_g\cos\theta_a. \tag{16}$$

Therefore, the restricted matrix $\Phi$ is

$$\Phi = \begin{bmatrix} x_p - e \\ z_p - \left\{ r\cos\theta_a + \sqrt{l_c^2 - (r\sin\theta_a - e)^2} \right\} \\ x_c - \left\{ e + \left(1 - \frac{l_g}{l_c}\right)(r\sin\theta_a - e) \right\} \\ z_c - \left\{ r\cos\theta_a + \frac{l_g}{l_c}\sqrt{l_c^2 - (r\sin\theta_a - e)^2} \right\} \\ x_r - r_G\sin\theta_a \\ z_r - r_G\cos\theta_a \\ \phi - \sin^{-1}\left(\frac{r\sin\theta_a - e}{l_c}\right) \end{bmatrix} = 0. \tag{17}$$

Using these constraints, the constraint matrix $C$ of the system is represented by

$$C = \frac{\partial \phi}{\partial x} = \begin{bmatrix} C_1 & | & C_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & | & r\sin\theta_a + \frac{r\cos\theta_a(r\sin\theta_a - e)}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & | & -\left(1-\frac{l_g}{l_c}\right) r\cos\theta_a \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & | & r\sin\theta_a + \frac{l_g}{l_c}\frac{r\cos\theta_a(r\sin\theta_a - e)}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & | & -r_G\cos\theta_a \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & | & r_G\sin\theta_a \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & | & -\frac{r\cos\theta_a}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \end{bmatrix}, \tag{18}$$

The generalized velocity $\upsilon$ is divided into the independent velocity component of $\upsilon$, i.e. the tangent velocity $\upsilon_2$, and the other component $\upsilon_1$, that is

$$\upsilon = \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix}, q := \upsilon_2 = \dot{\theta}_a. \tag{19}$$

Because $C\upsilon = 0$, then

$$C\upsilon = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix} = C_1\upsilon_1 + C_2\upsilon_2 = 0. \tag{20}$$

Hence

$$\upsilon_1 = -\left(C_1^T C_1\right)^{-1} C_1^T C_2 \upsilon_2. \tag{21}$$

Using Equation (21), the generalized velocity is rewritten as

$$\upsilon = \begin{bmatrix} -\left(C_1^T C_1\right)^{-1} C_1^T C_2 \\ I \end{bmatrix} \upsilon_2 = \begin{bmatrix} 0 \\ r\sin\theta_a + \frac{r\cos\theta_a(r\sin\theta_a - e)}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ -\left(1-\frac{l_g}{l_c}\right) r\cos\theta_a \\ r\sin\theta_a + \frac{l_g}{l_c}\frac{r\cos\theta_a(r\sin\theta_a - e)}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ -r_G\cos\theta_a \\ r_G\sin\theta_a \\ -\frac{r\cos\theta_a}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ 1 \end{bmatrix} \dot{\theta}_a = Dq. \tag{22}$$

Therefore the orthogonal matrix D is as follows:

$$D=\begin{bmatrix} 0 \\ r\sin\theta_a+\frac{r\cos\theta_a(r\sin\theta_a-e)}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ -\left(1-\frac{l_g}{l_c}\right)r\cos\theta_a \\ r\sin\theta_a+\frac{l_g}{l_c}\frac{r\cos\theta_a(r\sin\theta_a-e)}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ -r_G\cos\theta_a \\ r_G\sin\theta_a \\ -\frac{r\cos\theta_a}{\sqrt{l_c^2-(r\sin\theta_a-e)^2}} \\ 1 \end{bmatrix}. \tag{23}$$

### 2.3 Dynamical system with constraint: The single-cylinder model

The motion equation of the single-cylinder engine is derived using the constraint matrix and the orthogonal matrix. Using Equation (22), the generalized accelaration of the constraint system is

$$\dot{\upsilon}=\dot{D}q+D\dot{q}=\dot{D}\dot{\theta}_a+D\ddot{\theta}_a. \tag{24}$$

