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SIMPLE MODELS FOR PROCESS CONTROL

R. Gorez[1], P. Klán[2]
[1]UC Louvain, Belgium, [2]Institute of Computer Science Prague, Czech Rep.

Corresponding author: P. Klán, Institute of Computer Science,
182 07 Prague 8, Pod vodarenskou vezi 2, Czech Rep., pklan@cs.cas.cz

Abstract. A second–order–plus–dead–time model is derived from the data used for the characterization of a self–regulating process with a view to its PID control. This modeling approach provides guidelines for the design of the controller and tuning relations are proposed which provide flexibility in the choice of the control system responses and guarantee of stability and robustness. These features are illustrated by simulation results.

1 Introduction

Most industrial processes are stable, hence self-regulating, and they can be controlled by PI or PID controllers combining proportional, integral and derivative control terms. From the pioneering paper [9] by Ziegler and Nichols in the forties hundreds of design relations, tuning rules, and other methods for tuning PID controllers have been proposed in the literature ; tens of them are compiled in [5], and the most popular ones can be found in many textbooks on process control ; see e.g. [1, 6, 7]. Tuning rules proposed by Ziegler-Nichols in the forties, Cohen-Coon and Chien-Hrones-Reswick in the fifties, and by Åström-Hägglund in the nineties are based on a three-parameter characterization of the controlled system, that is the process itself and the various components in the measurement and control lines ; see [1, 2, 3, 9]. Such a three-parameter characterization of the controlled system is consistent with the use of a PID controller since tuning the latter involves also three parameters, the controller gain and the integral and derivative time constants. The three parameters of the process characterization are the process *static gain* and two parameters in the time domain. The latter may be an *apparent time constant* which provides a rough global representation of all the elementary storage processes in the controlled system and an *apparent dead time* which aims to some approximation of the unknown dynamics related to the interactions between all these individual processes. However a better characterization of the system dynamics is given by the *average residence time* which is the sum of the apparent time constant and dead time, so determining the time scale of the system dynamics, and by the *normalized dead time*, that is the ratio of the apparent dead time to the average residence time, which is a dimensionless parameter whose value in the range (0, 1) indicates the difficulty of controlling the process. The values of the three parameters can be easily determined by graphical methods from the process reaction curve (step response) or by a method of moments applied to the reaction curve or to the step response of a stable PI control loop.

However, the first-order plus dead-time (FOPDT) model which can be constructed from this three parameter characterization is not a realistic model neither for simulation nor even for controller design if the normalized dead time is small (below 0.5). In any control loop indeed, there are more than one single time constant, there may be additional time constants in the process itself and there are others in the various components of the control loop, especially in the actuator. Consequently, in the time domain the step response of the controlled system exhibits a smooth start-up whereas that of the FOPDT model shows an abrupt take-off. In the frequency domain the gain diagram of the FOPDT model has a slope of $-20\ dB/decade$ at frequencies beyond the inverse of the apparent time constant whereas the additional time constants hidden in the control loop result into a steeper descent of the gain diagram at high frequencies. These discrepancies between the true transfer function of the controlled system and its FOPDT model are minor for processes where the apparent dead time is relatively long ($\tau > 2/3$), they may be significant in other cases. Then, a second-order plus dead time (SOPDT) model would be more appropriate for designing a controller. Such a model can be obtained by various identification methods ; in the sixties already van der Grinten had proposed formulas determining the model parameters from the process reaction curve and graphical methods [8]. Here it is proposed to construct a SOPDT model with only three independent parameters closely related to that of the usual FOPDT model ; the values of these parameters can be determined from the same two moments used for determining the parameters of the FOPDT model, or directly from the latter or from any transfer function representing the process to be controlled.

2 Model building via the method of moments

Denoting by s the variable in the Laplace domain the transfer functions of FOPDT and SOPDT models of a self-regulating process can be written as

$$M_1(s) = K\frac{1}{1+T_a s}e^{-D_a s} \quad (2-1)$$

$$M_2(s) = K\frac{1}{(1+T_1 s)(1+T_2 s)}e^{-D_2 s} \quad (2-2)$$

where K is the static gain, and T_a and D_a are the apparent time constant and dead time for a FOPDT model while T_1, T_2, D_2 are the two time constants and the dead time of a SOPDT model. Such models can be used for designing a PID controller or any three-term feedback controller if the first terms of the Taylor–McLaurin expansion of the model are equal to that of the process. The transfer function of the latter can be expanded as

