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SNAPSHOT–BASED PARAMETRIC MODEL ORDER REDUCTION

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Abstract. Targeting large scale dynamical systems, the incorporation of manufacturing tolerances into Electronic Design Automation methods requires an extension of classical Model Order Reduction methods, such that the dependency of system matrices on certain geometry or material parameters is preserved within the reduced order model. Parametric Model Order Reduction methods are part of the current main research of many authors, but still have not reached a production ready stage. A major issue that will be addressed in this paper is the absence of an analytical relationship between parameter values and resulting system matrices in many real world applications. A second challenge is the preservation of passivity. Two examples from nanoscale IC design and microscale MEMS design will be given to demonstrate the effectivity of our approach.

1 Introduction

In modeling of micro mechanical electrical systems (MEMS), we use the finite element method (FEM) to obtain a spatial semi–discretization of the corresponding partial differential equations in form of linear time invariant second order descriptor systems

$$\left.\begin{array}{rcl} \mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{D}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t) & = & \mathbf{B}^{\text{in}}\mathbf{u}(t) \\ \mathbf{y}(t) & = & \mathbf{B}_1^{\text{out}}\mathbf{x}(t)+\mathbf{B}_2^{\text{out}}\dot{\mathbf{x}}(t)+\mathbf{F}\mathbf{u}(t) \end{array}\right\} \tag{1}$$

with system matrices $\mathbf{M},\mathbf{D},\mathbf{K}\in\mathbb{R}^{N\times N};\mathbf{B}^{\text{in}}\in\mathbb{R}^{N\times p};\mathbf{B}_1^{\text{out}},\mathbf{B}_2^{\text{out}}\in\mathbb{R}^{q\times N};\mathbf{F}\in\mathbb{R}^{q\times p}$.

Alternatively, first order descriptor systems, that is $\mathbf{M}\equiv\mathbf{0}$ and $\mathbf{B}_2^{\text{out}}\equiv\mathbf{0}$, can be obtained from the discretization of the heat equation, the discretization of Maxwell's equations for electro magnetic problems via partial element equivalent circuit method (PEEC, see [11]) and the formulation the circuit equations of RCL–networks via modified nodal analysis (MNA, see [12] and references therein) .

In models of micro- or nanoscale devices, the state space dimension N of (1) reaches magnitudes of $10^5\dots10^7$. Without Model Order Reduction (MOR), time domain simulations would be impossible, especially when the coupling of several large scale subsystems is required for system level simulation.

During the last decades, Krylov subspace projection based MOR methods proved as a practical way to reduce the state space dimension of large scale dynamical systems [1]. In a nutshell, these methods construct an orthonormal projection matrix $\mathbf{V}_n\in\mathbb{R}^{N\times n}$ to generate a reduced order system with state space dimension $n\ll N$, where the reduced system matrices are given as

$$\begin{array}{lll} \mathbf{M}_n:=\mathbf{V}_n^{\text{T}}\mathbf{M}\mathbf{V}_n, & \mathbf{D}_n:=\mathbf{V}_n^{\text{T}}\mathbf{D}\mathbf{V}_n, & \mathbf{K}_n:=\mathbf{V}_n^{\text{T}}\mathbf{K}\mathbf{V}_n, \\ \mathbf{B}_n^{\text{in}}:=\mathbf{V}_n^{\text{T}}\mathbf{B}^{\text{in}}, & \mathbf{B}_{1,n}^{\text{out}}:=\mathbf{B}_1^{\text{out}}\mathbf{V}_n, & \mathbf{B}_{2,n}^{\text{out}}:=\mathbf{B}_2^{\text{out}}\mathbf{V}_n. \end{array} \tag{2}$$

Note that the number of inputs p and outputs q is unchanged, allowing for a seamless replacement of the original system with the reduced system.

In our case, a rational Block–Arnoldi method is used in order to preserve the passivity of the system [2, 7]. Furthermore, the resulting projection matrix $\mathbf{V}_n$ yields a reduced system whose transfer function matches a certain amount of moments of the transfer function of the original system. This process is also known as implicit moment matching for multiple expansion points, where the so-called moments are defined as the Taylor coefficients of the transfer function in respect to the complex frequency variable and certain expansion points [1].

Confronted with a growing demand for tools incorporating manufacturing tolerances during simulation and design, Fraunhofer started a joint research project named CAROD (Computer–Aided Robust Design) [10]. One of its subgoals is the integration of Parametric Model Order Reduction (PMOR) methods into Electronic Design Automation (EDA) software. This should give a significant speedup for the computation of parameter sweeps needed for optimization tasks and sensitivity calculations.

