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Modelling of Hierarchical Manpower Systems and Determination of Control Strategies

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Abstract. Present paper describes modelling of hierarchical manpower systems by two different methodologies: a) system dynamics and b) discrete state space. Two various approaches contribute to the understanding of manpower system structure which is similar to the supply chain structure in manufacturing processes. Developed discrete state space model is applied for the determination of proper control strategies which should drive the system from initial states to goal states by prescribed trajectories and parameter limitations. Application of finite automaton enabled selection of acceptable strategies.

While the manpower planning problems have been extensively studied [8; 5; 16; 2; 13; 9; 12; 1; 10; 14] both in modelling and optimization there are practically no examples where the problem would consider time–variant boundaries with time–variant goal. This is somehow to be expected since the mathematical optimum in such cases is hard to provide. However, for the practical application the problem with time–variant boundaries and time–variant goal should be addressed. In previous research [8] it has been indicated, that the bare numerical approaches give little insight into the structure of gained solution. One major factor that impacts low acceptance of optimization methods in manpower planning is low undestanding of the system structure. In present paper, the approach with System Dynamics (SD) will be presented [4; 15], where the users get solid understanding of addressed system's structure. The relation of the SD model to the discrete time space will be presented where SD model contributes to the understanding of the system structure. Since one does not deal with technical optimization process, the understanding of each particular parameter as well as the model structure by user is of prime importance. One of the important contributions of the present paper is inclusion of the rules which determine acceptable strategies based on the time response of strategy. These rules could be qualitatively expressed by the users and should be considered at the provision of acceptable strategy. In our case the methods of Finite Automata (FA) were applied at the definition of criteria function.

1 Hierarchical Manpower Model Description

System dynamics [4] model shown in Fig. 1 represents transitions of manpower between individual ranks. The structure shows that a system with only one input, internal recruitment u is examined in a way where individual rank members are trained within the system rather. This would certainly hold for several top–most ranks of particular organization considered.

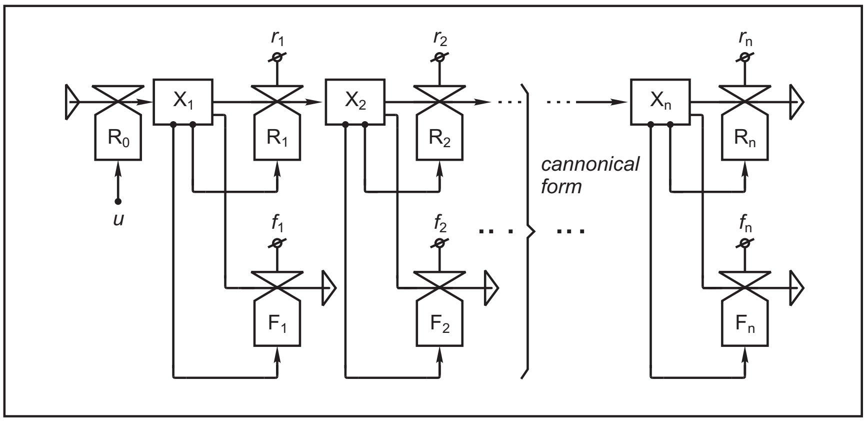

Figure 1: Structure of system dynamics model of rank transitions in the canonical form. Shown structure represents delay chain of first order delay elements.

The general form of the System Dynamics model definition in continuous time is:

$$x(t) = \int_{t_0}^{t} [R_{in}(t) - R_{out}(t)]\mathrm{d}t + x(t_0) \quad (1)$$

$$\frac{\mathrm{d}(x)}{\mathrm{d}t} = \text{x net change} = R_{in}(t) - R_{out}(t) \quad (2)$$

Stock variables $x_1, x_2, \dots, x_n$ (levels) describe the state of the system, in our case the number of members in rank $x_1, x_2, \dots, x_n$, while flow variables R and F (both are rates) represent the rates of change in stocks, such as transition rates R and fluctuation F.

