Non-linear distributed parameter observer applied to a denitrification reactor

I. Torres[1], C. Vilas[2], A. Vande Wouwer[2] and I. Queinnec[1]

[1]LAAS-CNRS, Université de Toulouse, France, [2]Service d'Automatique, Faculté Polytechnique de Mons, Belgique

Corresponding author: I. Torres,7 avenue du colonel Roche, F-31077 Toulouse, France, `itorres@laas.fr`

Abstract.   In this paper, the state observation of a denitrification reactor modelled by a parabolic PDE system is developed. The strategy is to design a distributed parameter Luenberger observer by a late lumping approach such as to keep the distributed nature of the system as long as possible in the construction of the state observer. The numerical implementation of the observer is based on the method of lines and the approximation of the spatial derivatives using a finite element discretization. It however results in a high order ODE system, which is then reduced to a lower order system involving only some dominant modes. Such dominant modes are obtained by modal decomposition.

Keywords: *Parabolic PDE, distributed parameter observer, method of lines, finite element method, modal analysis.*

1   Introduction

Biofiltration has proven to be a promising reaction system for wastewater [4], [9] or drinking water treatment [7], [2], but also in aquaculture or for control of air pollution. Such a device is compact, fairly simple to build and operate, and has shown good efficiency for biological treatment associated to low energy consumption. On the other hand, closed-loop control or supervision is quite limited due to the lack of on-line efficient and low cost instrumentation. Dealing with this problem then requires to implement state observation strategies.

Moreover, such biofiltration units are characterized by spatial distribution of micro-organisms which are fixed on a solid support [10]. Such distributed parameter systems (DPS) are represented by partial differential equations (PDE) to describe their distributed nature [3]. The state observation problem is formulated as an estimation of the complete spatial profiles of the state variables [15], [14]. Key problems related to the state observation of such PDE systems are:

- the location of measurement sensors [12], [13] and associated observability considerations [1],
- the choice of an observation strategy, e.g. a Kalman filter, a Luenberger observer, an asymptotic observer, etc [15].
- the choice of an early or late lumping approach [13], [15].

The main idea of this work is to design a distributed parameter Luenberger observer following the approach devised in [15] for a denitrification reactor represented by a parabolic PDE system and then, by using the method of lines, observer PDE equations are solved. Following a method of lines strategy, the observer PDEs are solved using the finite element method (FEM). In this way, a semi-discrete ordinary differential equation (ODE) system is obtained. Since the ODE system is usually of high dimension, a modal analysis is done such as being able to use only a selection of the dominant modes to integrate the solution.

The paper is organized as follow: in section 2 the denitrification reactor model is presented. In section 3 the distributed parameter observer is designed. In section 4 the solution of then observer PDE system, first by the method of lines and then by modal analysis, is discussed. In section 5 some results and simulations about state estimations of denitrification reactor are analyzed. Finally, in section 6 some conclusions are given.

2   Denitrification reactor model

The denitrification process under study is a biofilter, filled with a porous pouzzolane material. Nitrate and nitrite nitrogen issued from some wastewater are considered at the reactor input. An additional ethanol supply source may be used as a control input action or at least to ensure a sufficiently high ratio C/N such that carboneous component does not become the limiting source for the growth. Denitrification is performed in anaerobic conditions. The biological reaction is a two-stage reaction. The first stage is the denitration which transforms nitrate (NO_3) into nitrite (NO_2) while the second phase transforms nitrite into gaseous nitrogen (N_2). The same micro-organisms

population (bacteria) is involved in both stages, with ethanol as co-substrate. This biomass accumulates on the solid media surface due to filtration of bacteria present in the feeding water (if any) and to net growth. Thus, the biomass forms a biofilm around the filter particles, which thickens with time. One can then considers that all the biomass is fixed and does not move along the reactor. On the contrary, the soluble compounds (nitrate, nitrite and ethanol) are transported along the biofilter.