$$P(s) = P(0)+P'(0)s+P''(0)\frac{s^2}{2}+\ldots = K(1+p_1 s+p_2 s^2+\ldots), \quad (2-3)$$

where $P'(s), P''(s), \ldots$ are the derivatives of $P(s)$ with respect to the complex variable s and $K = P(0)$, $p_1 = \frac{P'(0)}{P(0)}, p_2 = \frac{P''(0)}{2P(0)}, \ldots$ The process gain can be determined from the final value in the process step response ; from well–known properties of the Laplace transform the coefficients $p_1, p_2, \ldots$ are given by the "moments", that is integrals, of the weighted complementary function of the normalized response. As for the Taylor–McLaurin expansion of the model transfer function it can be obtained by successive differentiations as in (2 – 3) or better by applying the algorithm in Appendix A. Then, comparing the coefficients in the expansions of the process and the model transfer functions yields the following relations :

$$K = \frac{r(\infty)}{\Delta u} \quad (2-4)$$

$$T_{ar} = T_a + D_a = T_1 + T_2 + D_2 = \int_0^\infty [1-\frac{r(t)}{r(\infty)}]\,dt \quad (2-5)$$

$$\frac{1}{2}(T_{ar}^2 + T_a^2) = \frac{1}{2}(T_{ar}^2 + T_1^2 + T_2^2) = \int_0^\infty t\,[1-\frac{r(t)}{r(\infty)}]\,dt \quad (2-6)$$

These relations provide values of the three parameters in the FOPDT model (2 – 1), hence the value of the normalized dead time

$$\tau = 1 - T_a/T_{ar}. \quad (2-7)$$

For determining the parameters of a SOPDT model additional relation or constraint is necessary ; here, it is proposed to set

$$T_1 = T_{ar}(1-\theta), \qquad T_2 = T_{ar}\,\theta\,(1-\theta), \qquad D_2 = T_{ar}\,\theta^2, \quad (2-8)$$

where θ is dimensionless parameter with values in the interval (0, 1), close to that of the normalized dead time τ. From the equality in (2 – 6) straightforward calculations indeed result into the following relationship between the two dimensionless parameters τ and θ :

$$1-\tau = (1-\theta)\sqrt{1+\theta^2}. \quad (2-9)$$

Then, the value of θ can be obtained from that of the normalized dead time τ via an iterative procedure

$$\theta = 1 - \frac{1-\tau}{\sqrt{1+\theta^2}} \quad (2-10)$$

starting from the initial approximation $\theta = \tau$; less than 5 iterations provide the value of θ with 4 exact decimals, which is largely beyond the accuracy required for controller tuning.

It should be noted that this SOPDT model is based on three parameters, the static gain of the process, the average residence time, that is the time scale, of the latter, and a dimensionless parameter θ indicating the difficulty of controlling the system. This is exactly the same simplicity as with the usual FOPDT model, and the tools used for parameter estimation are the same, with no more data being required. These data can be obtained from the process reaction curve or from the step response of any stable control system including the process to be controlled and a proper PI controller, as it is shown in Appendix B. Actually, the SOPDT model can be viewed as being derived from the FOPDT model but it is more realistic , both in the frequency domain and in the time domain. This SOPDT model indeed contains a main time constant $T_1 = (1-\theta)\,T_{ar} = T_a/\sqrt{1+\theta^2}$ which is close (slightly lower) to the apparent time constant $T_a = (1-\tau)\,T_{ar}$ of the FOPDT model, but the additional uncertain dynamics of the process is now modeled at the first order (in θ) by an additional time constant $T_2 = \theta\,(1-\theta)\,T_{ar}$ and at the second order by a dead time $D_2 = \theta^2\,T_{ar}$ whereas it is was modeled at the first-order by the apparent dead time $D_a = \tau\,T_{ar}$ in the FOPDT model. Last but not least, this SOPDT model provides guidelines for the design of a PID controller.

