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MODELLING DYNAMICS OF TWO-SIDED WEDGE MECHANISM WITH VARYING CONTACT

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Abstract. This paper is the result of research in multi-body contact dynamics emphasising the discontinuity approach for the analysis of varying force constraints problems of the worm gear drive systems by the special developed physical model called as Two-Sided Wedge like Mechanism. The discontinuity approach has been clarified the dynamically variable regimes of motion deals with worm gearmesh contact properties that covering sliding friction and backlash occurs between the worm and the mating gear. The two dynamical regimes of motion called as tractive and inverse-tractive are accepted. The qualitative force constraints analysis deals with the instantaneous changes of varying contact-cases and appropriate dynamical regimes that exhibit jump discontinuous events within continuous simulation process. The model configuration is presented by partitioning co-ordinates, which includes kinematics, force and inertial systems parameters. The residual form of mathematical model formulation has been presented by ODEs with Discontinuity kernel in the right-hand side. The numerical integration scheme for monitoring the possible switching events (or discontinuities) within continuous ODE solver has been carried out via additional construction of switching function and control conditions using a technique known as zero-crossing detection. Numerical example that illustrates the developed methodology is given for motor operated valve with worm gearbox.

1 Introduction

In many applications in mechanical engineering, worm gear drives are used to transmit power between rotating shafts utilizing the great effect of torque/force multiplication. The friction contact as well as backlash which change left and right-hand side contact of the meshing teeth has the influence on effectiveness and direction of internal force flow and also match durability, stress of contact elements. Therefore, the ability to incorporate them into multibody systems dynamics and to simulate non-linear friction contact and backlash via discontinuity effects for the internal force contour has become an essential topic.

A multibody systems (MBS) with varying contact properties are well discussed in the works [1,4] as a mechanical system consisting of bodies, which have rigid masses and inertias and can be linked together as joints, which reduce the total degrees of freedom. Typical joints constrain the relative displacement or rotation of the reference points of two bodies. On today is known investigations which modelling screw type joints and worm gear sets contact problems according to the simple one-sided contact wedge mechanical systems [5]. That model realizes restrictions upon capabilities to obtain dynamic characteristic of contact-case modification and efficiency of gear system in real time mode. Therefore the unique new design methodologies for the mathematical modelling and simulation of the varying force constraints and efficiency factors of the worm gear drive mechanical systems have been developed by more progressive model.

2 Model description and assumptions

The worm gear drive mechanical system can be modelled by a Two-Sided Wedge Mechanism (TSWM) as Designed Dynamical Model shown in the figure 1. The worm gearmesh contact is exhibited on a plane model by two slope active contact line (a-a) and (b-b) between conjugated wedge-like rigid and inertial bodies via the presence of backlash. This model has been developed to simplify visualization of internal contact conditions resembling a screw –like worm gear mesh with line- type contact. The model can be classified as a dynamically variable structural Multibody system with joint clearance depends on the type of contact line modification and appropriate type of force constraints. The inertial bodies m_1, m_2 are realised an assembly motion in generalized rectangular co-ordinates x_1, x_2 with relative motion to each other via one of two slope contact line ($a-a$) or ($b-b$) through presence of negligible clearance. An external force F1 and F2 acts along co-ordinate directions considering actuated and resistance toques which applied to the bodies.

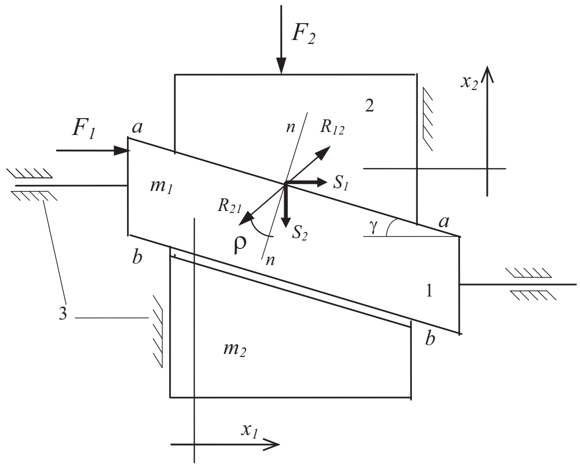


Figure 1. Two-Sided Wedge Mechanism. Design Model in Plane

The kinematic constraints on position, velocity and acceleration levels are stable for different contact cases and resulted in algebraic constraint equations

$$x_1 - x_2 tg\gamma = 0, \quad \dot{x}_1 - \dot{x}_2 tg\gamma = 0, \quad \ddot{x}_1 - \ddot{x}_2 tg\gamma = 0. \tag{1}$$

where $\dot{x}_1, \dot{x}_2\ \ddot{x}_1, \ddot{x}_2$ - linear velocity and acceleration components of appropriate bodies, $1/tg\gamma$ means constant kinematic ratio of wedge mechanism, γ - slope angle of contact line identifies the lead angle of the worm thread.