2 Parametric Model Order Reduction

Our starting point is the following parameter dependent version of (1):

$$\left.\begin{array}{rcl} \mathbf{M}(\mu)\ddot{\mathbf{x}}(t)+\mathbf{D}(\mu)\dot{\mathbf{x}}(t)+\mathbf{K}(\mu)\mathbf{x}(t) & = & \mathbf{B}^{\text{in}}(\mu)\mathbf{u}(t) \\ \mathbf{y}(t) & = & \mathbf{B}_1^{\text{out}}(\mu)\mathbf{x}(t)+\mathbf{B}_2^{\text{out}}(\mu)\dot{\mathbf{x}}(t)+\mathbf{F}(\mu)\mathbf{u}(t) \end{array}\right\} \tag{3}$$

Each of the system matrices is given as a polynomial in the parameter vector $\mu := (\mu_1 \dots \mu_k)^{\mathrm{T}} \in \mathbb{R}^k$ with matrix valued coefficients:

$$\mathbf{M}(\mu) := \sum_{i_1,\dots,i_k} \mu_1^{i_1} \cdot \dots \cdot \mu_k^{i_k} \mathbf{M}_{i_1,\dots,i_k}, \qquad \mathbf{D}(\mu) := \sum_{i_1,\dots,i_k} \mu_1^{i_1} \cdot \dots \cdot \mu_k^{i_k} \mathbf{D}_{i_1,\dots,i_k}, \qquad \text{etc.} \tag{4}$$

As the formulation (4) of the parameter dependent system matrices can be seen as a linear combination of constant matrices, we obtain a reduced system similar to (3) by applying the projection step (2):

$$\mathbf{M}_n(\mu) := \mathbf{V}_n^{\mathrm{T}} \left(\sum_{i_1,\dots,i_k} \mu_1^{i_1} \cdot \dots \cdot \mu_k^{i_k} \mathbf{M}_{i_1,\dots,i_k} \right) \mathbf{V}_n \tag{5}$$

$$= \sum_{i_1,\dots,i_k} \mu_1^{i_1} \cdot \dots \cdot \mu_k^{i_k} \, \mathbf{V}_n^{\mathrm{T}} \mathbf{M}_{i_1,\dots,i_k} \mathbf{V}_n \tag{6}$$

$$=: \sum_{i_1,\dots,i_k} \mu_1^{i_1} \cdot \dots \cdot \mu_k^{i_k} \mathbf{M}_{i_1,\dots,i_k;n} \tag{7}$$

The remaining reduced system matrices $\mathbf{D}_n(\mu), \mathbf{K}_n(\mu), \dots$ are constructed analogously.

For the construction of a suitable projection matrix $\mathbf{V}_n$, different authors have proposed an extension of the classical moment matching approach outlined in the previous section, where not only frequency moments are matched, but moments in respect to the parameters also [6, 4, 13, 5, 9].

But in order to make these methods applicable for our applications, we had to implement the snapshot generation step outlined in the following section.

3 Snapshot Generation

In applications where model generation is done with third party software given as a black box, the analytic relationship between parameters and system matrices necessary for a problem formulation like (3) is not known explicitly. But at least these tools allow for a generation of a series of *snapshots* of the full order model at fixed parameter values. In the context of the CAROD project, we investigated the suitability of matrix–valued finite differences for the generation of parameterized system matrices in the desired polynomial form (4) [8].

For the sake of clarity, we restrict ourselves to single-parameter systems in this paper. Given a parameter dependent matrix $\mathbf{A}(\mu)$, a nominal parameter value $\mu_0 \in \mathbb{R}$ and a step size $h > 0$, the corresponding *first order central difference quotient* is defined as

$$\delta_h^1 \mathbf{A}(\mu_0) := \frac{\mathbf{A}(\mu_0 + h/2) - \mathbf{A}(\mu_0 - h/2)}{h} \tag{8}$$

With three snapshots, i.e. given values of $\mathbf{A}(\mu)$ at μ_0, $\mu_0 - h/2$ and $\mu_0 + h/2$, we are now able to construct an approximation $\Delta^1 \mathbf{A}(\mu)$ to a truncated Taylor series of $\mathbf{A}(\mu)$:

$$\Delta^1 \mathbf{A}(\mu) := \mathbf{A}(\mu_0) + (\mu - \mu_0) \; \delta_h^1 \mathbf{A}(\mu_0) \tag{9}$$

This approach can be extended by composition of the finite difference operator δ_h^1, yielding the following second order approximation of $\mathbf{A}(\mu)$:

$$\delta_h^2 \mathbf{A}(\mu_0) := \delta_h^1 \left(\delta_h^1 \mathbf{A}(\mu_0) \right) = \frac{\mathbf{A}(\mu_0 + h) - 2\mathbf{A}(\mu_0) + \mathbf{A}(\mu_0 - h)}{h^2} \tag{10}$$

$$\Delta^2 \mathbf{A}(\mu) := \mathbf{A}(\mu_0) + (\mu - \mu_0) \; \delta_h^1 \mathbf{A}(\mu_0) + \frac{(\mu - \mu_0)^2}{2} \, \delta_h^2 \mathbf{A}(\mu_0) \tag{11}$$

By applying these finite difference operators to snapshots of the system matrices of a parameter dependent model, we are now able to generate a parameter dependent descriptor system (3) that can be reduced using the projection based PMOR approach outlined in the previous section.