3 Design of a PID controller

In 1965 A. Haalman proposed to tune PID controllers via the representation of the process to be controlled by a SOPDT model and the cancellation of the two process time constants by means of the two controller time constants [4]. This method is resumed here with the SOPDT model (2 – 2 but a design parameter μ is included in the controller gain, then allowing some adaptation of the control loop damping by a modification of the controller gain only. Setting

$$K_i = \frac{1}{\mu K D_2} \implies K_p = \frac{T_i}{\mu K D_2} \qquad (3-11)$$

where K_i and $K_p = K_i T_i$ are, respectively, the integral and proportional gains of the controller, the open–loop transfer function then becomes

$$P(s)C(s) = K K_i \frac{1}{s} e^{-D_2 s} = \frac{1}{\mu D_2 s} e^{-D_2 s}. \qquad (3-12)$$

In the Nyquist plane the plot of the open–loop transfer function (3 – 12) is a spiral which is starting along a vertical with abscissa $-1/\mu$, asymptotically from the zero frequency, and which is getting wound around the origin for increasing frequencies ; at all frequencies, the Nyquist plot is standing at the right-hand side of this vertical $-1/\mu$. Straightforward calculations then yield the ultimate frequency ω_u and the ultimate value μ_u bringing the closed–loop system to the stability limit

$$D_2\,\omega_u = \pi/2 \implies \omega_u = \pi/2D_2 \text{ and } \mu_u D_2\,\omega_u = 1 \implies \mu_u = 2/\pi \qquad (3-13)$$

Therefore, with a selected value of μ, the designer of the control system can determine the gain margin μ/μ_u, as also the cut-off frequency and the phase margin of the control loop

$$\mu D_2\,\omega_{co} = 1 \implies \omega_{co} = 1/\mu D_2 \text{ and } \Phi_M = \frac{\pi}{2} - D_2\,\omega_{co} = \frac{\pi}{2} - \frac{1}{\mu}\, rad\,, \qquad (3-14)$$

as shown by the next table.

Robustness with proposed controller settings

μ	1.3	1.5	2	3
Gain margin	2	2.4	3.1	4.7
Phase margin	46°	52°	61°	71°

Since there is a close relationship between the damping and the phase margin of the closed–loop system, selecting values of μ between 1.3 and 3 allows the designer to choose the types of closed–loop responses, from fast responses with a high overshoot ($1.3 < \mu < 2$) to over–damped slow responses ($2 < \mu < 3$). This choice can be achieved by varying the parameter μ, that is the controller gain only, without changing the controller time constants ; by the way, this is the natural reaction of control engineers and plant operators when they want to change on–line the behavior of the control system.

In [1]Åström-Hägglund have proposed a collection of transfer functions representative of typical industrial processes. Many simulation runs have been performed with various transfer functions of this collection. For all of them the shape of the open–loop Nyquist plot was similar to the spiral shape of the nominal transfer function (3 – 12), with the plot standing at the right-hand side of the vertical $-1/\mu$ or at least close to it at all frequencies. Therefore, the minimum distance from the actual Nyquist plot to the critical point $(-1,0)$ is greater than $1-1/\mu$, and the gain margin and the phase margin are greater than, respectively, μ and $\arccos 1/\mu$. Therefore, the proposed tuning method is not only simple and intuitive, but it also provides some flexibility in the choice of the closed–loop responses together with some guarantee on the control system robustness. This is illustrated by the simulation results presented in the next section.

4 Simulation results

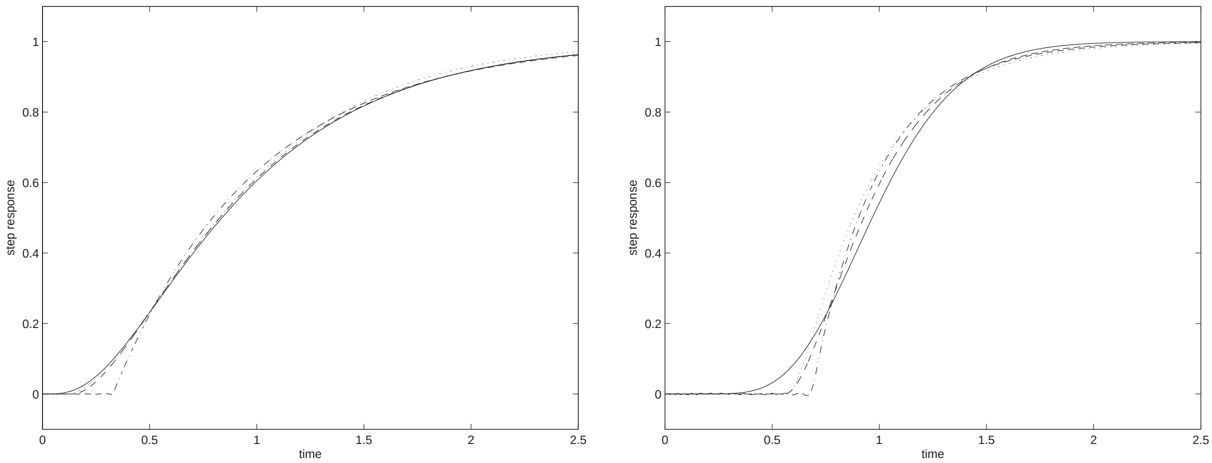
Simulation tests have been performed with the two following typical transfer functions :