Formulation of the System shown in Fig. 1 in discrete space where $\Delta t = 1$, takes a form of:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \end{cases} \tag{3}$$

Where matrix $\mathbf{A}$ from Eq. 3 is:

$$\mathbf{A} = \begin{bmatrix} 1 - r_1(k) - f_1(k) & 0 & 0 & \dots \\ r_1(k) & 1 - r_2(k) - f_2(k) & 0 & \dots \\ 0 & r_2(k) & 1 - r_3(k) - f_3(k) & \dots \\ 0 & 0 & r_3(k) & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{4}$$

The input $u(k)$ to the considered system is provided to x_1 such that

$$x_1(k+1) = [1 - r_1(k) - f_1(k)]\, x_1(k) + u(k) \tag{5}$$

Therefore $u(k)$ represents the new recruitment to the rank x_1 and in fact this is the only input to the considered system. The system dynamics model depends on input parameters (recruitment, promotions, and fluctuations).

2 Definition of strategy according to target functions

Trajectory function, which describes the way in which particular goal value should be attained is defined by the following rule:

$$g(t) = x_T + \frac{(x_0 - x_k)\,(k - t)}{k}\, e^{-pt} \tag{6}$$

where x_0, x_T represent initial and terminal state, k is simulation time and $p \in [0, \infty]$ importance factor. The set of curves which attain goal value in prescribed final time is dependant on the value of importance factor, which determine how fast the target value is achieved.

We want to minimize the distance to the target function defined by Eq. (6):

$$J = \mathscr{A}\Big(\min_{u,r,f} \Big[\sum_{k=1}^{t_k} \mathbf{w}\Big(\mathbf{z}(k) - \mathbf{x}(k)\Big)^2 \Big] \Big) \tag{7}$$

subject to:

$$\begin{array}{ccccc} \mathbf{u}_{min}(k) & \leq & \mathbf{u}(k) & \leq & \mathbf{u}_{max}(k) \\ \mathbf{r}_{min}(k) & \leq & \mathbf{r}(k) & \leq & \mathbf{r}_{max}(k) \\ \mathbf{f}_{min}(k) & \leq & \mathbf{f}(k) & \leq & \mathbf{f}_{max}(k) \end{array} \tag{8}$$

where $\mathscr{A}$ represents applied automaton which is used to classify acceptable strategies by elimination oscillations in rates. The set of states is
$S = \{S_0, S_1, S_2, S_3, S_4, S_5\}$, the comparison alphabet is $A = \{l, e, g\}$, the initial state is $i = \{S_0\}$ and the set of terminal states is $T = \{S_0, S_1, S_2, S_3, S_4, S_5\}$. The transition function of A_2, $\delta : S \times A \rightarrow S$ is defined by the following transition table:

	l	e	g
$\leftrightarrow S_0$	S_2	S_0	S_1
$\leftarrow S_1$	S_3	S_1	S_1
$\leftarrow S_2$	S_2	S_2	S_4
$\leftarrow S_3$	S_3	S_3	S_5
$\leftarrow S_4$	S_5	S_4	S_4
$\leftarrow S_5$	S_5	S_5	S_5

(9)

Where $\mathbf{w}$ is time–invariant vector of weights reflecting the importance of holding deviations for rank n as small as possible, $\mathbf{z}(k)$ represents goal trajectory of the system defined by Eq. 6, $\mathbf{u}_{min}(k)$ and $\mathbf{u}_{max}(k)$ vectors of lower and upper boundary for recruitment in rank x_1 respectively, $\mathbf{r}_{min}(k)$ and $\mathbf{r}_{max}(k)$ vectors of lower and upper boundary for transitions between ranks x respectively, $\mathbf{f}_{min}(k)$ and $\mathbf{f}_{max}(k)$ vectors of lower and upper boundary for fluctuations in rank x respectively. Note that all boundaries are time dependant which increases the complexity of addressed optimization problem.

Application of Pattern search numerical optimization with GPS Positive Basis 2N Pool method (MATLAB implementation) the proposed system enables efficient determination of proper strategies according to provided target trajectory functions [8].

Dynamic Programming Approach with Three States Example Let us consider the following dynamic programming [11] optimal strategy formulation:

$$\phi(k+1) = \min_{u,r,f} H\Big[\mathbf{x}(k), \mathbf{A}(k), \mathbf{B}(k), \mathbf{u}(k)\Big] \quad (10)$$

$$x(k+1) = G\Big(\phi(k)\Big) \quad (11)$$

$$\phi(0) = 0 \quad (12)$$

with

$$\begin{aligned} u(k) &\in U(k) \\ r(k) &\in R(k) \\ f(k) &\in F(k) \end{aligned} \quad (13)$$

where $\phi(k)$ is optimal set of parameters u, r, f at time k and H performance function, in our case, quadratic performance index. Initial condition for ϕ annotates, that at time $t = 0$ no optimal solution for ϕ exists meaning that at the stated initial conditions for $\mathbf{x}$ there is no strategy that would improve H; possible disproportion at time $t = 0$ could not be changed in any way. G is function declared by the state space definition of the system in Eq. 3. At each time-step one has to solve one optimization problem according to defined function.