Using the constraint matrix $C$ in Equation (18), the constraint dynamical system with the constraint reaction forces is

$$M\dot{v}=h+C^T\lambda, \tag{25}$$

where $\lambda$ is the Lagrange multipliers.Substituting Equation (24) into Equation (25), and multiplying both sides of the equation by $D^T$, the equation is rewriten as

$$D^TM\left(\dot{D}\dot{\theta}_a+D\ddot{\theta}_a\right)=D^T\left(h+C^T\lambda\right)=D^Th. \tag{26}$$

Because $CD=0$, the equation is simplyfied as

$$D^TMD\ddot{\theta}_a \quad = \quad D^T\left(h-M\dot{D}\dot{\theta}_a\right). \tag{27}$$

### 2.4 Engine modeling considering the twist of the crankshaft

The in-line two-cylinder engine has two pistons, and two pistons are connected by a crankshaft. Using the previous result, i.e. the single-cylinder engine model, the twistable crankshaft model and appropriate constraint conditions, the in-line two-cylinder engine model is derived.

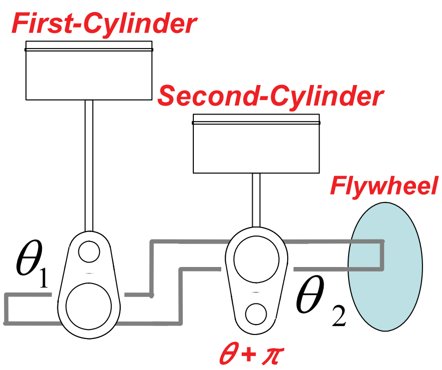


**Figure 2:** Two-cylinder engine model

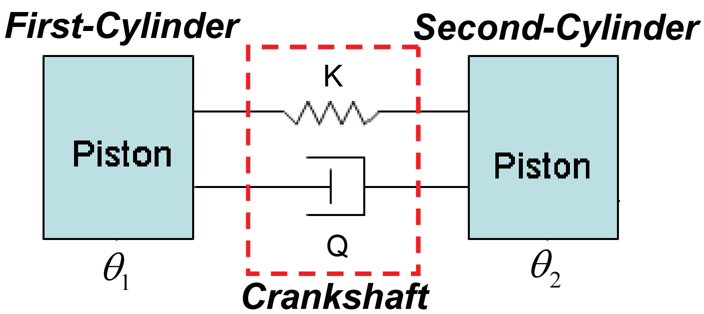


**Figure 3:** Crankshaft model of the two-cylinder engine

The agnle of the two pistons are basically shifted 180 degrees, but to consider the effect of the twist of the crankshaft, we let the angle of the first piston and the second piston $\theta_1$ and $\theta_2$, respectively(See Fig.2). Using Equation (27), the motion equation of the first-cylinder and the second-cylinder are derived as follows:

$$D_1^T M D_1 \ddot{\theta}_1 = -D_1^T M \dot{D}_1 \dot{\theta}_1 + D_1^T h_1, \tag{28}$$
$$D_2^T M D_2 \ddot{\theta}_2 = -D_2^T M \dot{D}_2 \dot{\theta}_2 + D_2^T h_2, \tag{29}$$

where M is the generalized mass matrix of the single-cylinder engine model, $D_i$ is the orthogonal complement matrices to the constraint matrix, $h_i$ is the generalized forces, the subscription 1 implies first-cylinder's one and 2 implies second-cylinder's one, respectively. Metal rigidity of the crank shaft is expressed by the spring constant K and the damping constant Q. Using these constants, the twist of the crankshaft is taken into consideration(See Fig.3). To consider the assumption of the crankshaft,the motion equation of the first-cylinder and the second-cylinder that are connected by the crankshaft are derived as follows:

$$D_1^T M D_1 \ddot{\theta}_1 = D_1^T h_1 - D_1^T M \dot{D} \dot{\theta}_1 - K(\theta_1 - \theta_2) - Q\left(\dot{\theta}_1 - \dot{\theta}_2\right), \tag{30}$$
$$D_2^T M D_2 \ddot{\theta}_2 = D_2^T h_2 - D_2^T M \dot{D} \dot{\theta}_2 - K(\theta_2 - \theta_1) - Q\left(\dot{\theta}_2 - \dot{\theta}_1\right). \tag{31}$$

