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CONVECTION IN A VOLCANIC CONDUIT

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Abstract. A common feature of basaltic volcanoes is that they can exhibit periods of persistent degassing, which can last from years to millennia (e.g. Stromboli, Izu-Oshima). During these degassing periods there is a constant flux of gas released to the atmosphere with relatively negligible volumes of magma being discharged. Some mechanism must be at work within the magmatic plumbing system which transports a steady supply of volcanic gases from deep below the earth's surface to the volcanic vent and, in particular, sufficient heat must also be transported to offset losses to the surrounding rock and prevent the system from closing. A deep magma chamber is the source of both gas and heat and we study a model whereby convection in the volcanic conduit keeps the system open and active with a continuous supply of fresh hot gas-rich magma from the underlying chamber.

1 Introduction

When a volatile-rich magma ascends from a magma chamber towards the earth's surface, a decrease in hydrostatic pressure causes gas exsolution and bubble formation. The bubbly mixture will then rise buoyantly to the surface and these gases will be released into the atmosphere. The degassed magma will now be more dense than the nondegassed magma and as a result will descend back to the chamber along the conduit walls and displace the gas-rich magma there. This creates a convective circulation within the conduit driven by the density difference between the degassed and nondegassed magmas. This proposed mechanism of continuous passive degassing has been considered in many studies in the literature, see for example [2], [3] and [4]. A simple model for basaltic magmas consists of Poiseuille flow in a concentric double walled pipe, where nondegassed magma ascends in the center and heavier degassed magma descends in the outer annulus and the flow is driven by the density difference between the ascending and descending magmas. These conduit convection models are simplistic and ignore the effects of bubbles, simply assuming a change in density when gases are released at the top of the conduit. Bubbles will cause buoyancy effects to increase as the magma ascends and the resultant density difference between degassed and nondegassed magma is in actuality a function of height above the chamber. Increased buoyancy, in turn, leads to an increased magma ascent velocity. A particularly important consequence of conduit convection models is the transport of heat from the chamber to the conduit which can prevent the magma from cooling by offsetting the heat loss to the surrounding rock en route to the surface. It was shown in [3] that the temperature decrease in the magma due to conductive heat loss to the conduit walls is negligible due to the continued supply of fresh hot magma from the chamber. Expanding the idea of these conduit convection models, we construct a comprehensive two-phase flow theory to examine the process of convective overturn in a conduit.

2 A Two-Fluid Model for Flow in a Conduit

We consider a two-dimensional model for the gas-liquid mixture in the conduit. For simplicity, we assume the presence of a single gas species, which we take to be water vapor. Equations for the conservation of mass and momentum of the homogeneous (single velocity) mixture in the conduit are

$$\rho_t + \nabla.(\rho \mathbf{u}) = 0, \tag{1}$$

$$\rho(\mathbf{u}_t + \mathbf{u}.\nabla\mathbf{u}) = -\nabla p - g\rho\mathbf{k} + \eta\nabla^2\mathbf{u}, \tag{2}$$

where we assume a single pressure such that $p_g = p_l = p$, $\mathbf{k}$ is the unit vector in the vertical direction and η is the viscosity of the mixture. The density of the gas-liquid mixture is given by

$$\rho = \alpha\rho_g + (1-\alpha)\rho_l, \tag{3}$$

where ρ_g and ρ_l are the gas and liquid densities respectively and α is the volume fraction of gas. We assume an ideal gas such that

$$\rho_g = \frac{Mp}{RT}, \tag{4}$$

where M and R are the molar mass and universal gas constant for H_2O respectively. We adopt a Boussinesq approach and define the liquid density as a function of the temperature by

$$\rho_l = \bar{\rho_l}[1 - \beta(T - \bar{T})], \tag{5}$$

where $\bar{\rho_l}$ is the density of the melt at temperature $\bar{T}$ and β is the coefficient of thermal expansion. Now, in addition to the gas phase, H_2O can be present dissolved in the liquid phase. If c denotes the mass fraction of dissolved water then conservation of the total mass of H_2O (both in gas form and dissolved in the melt) yields a third conservation equation

$$\frac{D}{Dt}[\alpha\rho_g + c(1-\alpha)\rho_l] = -[\alpha\rho_g + c(1-\alpha)\rho_l]\nabla.\mathbf{u}, \tag{6}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}.\nabla$. It can be easily shown, using (1), that the total mass of the silicate melt $(1-c)(1-\alpha)\rho_l$ is automatically conserved. Deep below the earth's surface pressures are sufficiently high that all gases are dissolved in the liquid melt (i.e. $\alpha = 0$) but as the magma ascends towards the earth's surface and the hydrostatic pressure decreases the magma becomes supersaturated in volatiles and bubbles will start to nucleate, so that c will decrease from its initial value $c|_{\alpha=0} = c_0$. We assume that an equilibrium exists between the amount of H_2O dissolved in the melt and the surrounding pressure which satisfies the experimental relationship

$$c = K\sqrt{p}, \tag{7}$$

where K is a solubility constant depending on magma composition. Following (5) the density of the liquid melt varies with temperature, however, at a given temperature the density $\bar{\rho_l}$ will depend on the quantity of dissolved gases via