In order to clearly present the approach by dynamic programming and possible consequence of application of quadratic performance index with boundaries let us consider the following Three States Example:

Consider three ranks, x_1, x_2, x_3 with initial conditions $x_1(0) = 100$, $x_2(0) = 50$, $x_3(0) = 80$, target values $z_1(k) = 70$, $z_2(k) = 40$ and $z_3(k) = 100$ with boundaries

$$\begin{aligned} 0.00000 \le\ & \mathbf{u}(k) \le 50.00000 \\ 0.06000 \le\ & \mathbf{r}(k) \le 0.20000 \\ 0.00010 \le\ & \mathbf{f}(k) \le 0.04000 \end{aligned} \quad (14)$$

At each time-step the following optimization problem is solved for $\phi(k+1)$:

$$\begin{aligned} \min_{u,r,f} \Big(& [z_1(k) - x_1(k) + f_1(k)x_1(k) - u(k) + x_1(k)r_1(k)]^2 + \\ & [z_2(k) - x_2(k) + f_2(k)x_2(k) - x_1(k)r_1(k) + x_2(k)r_2(k)]^2 + \\ & [z_3(k) - x_3(k) + f_3(k)x_3(k) - x_2(k)r_2(k) + x_3(k)r_3(k)]^2 \Big) \end{aligned}$$

Results obtained by the dynamic programming approach exercise oscillatory beharior which does not satisfy acceptable strategy criteria where oscillations in rates should be avoided. Important conclusion with regard to gained results is, that optimal strategy, which would consider only target values for ranks within prescribed boundaries of the parameters could provide undesired oscillatory solution. It has to be noted, that if the only goal would be, that the desired number of man in particular rank should be achieved as fast as possible this would be an optimal solution. One could argue, that the criteria function stated by Eq. (12) should incorporate the terms with regard to $\mathbf{u}$ and other rate elements. However, even incorporating mentioned terms in criteria function would not necessarily provide non–oscillatory solutions since the weight put on particular term determines what is more important, whether: a) distance from the target value or b) oscillation in the strategy rate elements. Put in other words, what is more important, to achieve target values as fast as possible or the path to get there? There are many approaches to define optimal solutions in hierarchical system, [8; 13] however, optimal solution could hardly be provided especially if all the of the parameters in state space as well as parameter boundaries are time-variant as in our case and basically in any other similar real–world case.

3 Conclusion

Successful application of sophisticated optimization approaches to the manpower planning depends on the user understanding of the process considered. In this regard SD has been identified as proper approach to modelling hierarchical manpower system. Systemic view to the developed system provided an understanding of the problem addressed to the Decision Group. Since the problem addressed is much larger as regularly expected user interface should be carefully designed in order to provide easy definition of addressed complex optimization problem. General observation at the literature review showed, that there is no single method, that would provide optimal solution of the described problem. One reason is in weights that could be arbitrarily put to particular part of the optimization problem. Initially it would seem reasonable to define the control problem only as minimization of the distance to the target function with consideration of parameter boundaries. Here the rationale is, that target trajectory should be reached with no important costs when rate elements are within prescribed boundaries. This kind of problem formulation would actually yield the optimal solution if one would like to achieve target values in shortest possible time. Important finding is however, that statement of the problem, where only minimization of the distance to the target function by considering boundaries is not sufficient, resulting in possible undesired oscillatory solutions. In order to complete the definition of the control problem, the acceptable strategies were described and FA were developed accordingly. The differences between two differently stated control problems were shown on examples. Application of FA provided proper results where gained strategies did not show undesired oscillation patterns. Certainly, there is a cost that is paid for providing the proper shape of the strategy which was shown by different values of quadratic performance index meaning, that more time is needed to achieve desired goal. Developed approach could be transferred to other similar hierarchical systems such as supply chains [6] or chain production systems [7]. As the further research, the algorithms above optimization algorithm such as GA and PS should be considered, which would automate the tuning of the algorithm as well as other computational approaches [3].

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