The dynamics of the biofilter can be deduced from mass balance considerations for the four different components, considering the following assumptions:

- The detachment of biofilm and particles retained by filtration is neglected;
- Once the biofilm reaches a critical 'per unit' surface thickness, the deeper part of the biofilm is considered as inactivated, and a maximum active biomass concentration X_{amax} is reached [1]. Then, after some transition period, the growth of micro-organisms just balances the death and inactivation process;
- The decay of biomass is neglected, hidden in the notion of maximum active biomass concentration.
- Radial dispersion is negligible. Axial dispersion obeys Fick's diffusion law.

Remark 1 *Diffusion phenomenon has been often neglected to simplify the PDE system [2], [5]. It is however a key parameter for two main reasons. First, if the boundary layer is destroyed by surface irregularities of the biofilm, substrate diffusion in the biofilm is strongly influenced by fluid turbulence. The gradient of concentration inside the biofilm is reduced [6]. Secondly, neglecting the diffusion phenomenon results in hyperbolic PDEs, instead of parabolic PDEs, which have distinctive properties as compared to parabolic PDEs and are usually more difficult to solve numerically.*

The denitrification reactor is then modelled by the following parabolic PDE system:

$$\frac{\partial x_1(z,t)}{\partial t} = D_f \frac{\partial^2 x_1(z,t)}{\partial z^2} - \frac{v}{\varepsilon}\frac{\partial x_1(z,t)}{\partial z} - \frac{1-Y_{h_1}}{1.14 Y_{h_1}\varepsilon}\mu_1(x_1,x_3)x_4(z,t) \tag{1}$$

$$\frac{\partial x_2(z,t)}{\partial t} = D_f \frac{\partial^2 x_2(z,t)}{\partial z^2} - \frac{v}{\varepsilon}\frac{\partial x_2(z,t)}{\partial z} + \frac{1-Y_{h_1}}{1.14 Y_{h_1}\varepsilon}\mu_1(x_1,x_3)x_4(z,t) - \frac{1-Y_{h_2}}{1.71 Y_{h_2}\varepsilon}\mu_2(x_2,x_3)x_4(z,t) \tag{2}$$

$$\frac{\partial x_3(z,t)}{\partial t} = D_f \frac{\partial^2 x_3(z,t)}{\partial z^2} - \frac{v}{\varepsilon}\frac{\partial x_3(z,t)}{\partial z} - \frac{1}{Y_{h_1}\varepsilon}\mu_1(x_1,x_3)x_4(z,t) - \frac{1}{Y_{h_2}\varepsilon}\mu_2(x_2,x_3)x_4(z,t) \tag{3}$$

$$\frac{\partial x_4(z,t)}{\partial t} = (\mu_1(x_1,x_3)x_4(z,t) + \mu_2(x_2,x_3)x_4(z,t))\left(1 - \frac{x_4(z,t)}{X_{amax}}\right) \tag{4}$$

for $0 < z \leq L$, where z is the axial space variable, $x_1(z,t)$, $x_2(z,t)$, $x_3(z,t)$ and $x_4(z,t)$ represents the nitrate $(g[N]/m^3)$, nitrite $(g[N]/m^3)$, ethanol $(g[DCO]/m^3)$ and active biomass concentrations. $x_{1,in}(t)$, $x_{2,in}(t)$, $x_{3,in}(t)$ and $x_{4,in}(t)$ represent the nitrate, nitrite, ethanol and active biomass at reactor input, respectively. D_f, v, Y_{h_1}, Y_{h_2}, μ_1 and μ_2 represent the diffusion term, the flow speed m/h (the ratio between the feeding rate (m^3/h) at reactor input and the biofilter transverse surface (m^2)), micro-organisms yield coefficients and population specific rates which transform nitrate into nitrite, then nitrite into gas nitrogen $(1/h)$.