$$\bar{\rho_l} = \frac{\rho_w\rho_m}{\rho_w - c(\rho_w - \rho_m)}, \tag{8}$$

where ρ_m is the density of the silicate melt and ρ_w is the density of water. Finally, conservation of energy is given by

$$\rho\frac{Dh}{Dt} - \frac{Dp}{Dt} = k_T\nabla^2 T, \tag{9}$$

where the mixture enthalpy is

$$\rho h = \alpha\rho_g h_g + (1-\alpha)\rho_l h_l, \tag{10}$$

and the individual phase enthalpies are given by

$$h_g = \bar{h}_g + c_{p_g}(T - \bar{T}), \tag{11}$$

$$h_l = \bar{h}_l + c_{p_l}(T - \bar{T}), \tag{12}$$

such that $h_g = \bar{h}_g$ and $h_l = \bar{h}_l$ when $T = \bar{T}$, c_{p_g} and c_{p_l} are the gas and liquid specific heats at constant pressure and k_T is the thermal conductivity of the mixture. Equations (1)-(12) give us twelve equations for the twelve unknowns $\mathbf{u}$, ρ, p, α, ρ_g, ρ_l, T, c, $\bar{\rho_l}$, h, h_g and h_l.

2.1 A Simplified Model

The mass fraction of water dissolved in the magma is typically small ($c_0 \approx 0.05$) and if we also assume that density variations are primarily driven by gas exsolution and not thermal effects we can write the mixture density as

$$\rho \approx \rho_m - \alpha\Delta\rho,$$

where $\Delta\rho = \rho_m - \rho_g$. Following a standard Boussinesq-type approach, we further assume that buoyancy effects due to gas exsolution will only be important in the gravity term of the momentum equation and our model can be concatenated into four equations for the four unknowns $\mathbf{u}$, p, T and α:

$$\nabla.\mathbf{u} = 0,$$

$$\rho_m(\mathbf{u}_t + \mathbf{u}.\nabla\mathbf{u}) = -\nabla(p + g\rho_m z) + \alpha\Delta\rho g\mathbf{k} + \eta\nabla^2\mathbf{u},$$

$$T(\alpha, p) = \frac{\alpha M p}{R\rho_m(c_0 - K\sqrt{p})},$$

$$\rho_g L\frac{D\alpha}{Dt} + \alpha(1-\alpha)L\frac{D\rho_g}{Dt} + [\alpha\rho_g c_{pg} + (1-\alpha)\rho_m c_{pl}]\frac{DT}{Dt} = \frac{Dp}{Dt} + k_T\nabla^2 T,$$

where L is the latent heat of exsolution. Dimensionless variables are defined by

$$\nabla = \frac{1}{d}\nabla', \qquad \mathbf{u} = \frac{\kappa_l}{d}\mathbf{u}', \qquad t = \frac{d^2}{\kappa_l}t', \qquad z = dz', \qquad \rho_g = \frac{M\rho_m g l}{RT_c}\rho_g',$$

$$T = T_a + \Delta T\theta, \qquad p = p_a + \rho_m g(l-z) + \frac{\eta\kappa_l}{d^2}P$$

where d and l are the radius and length of the conduit respectively, T_a and T_c are the surface and chamber temperatures such that $\Delta T = T_c - T_a$, $\kappa_l = \frac{k_T}{\rho_m c_{pl}}$ is the thermal diffusivity of the mixture and p_a is atmospheric pressure. Neglecting terms that are relatively small, and dropping primes, yields the simplified model

$$\nabla.\mathbf{u} = 0, \tag{13}$$

$$\frac{1}{\sigma}\frac{D\mathbf{u}}{Dt} = -\nabla P + \alpha R\mathbf{k} + \nabla^2\mathbf{u}, \tag{14}$$

$$\frac{D\theta}{Dt} = \nabla^2\theta, \tag{15}$$

where σ and R are the Prandtl and Rayleigh numbers respectively. The void fraction and dimensionless temperature are related by

$$\alpha = \frac{[1-\lambda(1-\theta)][1-\mu\sqrt{1-\varepsilon z}]}{\Gamma(1-\varepsilon z)}, \tag{16}$$

where $\varepsilon = \frac{d}{l}$, $\lambda = \frac{\Delta T}{T_c}$, $\mu = \frac{K\sqrt{\rho_m g l}}{c_0}$ and $\Gamma = \frac{Mgl}{c_0 R T_c}$. The model has thus been reduced to three equations for $\mathbf{u}$, P and θ. Equations (13)-(15) is a standard thermal convection model with the void fraction replacing the temperature as the driving force. However, we can use (16) to rewrite the momentum equation in the more familiar form

$$\frac{1}{\sigma}\frac{D\mathbf{u}}{Dt} = -\nabla\tilde{P} + \theta\tilde{R}\mathbf{k} + \nabla^2\mathbf{u}, \tag{17}$$

where $\tilde{P}$ and $\tilde{R}$ are a modified pressure and Rayleigh number.