The force constraint equation has been derived by kinetostatic analysis in the form

$$S_1 = -\psi_j S_2, (j = 1,2) \tag{2}$$

which identifies different relationships between reduced reactions $\vec{S}_1, \vec{S}_2$ (fig.1) that act on body 1 and 2 respectively along coordinate directions by the set of so-called Force Transfer Function (FTF) $\psi_j = \{\psi_1, \psi_2\}$ and written as

$$\psi_1 = tg(\gamma + \rho), \psi_2 = tg(\gamma - \rho) \tag{3}$$

where ρ is contact sliding friction angle. Those two FTFs are handled two different dynamical regimes of motion with qualitative level of internal force/power flow and efficiency of drive system. The first regime is named as tractive regime of motion, undergoing contact on line $(a-a)$ within internal force direction $\vec{S}_1 > 0, \vec{S}_2 < 0$ that means a direct force flow from body 1 to 2 with force ratio ψ_1 (3). The second regime is named as inverse -tractive regime of motion, undergoing contact on line $(b-b)$ within resulted internal force directions $\vec{S}_1 < 0, \vec{S}_2 > 0$ that means inverse force flow from body 2 to 1. The inverse-tractive regime reproduce force relationship by FTF ψ_2 (3). The set of FTF $\psi_j = \{\psi_1, \psi_2\}$ has been attributed as Discontinuity Kernel with two components that undergoes jump discontinuity during simulation.

For the worm gear drive is more preferable motion in tractive regime via higher efficiency level and lower power losses before inverse-tractive regime.

3 Mathematical problem formulation

The mathematical problem formulation results in residual ODE form with unpredictable discontinuities in the right-hand side within the set of the Force Transfer Functions $\psi_j (j = 1,2)$ and is written as

$$\ddot{x}_1(t) = \frac{F_1 - F_2\psi_j}{m_1 - m_2\psi_j tg\gamma}, (\psi_j = \{\psi_j, j = 1,2\}),\ x_1(0) = x_{10},\ \dot{x}_1(0) = \dot{x}_{10} \tag{6}$$

That equation describes an assembly motion within the space of independent state co-ordinates $x_1, \dot{x}_1$ in different regimes deals with discontinuities in the right-hand side This problem needs an additional controller "switching" function $\varphi(t) = \Phi(t, x_1, \dot{x}_1)$ which depending on state variables. This switching function has been constructed on acceleration constraint equation (1) and introduced into following control block

$$\varphi(t) = \ddot{x}_1^{own} - \ddot{x}_2^{own} tg\gamma \quad , \quad \psi_j = \begin{cases} \psi_1 & if \quad \varphi(t) \geq 0, \\ \psi_2 & if \quad \varphi(t) < 0, \end{cases} \tag{7}$$

where $\ddot{x}_1^{own} = F_1 / m_1$ - "own" acceleration of body 1, $\ddot{x}_2^{own} = F_2 / m_2$ - "own" acceleration of body 2.

The switching function is continuous nonlinear function undergoing zero-crossing condition $\varphi(\tau_k) = 0, (k = 1,2...)$ at the time point $t = \tau_k$ that fixed discontinuity event within system's force contour called as critical dynamic state. The internal force contour components at the critical state at time $t = \tau_k$ are equal zero $S_1(\tau_k) = S_2(\tau_k) = 0$ that means contact-case modification or instantaneous re-establishing contact during assembly motion. Problem has been tested using the resulted acceleration values $\ddot{x}_1(t)$ for unknown internal reaction that can be obtained from

$$S_1(t) = F_1 - m_1 \ddot{x}_1(t) \tag{8}$$

After computing $S_1(t)$ one can find $S_2(t)$ by using the dependence (2)

$$S_2(t) = S_1(t) / \psi_j, (j = 1,2) \tag{9}$$

4 Simulation results of real life mechanical system

For the simulation study we will consider AC Motor Operated Valve (MOV) with worm gear drive.
The input Worm gear geometry parameters includes reduction ratio u=40:1; diameter of worm d_1=50 mm, diameter of wheel $d_2 = 160mm$, worm lead angle γ = 4,76 deg.

The key data below outlines motor actuated force F_1 that means motor torque acts on the worm shaft according to the start AC motor characteristic by function $F_1 = \frac{M(\omega)}{R_1} = \frac{2F_{max}}{\frac{V_0 - \dot{x}_1}{V_0 - V_{max}} + \frac{V_0 - V_{max}}{V_0 - \dot{x}_1}}$, where $V_0 = \frac{\omega_0}{R_1^2}$=7.85 m/s synchronous linear velocity; $V_{max} = \frac{\omega_{max}}{R_1^2}$=6.5 m/s maximal linear velocity; $\dot{x}_1 = \frac{\omega}{R_1^2}$ - current linear velocity of driving body, ($R_1 = d_1 / 2$)- Motor characteristics: power P=7,5 kW, rotation frequency n_0=3000 rpm; n_n=2900 rpm, M_{max}/M_n=2,2,s_{max}=17%; Rotor inertia moment I_{rot}= 0.0069 kgm$^{2.}$. Calculated reduced values are: F_{max} =2178 N ($F_{max} = M_{max} / R_1$). Inertial parameters: the loaded mass is vary in the range m_2=[100; 200,300]kg., driving mass m_1 =11.8 kg ($m_1 = I_{rot} / R_1^2$).