The nitrate and nitrite growth specific rates are described, respectively, by the model of Monod with two substrate limitations:

$$\mu_1(x_1,x_3) = \eta_g \mu_{1_{max}} \frac{x_1}{x_1+K_{NO_3}}\frac{x_3}{x_3+K_C}$$
$$\mu_2(x_2,x_3) = \eta_g \mu_{2_{max}} \frac{x_2}{x_2+K_{NO_2}}\frac{x_3}{x_3+K_C}$$

where η_g, $\mu_{1_{max}}$, $\mu_{2_{max}}$, K_{NO_3}, K_{NO_2} and K_C are the correction factor for the anaerobic growth, the maximum specific growth rates of biomass on nitrate and nitrite and the affinity constants with respect to nitrate, nitrite and ethanol, respectively.

Associated to the dynamic equation for the denitrification process, appropriate initial and boundary conditions are given by:

- initial spacial profile at $t = 0$ for $0 \leq z \leq L$:

$$x_1(z, t = 0) = x_{1,0}(z) = x_{1,in}(0) \; g[N]/m^3 \tag{5}$$

$$x_2(z, t = 0) = x_{2,0}(z) = x_{2,in}(0) \; g[N]/m^3 \tag{6}$$

$$x_3(z, t = 0) = x_{3,0}(z) = x_{3,in}(0) \; g[DCO]/m^3 \tag{7}$$

$$x_4(z, t = 0) = x_{4,0}(z) = X_{amax} \; g[DCO]/m^3 \tag{8}$$

- boundary conditions at $z = 0$ (input) for $t > 0$

$$\frac{\partial x_1}{\partial z} = \frac{v}{D_f}(x_1 - x_{1,in}) \tag{9}$$

$$\frac{\partial x_2}{\partial z} = \frac{v}{D_f}(x_2 - x_{2,in}) \tag{10}$$

$$\frac{\partial x_3}{\partial z} = \frac{v}{D_f}(x_3 - x_{3,in}) \tag{11}$$

$$x_4(z, t) = x_{4,in}(t) \tag{12}$$

- boundary conditions at $z = L$ (output) for $t > 0$

$$\frac{\partial x_1}{\partial z} = 0 \tag{13}$$

$$\frac{\partial x_2}{\partial z} = 0 \tag{14}$$

$$\frac{\partial x_3}{\partial z} = 0 \tag{15}$$

Remark 2 *Initial conditions express that biomass and substrate are homogeneously distributed along the biofilter. Substrate concentrations are set equal to the corresponding influent concentrations [1].*

For numerical application, one considers the following nominal values borrowed from [1]:

$$Y_{h_1} = 0.56 \qquad\qquad Y_{h_2} = 0.54$$
$$\mu_{1max} = 0.36 \; 1/h \qquad\qquad \mu_{2max} = 0.32 \; 1/h$$
$$K_{NO_3} = 1.5 \; g[N]/m^3 \qquad\qquad K_{NO_2} = 1.0 \; g[N]/m^3$$
$$K_C = 40 \; g[DCO]/m^3 \qquad\qquad X_{amax} = 800 \; g[DCO]/m^3$$
$$n_g = 0.8 \qquad\qquad \varepsilon = 0.52$$
$$x_{1,in}(t) = 16.93 \; g[N]/m^3 \qquad\qquad x_{2,in}(t) = 0 \; g[N]/m^3$$
$$x_{3,in}(t) = 101.5 \; g[DCO]/m^3 \qquad\qquad x_{4,in}(t) = 0 \; g[DCO]/m^3$$
$$D_f = 0.4756 \; m^2/h \qquad\qquad v = 4 \; m/h$$

The system (1)-(4) can be rewritten in matrix form as:

$$\frac{\partial x}{\partial t} = A_1 \frac{\partial^2 x}{\partial z^2} + A_2 \frac{\partial x}{\partial z} + F(x) \tag{16}$$

$$x(z, t = 0) = x_0(z) \tag{17}$$

where $x = [x_1 \, x_2 \, x_3 \, x_4]^T$ is the state vector and matrices A_1, A_2 and $F(x)$ are diagonal square matrices $\in \mathbb{R}^{4 \times 4}$.