$$P_1(s) = \frac{1}{(1+Ts)(1+\alpha Ts)(1+\alpha^2 Ts)(1+\alpha^3 Ts)}, \quad \alpha = 0.4 \qquad (4-15)$$

$$P_2(s) = \frac{1}{(1+Ts)^n}, \quad n = 10 \qquad (4-16)$$

with the same average residence time $T_{ar} = 1$ for the two transfer functions. The table below shows the parameter values of the FOPDT and SOPDT models for the two processes, and the responses of each process and its FODPT and SOPDT models are shown in Fig. 4 – 1.

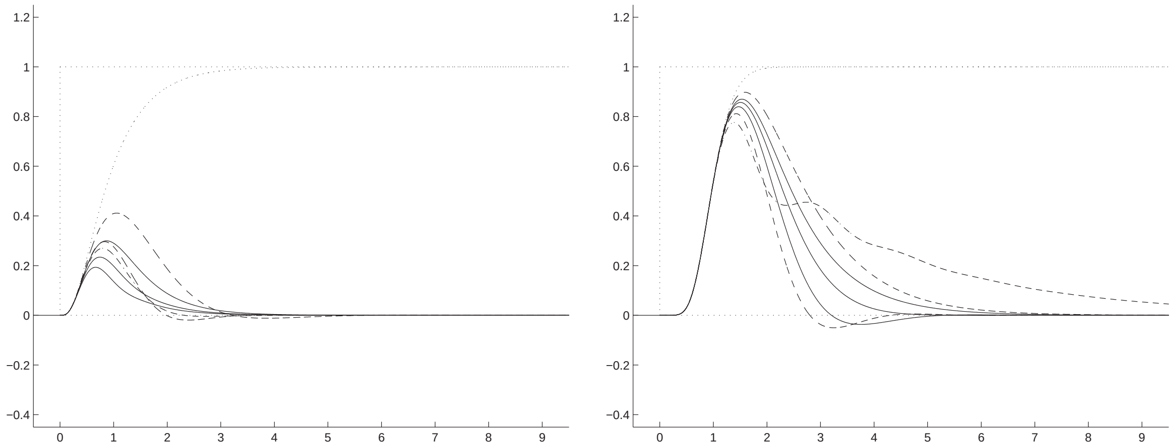
Process	FOPDT parameters			SOPDT parameters			
	T_a	D_a	τ	θ	T_1	T_2	D_2
$P_1(s)$	0.67	0.33	0.33	0.37	0.63	0.23	0.14
$P_2(s)$	0.32	0.68	0.68	0.75	0.25	0.19	0.56



Full lines — Process
Broken lines — Proposed SOPDT model
Dash–dotted lines — FOPDT model
Dotted lines — van der Grinten SOPDT model

Figure 4 – 1: Step responses of the process and its models. Left : $P_1(s)$, right : $P_2(s)$.

Clearly, the response of the process is better represented by the SOPDT model than by the FOPDT model, especially for the fist process whose normalized dead time is low ($\tau = 0.33$ for $P_1(s)$ versus 0.68 for $P_2(s)$). Obviously, the first process is also easier to control than the other one thanks to its low normalized dead time. The responses to a unit–step change of the process load (disturbance at the process input) are shown in Fig. 4 – 2 for these two processes and PID controllers tuned according as proposed here or with the well–known Ziegler–Nichols settings or with the Åström–Hägglund settings, the one for slow responses without overshoot, the other for faster responses with some overshoot. It turns out that the controller tuning based on the SOPDT model proposed here provides better responses with a wide flexibility in the choice of the response damping ; thanks to a proper choice of the design parameter, that is of the controller gain, it is possible to cover continuously the whole range between a slow over–damped response and a fast under–damped response, without changing the controller time constants unlike the Åström–Hägglund settings.



Full lines	Proposed controller settings
Broken lines	Åström–Hägglund settings
Dash–dotted lines	Ziegler–Nichols settings
Dotted lines	process reaction curve

Figure 4 – 2: PIDcontrol. Responses to a load unit–step change. Left : $P_1(s)$, right : $P_2(s)$.