The resistance force F_2 has to be constant over the observed time period and can be set in the range F_2 =[-5000; -10000; -15000] N. The contact friction angle analytically defines by following function $\rho(\dot{x}_1) = \left(a(\dot{x}_1 / \cos\gamma)^b + c\right)^{-1}$ with approximation coefficients a=0,161; b=0,535; c=0,088 taken from [6].

The problem has been simulated by Matlab function ODE 23s with zero initial conditions $x_1(0) = 0, \dot{x}_1(0) = 0$ and initial start value of switching function $\varphi(t = 0)$ with the proper value of FTF $\psi_j(j = 1,2)$ . .

The main attention addresses to the simulation of systems acceleration variable $\ddot{x}_1(t)$ versus time and internal reactions $S_1(t), S_2(t)$ with depicted maximal force values S_1^m, S_2^m . The qualitative dynamic analysis can be provided by calculation of dynamic load factors written as

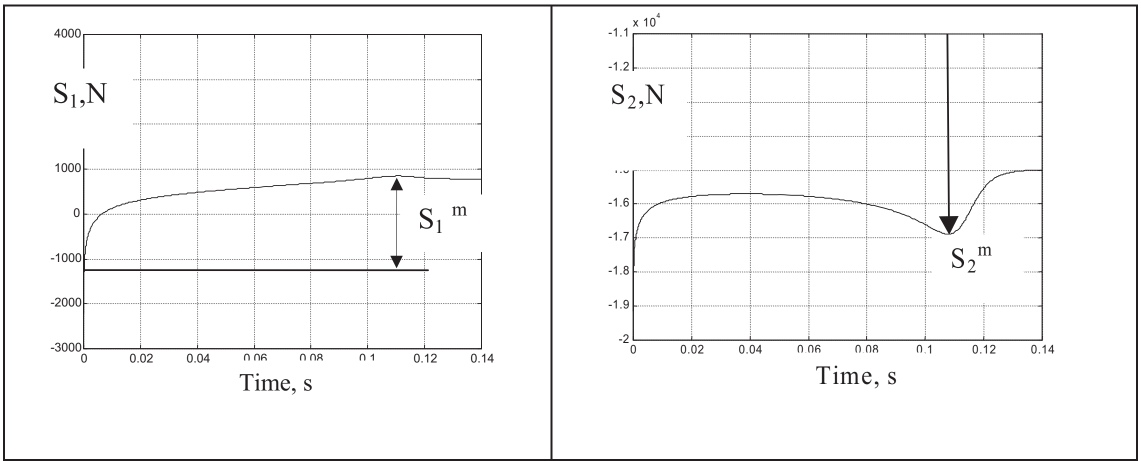
$$K_{D1} = \left|S_1^m\right| / F_{t1}, K_{D2} = \left|S_2^m\right| / F_{t2}, \tag{10}$$

where F_{t1}, F_{t2} are the static tangential forces acts at the worm gear mesh., which are calculated as $F_{t1} = 2T_1 / d_1 = 2 \cdot 24,75 / 0,05 = 990N$, $F_{t2} = 2T_2 / d_2 = 2T_1 u\eta / d_2 = 2 \cdot 24,5 \cdot 40 \cdot 0,67 / 0,16 = 8303,7N$.

The given range of input model parameters m_2, F_2 offered to provide test series, which are summarised in the Table 3. One of the test cases for a driven mass equals m_2=200 kg and external resistant force F_2 = -15000 N is shown on plotted graphs (Fig.2).

Test number	Model varying parameters m_2 F_2	Simulation results S_1^m [N]	K_{D1}	S_2^m [N]	K_{D2}
1	100 kg -5000 N	800	0,81	-6300	0,76
2	100 kg -10000 N	1400	1,40	-11300	1,37
3	100 kg -15000 N	1800	1,84	-16500	2,00
4	200 kg -5000 N	900	0,91	-7600	0,92
5	200 kg -10000 N	1500	1,50	-12100	1,36
6	200 kg -15000 N	2000	2,01	-16900	2,00

Table 3 Maximal values of internal forces and dynamic factors for the worm gear drive of MOV



a) Plots of $S_1(t)$ and maximal value S_1^m =2000N

b) Plots of $S_2(t)$ and maximal value S_2^m =-16900 N

Figure 2. Resulted simulation trajectory of $S_1(t)$ and $S_2(t)$ for F_2=-15000 N , $m_2 = 200kg$

5 Conclusions

The presented above results shows that the worm gear drive system of MOV for all the cases of increased loaded mass m_2 (connected to the driven shaft) and increased resistance force F_2 have been reproduced tractive regime of motion that means direct force flow (S_1>0) and appropriate efficiency η with greater advantage of the gears transmissions. This work demonstrates the usefulness of switching function and discontinuity approach within force constraint for control and distinguishing different dynamical regimes of motion of the worm gear drive mechanical systems that aimed to improve dynamical design according to developed Wedge Model.

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