In the following, for state estimation purposes, it is assumed that nitrate and nitrite concentrations are measured both at the input and the output of the biofilter, but also at $p - 1$ internal location along the biofilter. The biomass concentration is not accessible, and the ethanol concentration is measured only at the reactor input. In this way, the measured output is defined as $y = [x_1(z_1, t) \, \ldots \, x_1(z_p, t) \, x_2(z_1, t) \, \ldots \, x_2(z_p, t)]^T$, where z_p represents the output.

3 Non-linear distributed parameter observer

A nonlinear distributed parameter observer, with a formulation analog to the Luenberger observer, is designed so as to assign the error dynamics as proposed in [15]. Since DPS dynamics are characterized by an infinite number of modes, the observer design would in principe require the specification of a large (theoretically infinite) number of tuning parameters.

In order to bypass this high-dimensional design problem, a late lumping approach to the construction of distributed parameter observers (DPO) was developed in [15]. In this way, the structure of the DPO follows from a direct extension of Luenberger's approach to infinite dimensional systems like:

$$\frac{\partial \hat{x}}{\partial t} = A_1 \frac{\partial^2 \hat{x}}{\partial z^2} + A_2 \frac{\partial \hat{x}}{\partial z} + F(\hat{x}) + K(\hat{x})(y - \hat{y}) \tag{18}$$

$$\hat{x}(z, t = 0) = \hat{x}_0(z) \tag{19}$$

where $\hat{x} = [\hat{x}_1 \, \hat{x}_2 \, \hat{x}_3 \, \hat{x}_4]^T$ is the estimated state vector and $K(\hat{x})$ is the correction term, a rectangular matrix $\in \mathbb{R}^{4 \times 2p}$.

The design of operator K is based on the estimation error equations $e(z, t) = \hat{x}(z, t) - x(z, t)$. Therefore, we obtain:

$$\frac{\partial e}{\partial t} = A_1 \frac{\partial^2 e}{\partial z^2} + A_2 \frac{\partial e}{\partial z} + F(\hat{x}) - F(x) + K(\hat{x})(y - \hat{y}) \tag{20}$$

$$e(z, t = 0) = \hat{x}_0(z) - x_0(z) \tag{21}$$

The linearization of $F(x)$ along the estimated trajectory $\hat{x}(z, t)$ can be done to obtain [15]:

$$\frac{\partial e}{\partial t} = A_1 \frac{\partial^2 e}{\partial z^2} + A_2 \frac{\partial e}{\partial z} + \left. \frac{\partial F(x)}{\partial x} \right|_{\hat{x}} e + K(\hat{x})(y - \hat{y}) \tag{22}$$

This linearization is justified as soon as the estimation error is assumed sufficiently small, i.e.:

$$\|e(z, t = 0)\| = \|\hat{x}_0(z) - x_0(z)\| << 1 \tag{23}$$

Physical knowledge about the system is used to design the correction term $K(\hat{x})(y - \hat{y})$. Considering the ith PDE, the ith correction term is constructed in terms of error profile $e(z, t)$ and a tuning parameter row vector $\alpha_i \in \mathbb{R}^{1 \times 2}$, i.e.:

$$k_i^T(y - \hat{y}) = \left[-\left(\left. \frac{\partial f_i(x)}{\partial x_1} \right|_{\hat{x}} + \alpha_{i,1} \right) \; -\left(\left. \frac{\partial f_i(x)}{\partial x_2} \right|_{\hat{x}} + \alpha_{i,2} \right) \; 0 \; 0 \right] e(z, t) \tag{24}$$

for $i = 1, 2, 3, 4$. Initial profile $\hat{x}_0(z)$ as well as error profile $e(z, t)$ along the space in equations above are evaluated by linear interpolation of measurement states.