3 Velocity Profile

The problem is symmetric about the centerline of the conduit, denoted by $x = 0$, so it is sufficient to analyze the region between the centerline and the conduit wall located at $x = 1$, Figure 1. We assume that the upwelling ($w > 0$) region is contained in $0 \leq x < s$ and the downwelling ($w < 0$) region in $s < x < 1$ and s can be determined by conserving the volume of the mixture. We require $w = 0$ at the interface between the two regions, i.e. $w(s) = 0$. We further assume that the ascending gas-rich magma being transported from the chamber is at the chamber temperature $\theta = 1$ and the descending degassed magma is at the surface temperature $\theta = 0$, with a thermal boundary layer separating the two regions. To ensure symmetry across the centerline we set $\frac{\partial w}{\partial x} = 0$ at $x = 0$. The no-slip condition at the conduit wall $x = 1$ implies $w = 0$ there. We analyze the basic state when the system has reached a steady state and $\mathbf{u} = (0, w(x))$. Mass conservation (13) is thus automatically satisfied. If follows from (17) that $\tilde{P} = \tilde{P}(z)$ and

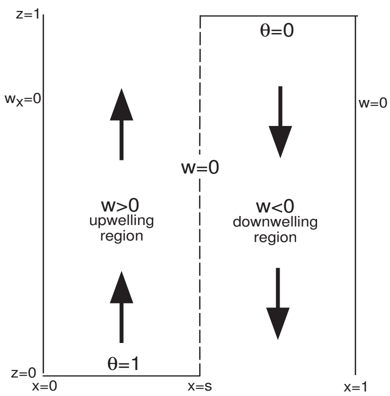

Figure 1: Conduit schematic

$$\frac{d^2w}{dx^2} = \frac{d\tilde{P}}{dz} - \tilde{R}\theta.$$

This can be solved in both the upwelling($\theta = 1$) and downwelling ($\theta = 0$) regions to obtain

$$w = \begin{cases} \frac{1}{2}(x^2 - s^2)(\tilde{P}_z - \tilde{R}) & \text{for } 0 \leq x \leq s \\ \frac{1}{2}\tilde{P}_z[x^2 - x(1+s) + s] & \text{for } s \leq x \leq 1 \,. \end{cases}$$

Now to ensure that the derivative of w is continuous across the interface $x = s$ we require

$$\tilde{P}_z = \frac{2s\tilde{R}}{1+s},$$

and this yields $\frac{dw}{dx}|_{x=s} = -\frac{s(1-s)}{(1+s)}\tilde{R} < 0$ as expected. The position of the upwelling-downwelling interface can be easily obtained by conserving the volume of the mixture via

$$\int_{x=0}^{x=1} w\,dx = 0.$$

4 Thermal Boundary Layer

The next step is to consider the region around $x = s$ where the temperature undergoes a rapid change from $\theta = 1$ to $\theta = 0$. We make the standard boundary layer assumption that heat diffusion in the lateral direction is more rapid that that in the vertical direction and rescale $z = \frac{1}{\varepsilon}Z$, where $\varepsilon \ll 1$. The heat equation (15) now becomes

$$\varepsilon w \frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial x^2},$$

and $w \sim \tilde{R} \gg 1$. In the boundary layer region we define $x = s + \delta X$, where δ is the width of the layer, and the matching boundary conditions

$$\theta \to 0 \quad \text{as} \quad X \to +\infty, \qquad \theta \to 1 \quad \text{as} \quad X \to -\infty,$$

must be satisfied. Defining the shock width as $\delta = (\varepsilon\tilde{R})^{-\frac{1}{3}}$, and letting $\xi = \frac{1+s}{s(1-s)}Z$ for convenience, we rewrite the thermal boundary layer problem as

$$-X\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial X^2}, \tag{18}$$

with boundary conditions

$$\theta = 0 \quad \text{when } X > 0, \xi = 1 \quad \text{and as} \quad X \to +\infty, \tag{19}$$

$$\theta = 1 \quad \text{when } X < 0, \xi = 0 \quad \text{and as} \quad X \to -\infty, \tag{20}$$

where, for convenience, the rescalings $\xi \to \gamma\xi$ and $X \to \gamma^{\frac{1}{3}}X$ have been applied. An identical problem to that presented by (18)-(20) was solved in [1] in the context of a salt-finger problem. The heat fluxes at the upwelling-downwelling interface can be obtained by solving the problem in both regions separately and ensuring the two solutions match up at the interface.

5 Summary and Future Work

We have presented a two-fluid model for convection in a volcanic conduit. The model reduces to a standard thermal convection problem with the gas volume fraction providing the driving force. An analysis of a basic flow with vertical acceleration alone yields a thermal boundary layer problem identical to a problem in salt-finger analysis presented in [1]. The next step in our analysis is to consider the viscous boundary layer at the upwelling-downwelling interface where the acceleration terms in (17) are important. We also intend to investigate the change in viscosity between the degassed and nondegassed magmas and compare our results with observed gas fluxes at Stromboli volcano.

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