In addition, the crankshaft is also attached a flywheel, its inertia moment J is included in the second cylinder engine model. The motion equation of the second-cylinder in consideration of the flywheel is

$$\left(J + D_2^T M D_2\right) \ddot{\theta}_2 = D_2^T h_2 - D_2^T M \dot{D} \dot{\theta}_2 - K(\theta_2 - \theta_1) - Q\left(\dot{\theta}_2 - \dot{\theta}_1\right). \tag{32}$$

Consequently, the motion equation of the two-cylinder engine considering the twist of the crankshaft are derived as Equation (30) and (32)

# 3 Torque estimation using the modeling result

To estimate the engine torques using Equation (30) and (32), forcusing on the pressure in the generalized forces term Equation (4) and the fact that the engine torques are generated by it, Equation(30) and (32) are transformed into as follows:

$$P_1 \upsilon(\theta_1) = a(\theta_1) \dot{\theta}_1 \ddot{\theta}_1 - b(\theta_1) + f(\theta_1) + c(\theta_1) \dot{\theta}_1^{\ 2}, \tag{33}$$
$$P_2 \upsilon(\theta_2) = a(\theta_2) \dot{\theta}_2 \ddot{\theta}_2 - b(\theta_2) + f(\theta_2) + c(\theta_2) \dot{\theta}_2^{\ 2} + J\dot{\theta}_2, \tag{34}$$

where

$$\upsilon(\theta_a) = \left( r\sin\theta_a + \frac{r\cos\theta_a (r\sin\theta_a - e)}{l_c^2 - (r\sin\theta_a - e)^2} \right) A, \tag{35}$$
$$f(\theta_a) = \left( r\sin\theta_a + \frac{\cos\theta_a (r\sin\theta_a - e)}{\sqrt{l_c^2 - (r\sin\theta_a - e)^2}} \right)^2 c_p \dot{\theta}_a + c_r \dot{\theta}_a, \tag{36}$$
$$a(\theta_a) = D^T M D, \tag{37}$$

$$\begin{aligned} b(\theta_a) = {} & (\left( r\sin\theta_a + \frac{l_g}{l_c} \frac{r\cos\theta_a (r\sin\theta_a - e)}{\sqrt{l_c^2 - (r\sin\theta_a - e)^2}} \right) m_c g \\ & + \left( r\sin\theta_a + \frac{r\cos\theta_a (r\sin\theta_a - e)}{\sqrt{l_c^2 - (r\sin\theta_a - e)^2}} \right) m_p g + \frac{m_r}{4} g r_g \sin\theta_a - T_l), \end{aligned} \tag{38}$$

$$c(\theta_a) = D^T M \dot{D}. \tag{39}$$

In Equation (33) and (34), both side of the equations are meant engine torques. The left hand side of the equations are considered as the true values in the sense that the terms are able to measure, and right hand side of the equations are the equation to estimate the torques. Therefore, to estimate the engine torques, the crank velocities and accelerations, that are not measure are needed, so the High-Gain-Observer type differentiator that has a better performance to noisy signals such as the crank angle is used to derive the velocities and the accelerations. Hence, if the crank angles $\theta_1$ and $\theta_2$ are able to measure, the engine torques are estimated by Equation (33) and (34).