Remark 3 *The correction term (24) is used to compensate the nonlinearities of ith equation. The resulting observer system is asymptotically stable as soon as $\alpha_{i,j}$ are positive elements high enough. Since measurements about ethanol and biomass are not available inside the reactor, their error profile can not be calculated. Therefore, error related to these two variables are not considered and $\alpha_{i,3}$ and $\alpha_{i,4}$ are set to zero.*

4 PDE system solution

Once the observer has been developed by the late lumping approach, the implementation of the observer PDE system requires spatial discretization. In this paper, the method of lines is used to solve the observer PDEs numerically. More specifically, the FEM is selected for spatial approximation, and the resulting system of discretized equations is integrated in time using an appropriate ODE solver. In this way, a high order semi-discrete ODE system is obtained. In order to reduce the order of the ODE system to integrate, a modal analysis may be developed such as to consider the dominant modes only for numerical integration. This is potentially important because for very high order systems, it may reduce the cost to compute the final solution in a significant way. Furthermore, this reduction strategy can be used to design control laws which use expensive synthesis methods as, for exemple, those envolving an optimization problem with LMI constraints.

4.1 Method of lines

The idea behind the method of lines is the substitution of derivatives by an approximation with respect to one independent variable (generally space) and numerical integration (time integration) of the resulting semi-discrete ODE system (discrete in space - continuous in time) [16]. In this work, we approximate the spatial derivatives by using FEM on the spatial domain divided into N finite discrete elements uniformly distributed. Then, we obtain the matrices FEM: $M_M \in \mathbb{R}^{N \times N}$, $M_T \in \mathbb{R}^{N \times N}$, $M_D \in \mathbb{R}^{N \times N}$ and $M_B \in \mathbb{R}^{N \times N}$ to approximate the spatial derivatives like [11]:

$$\frac{\partial}{\partial z} = M_M^{-1} M_T$$

$$\frac{\partial^2}{\partial z^2} = -M_M^{-1} (M_D + M_B)$$

and the vector $g_i \in \mathbb{R}^N$ to relate non-homogeneous boundary conditions. Therefore, we obtain the following semi-discrete ODE system for each state variable x_i:

$$\frac{d\hat{x}_i}{dt} = -M_{Mi}^{-1}(a_{1(i,i)}M_{Di} + a_{2(i,i)}(M_{Ti} + M_{Bi}))\hat{x}_i + M_{Mi}^{-1}g_i + f_i(\hat{x}) + k_i(\hat{x})(y - \hat{y}) \tag{25}$$

for $i = 1, 2, 3, 4$. The resulting system of semi-discrete ODEs is integrated in time to compute the final solution.

Remark 4 *M_{Bi} matrix involves information about homogeneous boundary conditions and g_i vector contains information about non-homogeneous boundary conditions. Because the system boundary conditions (9)-(11) are non-homogeneous and $x_{i,in}$ can change in time, this term is related to g_i vector, specifically to its first element (the element at input of the semi-discrete ODE system).*

4.2 Modal analysis

A convenient and useful form of analysis of second-order equations is through modal decomposition. This form of analysis is possible when the second order equation like (18) has a spatial operator which can be made self-adjoint and which has a real discrete spectrum of eigenvalues [8].

Let us consider the state variable $x_i(\xi, t) : \mathbb{R}^N \times \mathscr{T} \to \mathbb{R}$, where $\mathscr{T}$ being the semiopen time interval $[0, \infty)$, expanded in a truncated Fourier series, so that:

$$x_i(\xi, t) \approx \sum_{k=1}^{N} \phi_i^k(\xi) m_i^k(t) \tag{26}$$

where $\phi_i^k(\xi)$ and $m_i^k(t)$ are a set of functions to be determined.

Let us define a spatial operator as:

$$\mathscr{A}_i x_i = a_{1(i,i)} \frac{\partial^2 x_i}{\partial z^2} \tag{27}$$

We solve the following eigenproblem by using the FEM approximation [11]:

$$\mathscr{A}_i \Phi_i = -M_{Mi}^{-1}(a_{1(i,i)}M_{Di} + a_{2(i,i)}M_{Bi})\Phi_i = \Lambda_i \Phi_i, \quad i = 1, 2, 3, 4 \tag{28}$$