4 Passivity Preservation

When several stable subsystems are coupled, the total system is not stable in general. But if the subsystems are passive, stability of the total system is guaranteed. This is why the preservation of passivity is important for us.

In our applications, the system matrices $\mathbf{M}, \mathbf{D}, \mathbf{K}$ are symmetric positive semidefinite, $\mathbf{F} \equiv \mathbf{0}$ and either $\mathbf{B}^{\text{in}} = (\mathbf{B}_1^{\text{out}})^{\mathrm{T}}$ with $\mathbf{B}_2^{\text{out}} \equiv \mathbf{0}$ or $\mathbf{B}^{\text{in}} = (\mathbf{B}_2^{\text{out}})^{\mathrm{T}}$ with $\mathbf{B}_1^{\text{out}} \equiv \mathbf{0}$. It can be shown, that these properties are sufficient for passivity of the non–parametric system (1), see e.g. [12]. Thus, the reduced system matrices obtained from (2) are symmetric positive semidefinite by construction and passivity is preserved.

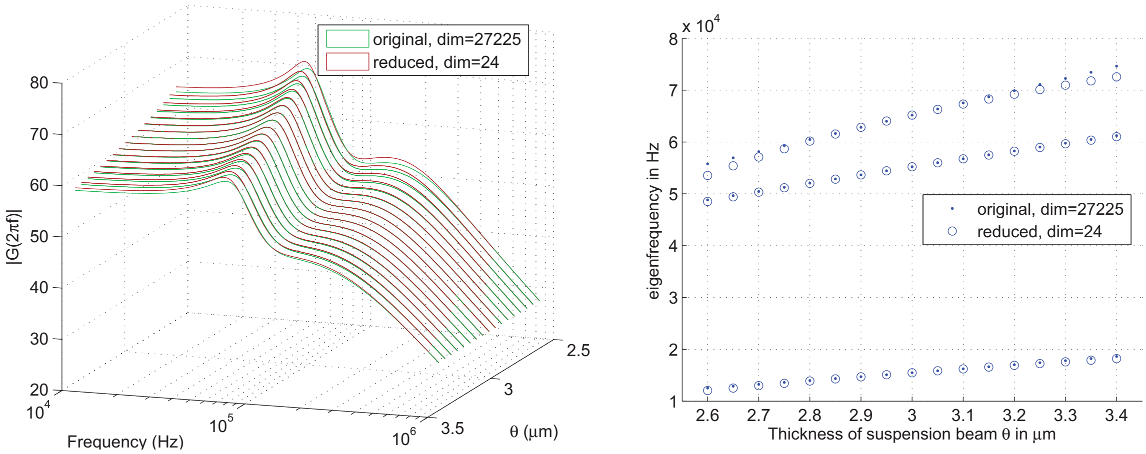
Consequently, if the parameter dependent system (3) is passive for a certain parameter range $[\mu_{\min}, \mu_{\max}] \subset \mathbb{R}$, the reduced parameter dependent system obtained via projection (5) is passive as well. However, for our snapshot based approach this statement does not hold in general, even if the snapshots themselves are passive.

But there is hope if the parameter dependence is steady and the disturbance of the parameter dependent system matrices is small. It is well known that positive semidefiniteness of an arbitary real symmetric matrix $\mathbf{A}$ is equivalent to its eigenvalues being non-negative. Provided that the underlying eigenvalue problem is well conditioned, positive semidefiniteness and therefore passivity will be preserved.

5 Numerical Examples

Our first example is a micro mechanical acceleration sensor modeled with ANSYS®, where the thickness θ of the suspension beam of the seismic mass has been varied between $2.6\ldots3.4\mu m$. Three snapshots where created for the generation of a parameterized model via first order central differences with nominal value $\theta_0 = 3\mu m$ and step size $h = 0.01\mu m$. The state space dimension of the full oder model has been reduced from 27225 to 24.

Figure 1 is a comparison of the full vs. the reduced order model at different parameter values where 1(a) shows the frequency response of the applied force versus the resulting displacement of the sensor's seismic mass and 1(b) is a plot of the first three dominant eigenfrequencies. For the considered frequency range of $0\ldots10^6$Hz, the relative error of the reduced transfer function stays below 10^{-2} for $\theta \in [2.8, 3.1]$ and below 10^{-1} for $\theta \in [2.6, 3.4]$.