## 4 Simulation result

The simulation of the engine model considering the twist of the crankshaft is performed. The cylinder pressure is mimicked by the Wiebe function, and the spring constant $K$ is set to 100000[N/m] and the damper constant $Q$ is set to 1000[Ns/m]. Simulation results are shown in Figure 4 to 7. From the simulation results, the crank angle velocities, the angular accelerations, and the torques are changed with the cylinder pressure. In addition, the gap is generated between the angle and the angular velocity of the first-cylinder and the second-cylinder by the twist of the crankshaft. These results are shown that the motion equation of the engine considering the twist of the crankshaft is obtained.

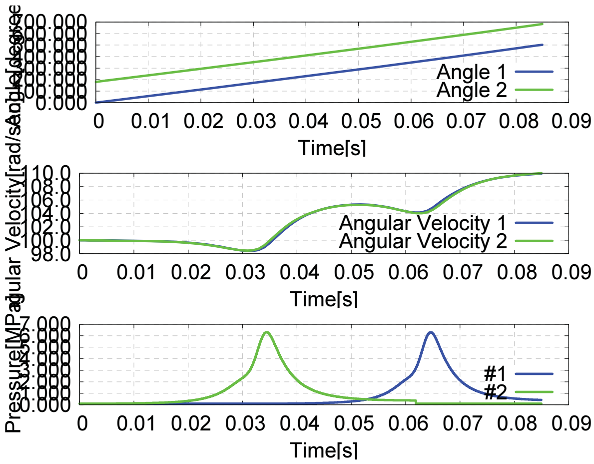


**Figure 4:** Angles(upper) and Angular velocities(middle) and cylinder pressures(lower) in considering the twist of the crankshaft on the two-cylinder engine

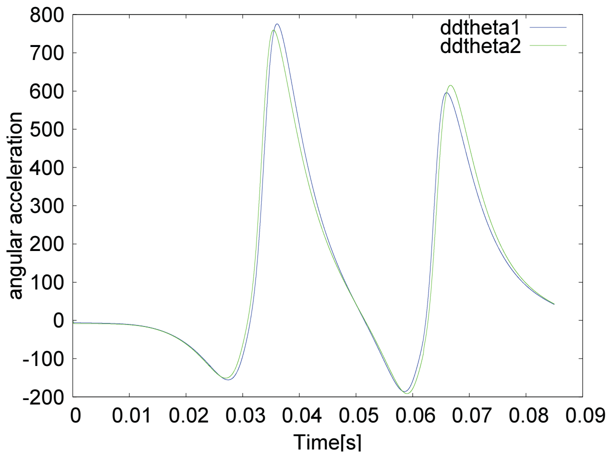


**Figure 5:** Angular accelerations in considering the twist of the crankshaft of the two-cylinder engine

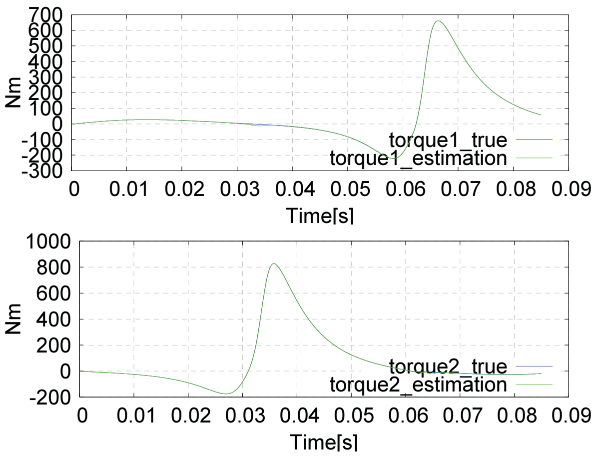


**Figure 6:** Torques in considering the twist of the crankshaft of the two-cylinder engine(True one and estimated one of the first-cylinder(upper) and the second-cylinder(lower))