Here $\Lambda_i \in \mathbb{R}^{N \times N}$ is a real eigenvalue diagonal matrix and $\Phi_i \in \mathbb{R}^{N \times N}$ is a real eigenfunction matrix of the Laplacian operator defined above. Since (28) is a homogeneous self-adjoint differential equation with homogeneous boundary conditions, the eigenfunctions are orthogonal [8]. Therefore, by defining the spatial projection operator $P_i = \Phi_i^T M M_i$, we can rewrite (25) as:

$$\frac{dm_i(t)}{dt} = \Lambda_i m_i(t) + a_{2(i,i)} \Phi_i^T(z) M_{Ti} \Phi_i(z) m_i(t) - P_i \left(M_{Mi}^{-1} g_i + f_i(\hat{x}) + k_i(\hat{x}) (y - \hat{y}) \right) \tag{29}$$

where $m_i \in \mathbb{R}^N$ are the time dependent modes of the state variable x_i. In the equations above, the state variables are approximated by using Fourier series (26), the eigenfunctions Φ_i and the modes m_i calculated by (28) and (29) respectively, as $\hat{x}_i(\xi,t) = \Phi_i(\xi) m_i(t)$ [11].

The modal decomposition has the advantage that the N modes related to the spatial discretization of the estimated state vector $\hat{x}$ from equation (29) may be organized from the slow modes to the fast ones. Therefore, we can consider, for the observer implementation, only the modes who involve the main information about the states to estimate. Only n dominant modes, associated to the first $n \leq N$ eigenvalues and eigenfunctions, issued from each Λ_i and Φ_i matrices respectively, are integrated in the observer. The influence of the modal reduction is discussed through the numerical illustrations.

5   Results and simulations

At this moment we have designed a distributed parameter observer for a denitrification biofilter and we have proposed a strategy to solve the PDE system based on FEM and modal analysis. It has been previously shown in [1] that, except during the initial colonization step, the biomass concentration remains almost constant at X_{amax}, even after a washing out, so its estimation may be bypassed. Therefore, for sake of simplicity, we have reduced the system (1)-(4) by taking only the three first PDEs and by considering $X_a(z,t) = X_{amax}$.

To evolve the observer, a data base has been generated using the simulation of model (1)-(4) with the default parameters, the boundary conditions and the initial conditions described in section 2. We then consider that the measured variables are the nitrate and the nitrite at input, in the middle and at output reactor, i.e. $p = 2$, with a sample period of 1 minute. The observer starts 3 minutes after the process beginning.

Since the mathematical model used to design the observer does not represent, generally, the true real denitrification biofilter, an uncertain model is considered by adding some uncertainty on the terms μ_1 and μ_2. Specifically, we use the values $\mu_{1max} = 0.34$ and $\mu_{2max} = 0.30$ in the observer model, which induce variations in system simulation larger than variations in the observer. Furthermore, in order to change the system dynamics a step signal is used in $x_{1,in}$ boundary condition as:

$$x_{1,in} = \begin{cases} 14.93 & ; t < 30 \, minutes \\ 16.93 & ; t \geq 30 \, minutes \end{cases}$$

For the distributed parameter observer, the tuning parameter matrix $\alpha \in \mathbb{R}^{3 \times 2}$ considers three state variables to estimate and two measurement variables. In order to select the components of matrix α, the cancellation of non-linear terms was tested and the selection of values in progressive power of 10 was made. The matrix α proposed is then:

$$\alpha = \begin{bmatrix} 0.85 & 0 \\ 0 & 100.0 \\ 3.5 & 3.5 \end{bmatrix}$$

Finally, the initial state is constructed by interpoling nitrate and nitrite initial measurements and by initializing the ethanol to the input value.