(a) Frequency response of acceleration sensor

(b) Dominant eigenfrequencies of acceleration sensor

Figure 1: Comparison of original and reduced model of acceleration sensor

As can be seen, the transfer characteristics and the shift of dominant eigenfrequencies are well approximated over a wide parameter range and passivity is preserved for $\theta \in [2.6, 3.4]$.

Our second example is a model for the electromagnetic interaction of three parallel nano–scale transmission lines. The lines have a length of 8mm and the width ω of line 1 has been selected as a parameter to be varied between $0.12\ldots0.32\mu m$. Five equivalent RCL–networks have generated using the PEEC method and the characteristic line parameters calculated with Simlab PCBMod® for a 2D profile of the transmission lines. The resulting parametric first order descriptor system was created with second order central differences. It has 6 inputs, 6 outputs and its state space dimension has then been reduced from 1203 to 90.

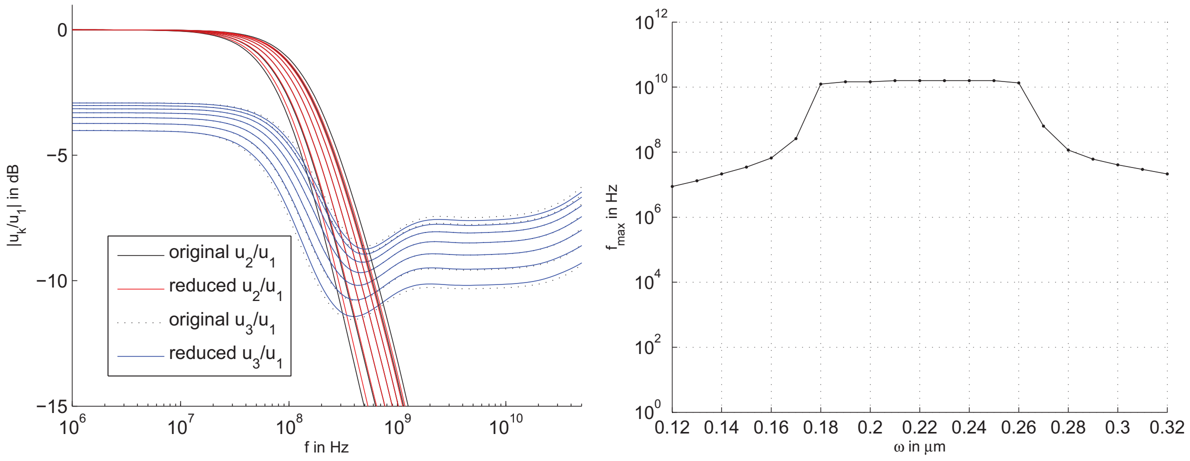
For different values of ω, figure 2(a) shows a comparison of the frequency response for input/output pair (1,2) related to the damping on line 1 and the pair (1,3) related to the crosstalk between line 1 and 2. In figure 2(a) we computed the maximum frequency for each $\omega = 0.12, 0.13, \ldots, 0.32\mu m$ where the relative error of the frequency response is below 1e-2 for all input/output pairs.

The plots show that in the neighborhood of the nominal value $\omega_0 = 0.22\mu m$, there is a good approximation of the parameter dependency up to 10GHz. Passivity is preserved for the whole parameter range.

6 Conclusions

We have shown that it is possible to obtain accurate and passive parameter dependent reduced order models from snapshots of full order systems at fixed parameter values with the help of finite differences.

Passivity has been checked a–posteriori in our examples. But an algorithm that automatically adjusts the set of expansion points and the number of Arnoldi–iterations needed for passivity preservation and a certain accuracy in a given parameter range would make parametric model order reduction more user friendly.



(a) Frequency response for damping on line 1 and crosstalk between line 1 and 2 for $\omega = 0.13, 0.16, 0.19, 0.22, 0.25, 0.28, 0.31\mu m$

(b) Maximum frequency for which relative error of frequency response for all input/output pairs is below 1e-2

Figure 2: Comparison of original and reduced model of transmission line model

For the sake of clarity we restricted ourselves to one–parameter systems in this paper. While the finite difference approach for multi–parameter systems is a straight forward extension to multivariate Taylor series, second and higher order finite differences will introduce cross dependencies that make model generation very resource intense. For example, five parameters would lead to 47 snapshots [8], resulting in a data set of 1.5 Gigabytes for the acceleration sensor. Regarding future research, a possible tool to reduce the number of snapshots could be an approach based on Sparse Grids [3].

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