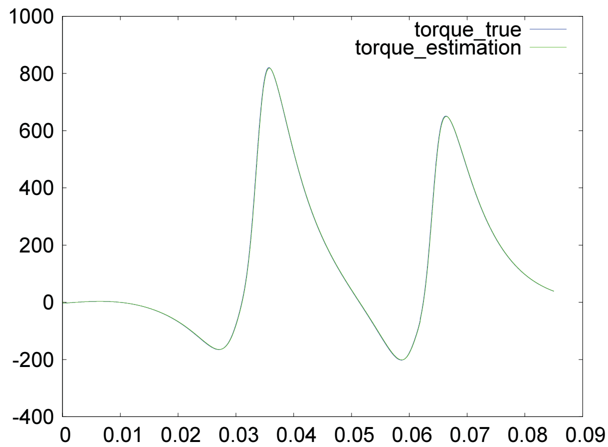


**Figure 7:** Total torques in considering the twist of the crankshaft of the two-cylinder engine

# 5 Conclusions and Future works

In this paper, considering the twist of the crankshaft, the engine model [6] is enhanced more realistic, and is applied to the high precision engine torque estimation. The propesed torque estimation equation needs the every crank angle on the every piston. But in the real engines, it is difficult to measure the every crank angle, and the crank angle can be measured is either the first-cylinder's one or the last-cylinder's one. Therefore, the other angles that are not able to measure are also estimated from the angle that is able to measure. As a future work, using the advantage of the model, i.e. accuracy and nonlinearity, we will tackle to design an observer that estimates torques and unmeasureable angles. As another future work, the proposed nonlinear model and the torque estimation method will be verified experimentally using the engine for the radio control car(Figure 8 and Figure 9).

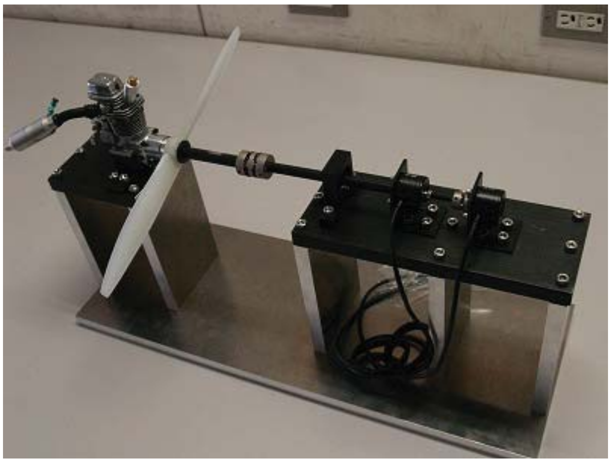

**Figure 8:** Real machine of engine

**Figure 9:** Four-stroke engine

# 6 References


[1] Ingemar Andersson, Tomas McKelvey, "Torque Ration Concept for Combustion Phasing Detection of a Spark Ignited Engine", IEEE Conference on Decision and Control, P1703-1708

[2] Yosuke Kitahara, Satoru Katohno,"Engine Torque Estimation based on Crankangle-Timing Model with Exact Robust Differentiator",CACS,(2007)

[3] C. Kallenberger, H. Haris, A. M Zoubir, "EVALUATION OF TORQUE ESTIMATION USING GRAY-BOX AND PHYSICAL CRANKSHAFT MODELING", ICASSP, P1529-P1532, (2008)

[4] W. Blajer, ”A geometrical interpretation and uniform matrix formulation of multibody systems dynamics”, ZAMM Z. Angew. Math. Mech, vol. 81, no. 4, 2004, pp 247-259.

[5] Luma K. Vasiljevic, Hassan K. Khalil,"Differention with High-Gain Observers the Presence of Measurement Noise", Proc. of CDC'06,D,P13-15

[6] ENYA METAL PROCUCTS CO., LTD. ,"CATALOG of 41-4CD TN DIESEL ENGINE", `http://www.enya-engine.com/41-4CDManual_E.pdf`