Because transport and diffusion terms are identical for the three PDEs (1)-(3), the FEM matrices are the same for the three Laplacian operators and therefore, we take $\Lambda_i = \Lambda$ and $\Phi_i = \Phi$. The examination of matrix Λ exhibits that the amplitude of eigenvalues varies from order 10^0 to order larger than 10^3. It can be checked on figures 1 and 3 that all the modes act on the dynamics of the estimated nitrate and the estimated ethanol. On the other hand, figure 2 shows that only the few first significantly modes act on the dynamics of the estimated nitrite.
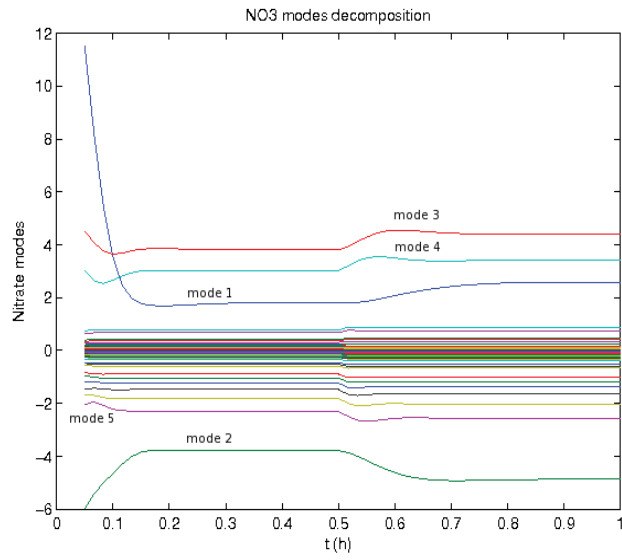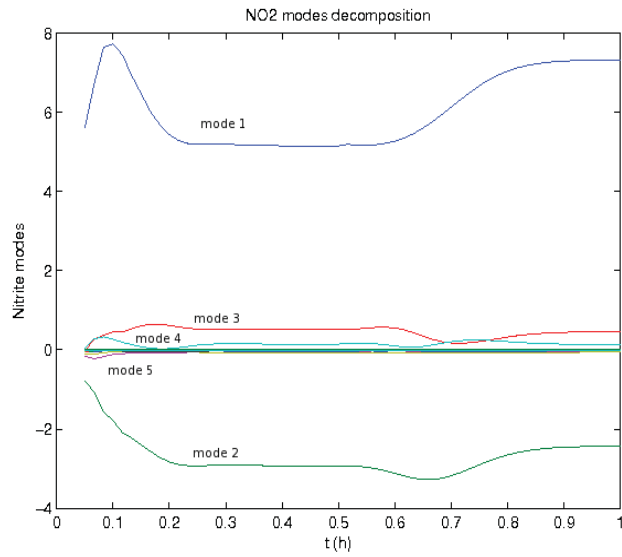
Figure 1: Nitrate dominant modes.

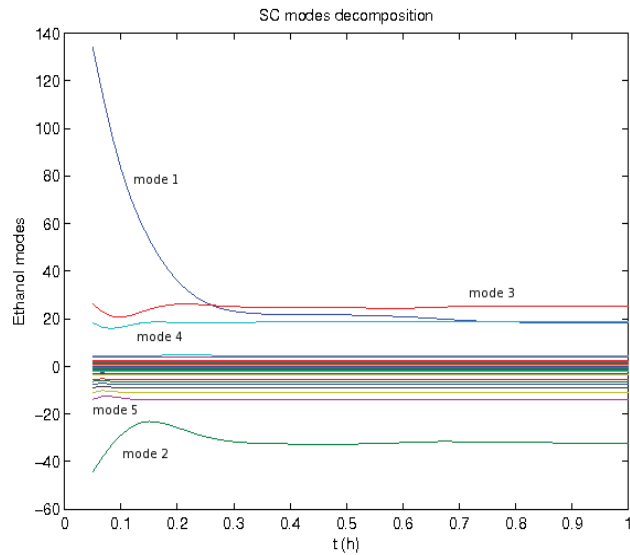**Figure 2:** Nitrite dominant modes.

Figure 3: Ethanol dominant modes.