5 Conclusions

With a view to the design and tuning of controllers a SOPDT model has been derived from the usual FOPDT characterization most often used for PID control of self–regulating processes. This modeling approach allows the use of tuning methods based on SOPDT models ; this provides some flexibility in the choice of the control loop behavior together with some guarantee on the stability and robustness of the latter. These features are proven by simple calculations and they are illustrated by simulation results. Actually, modeling is used here not for simulation purposes but in order to provide some guidelines for the design of controllers.

6 References

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Appendix A: Given a transfer function

$$G(s) = K\,\frac{1+n_1 s+n_2 s^2+\dots}{1+d_1 s+d_2 s^2+\dots}$$

the coefficients of its Taylor–McLaurin expansion

$$G(s) = K\,(1+g_1\,s+g_2\,s^2+\dots)$$

can be determined without any derivative calculation, just by multiplying the above expansion of $G(s)/K$ by the denominator of the rational fraction and identifying the coefficients of the same degree terms in this product and in the numerator of the latter. This provides the following set of linear equations :

$$\begin{aligned} n_1 &= g_1 + d_1 \\ n_2 &= g_2 + d_1 g_1 + d_2 \\ \ldots &= \ldots\ldots\ldots\ldots \end{aligned}$$

allowing a sequential determination of the coefficients $g_1, g_2, \ldots$ If the process transfer function includes non–rational factors, for example the exponential factor associated to a dead time, one has simply to include the Taylor-McLaurin expansions of these factors into the numerator or into the denominator of the rational fraction as appropriate. In particular, the transfer function

$$G(s) = K \frac{1 - T_0 s}{(1 + T_1 s)(1 + T_2 s) \ldots (1 + T_n s)} e^{-Ds}$$

can be written down in the form

$$G(s) = K \frac{1 - (T_0 + D)s + D^2 s^2/2 + \ldots}{1 + (T_1 + T_2 + \ldots + T_n)s + (T_1 T_2 + T_2 T_3 + \ldots + T_n T_1)s^2 + \ldots}$$

Then, straightforward calculations yield

$$\begin{aligned} -g_1 &= D + T_0 + T_1 + T_2 + \ldots + T_n = T_{ar} \\ g_2 &= \frac{1}{2} T_{ar}^2 + \frac{1}{2}(T_1^2 + T_2^2 + \ldots + T_n^2 - T_0^2). \end{aligned}$$

Appendix B: Thanks to the algorithm proposed in Appendix A it is possible to obtain the required characteristics of the process to be controlled from the step response of any stable control loop including this process and a PI controller with known parameters. Denoting by $P(s)$ and $C(s)$ the transfer functions of the process and of the PI controller, and by $G(s)$ and $Q(s)$ the closed–loop transfer functions relating, respectively, the process output variable y and the controller output variable u to the controller reference signal w, and using the following Taylor–McLaurin expansions of these transfer functions :

$$\begin{aligned} P(s) &= K(1 + p_1 s + p_2 s^2 + \ldots), \\ C(s) &= K_p \frac{1 + T_i s}{T_i s}, \\ G(s) &= \frac{P(s)C(s)}{1 + P(s)C(s)} = 1 + g_1 s + g_2 s^2 + \ldots, \\ Q(s) &= \frac{1}{K} \frac{C(s)}{1 + P(s)C(s)} = \frac{1}{K}(1 + q_1 s + q_2 s^2 + \ldots), \end{aligned}$$

straightforward calculations yield the following relations for the process characteristics :

$$\begin{aligned} K &= \frac{y(\infty}{u(\infty)} = \frac{w(\infty}{u(\infty)} = \frac{T_i}{-g_1 K_p} \\ T_{ar} &= -p_1 = q_1 - g_1 \\ T_a &= \sqrt{2(g_2 - q_2) + T_{ar}(T_{ar} + 2g_1)} \end{aligned}$$

where

$$g_1 = \int_0^\infty [1 - \frac{y(t)}{y(\infty)}]\,dt \quad g_2 = \int_0^\infty t\,[1 - \frac{y(t)}{y(\infty)}]\,dt$$

$$q_1 = \int_0^\infty [1 - \frac{u(t)}{u(\infty)}]\,dt \quad q_2 = \int_0^\infty t\,[1 - \frac{u(t)}{u(\infty)}]\,dt$$

where $y(t)$ and $u(t)$ are, respectively, the process and controller responses to a step of magnitude $w(\infty)$ in the controller reference variable.

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