This is illustrated on figures 4-6, where the simulated state are presented in solid blue and the estimated values are presented in dashed black, for which 50 modes are used to estimate nitrate and ethanol, while only the first 10 dominant modes are used for the nitrite.
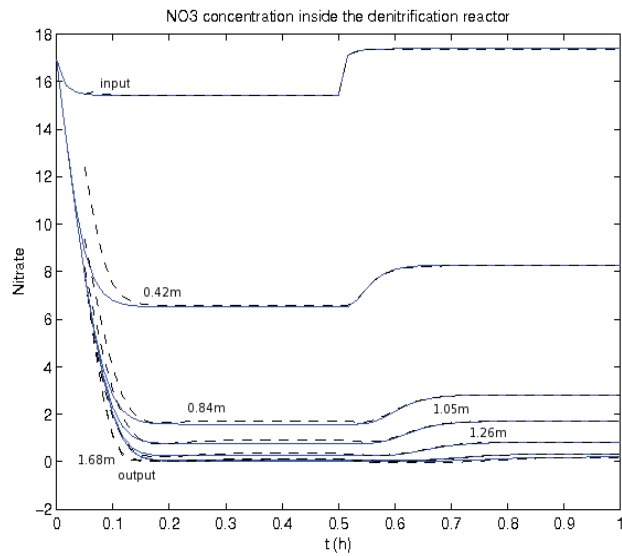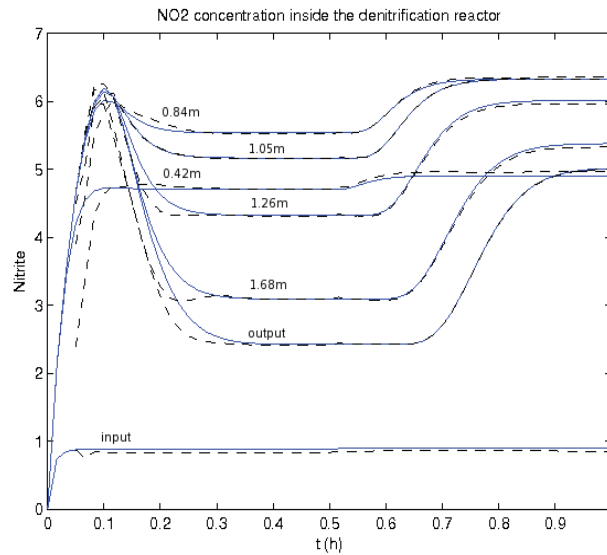

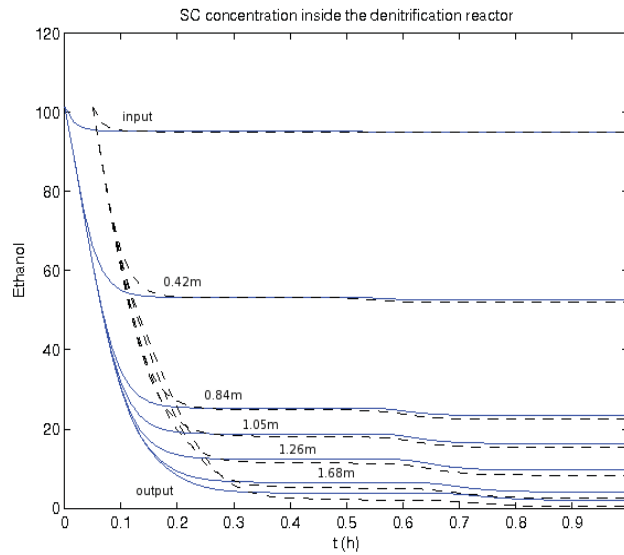**Figure 4:** Nitrate estimation with 50 modes.

Figure 5: Nitrite estimation with 10 modes.

Figure 6: Ethanol estimation with 50 modes.

As expected, nitrate and nitrite space profile estimation along time provides a better fitting to the generated data base than the ethanol space profile estimation since measurements in three locations (input, one internal point and output) are available for those variables. The ethanol space profile along time is however correctly estimated. In accordance with the dominant mode analysis, it is sufficient to consider 10 modes for nitrite integration. On the other hand, one can not reduce the number of modes used for nitrate and ethanol estimations while keeping admissible quality of estimations. Figures 7 and 8 show in solid blue the simulated state values and in dashed black the estimated values by using 40 modes to estimate nitrate and ethanol concentration profiles. It is clear that a little diminution in the number of modes results in a considerable degradation of the state estimation.
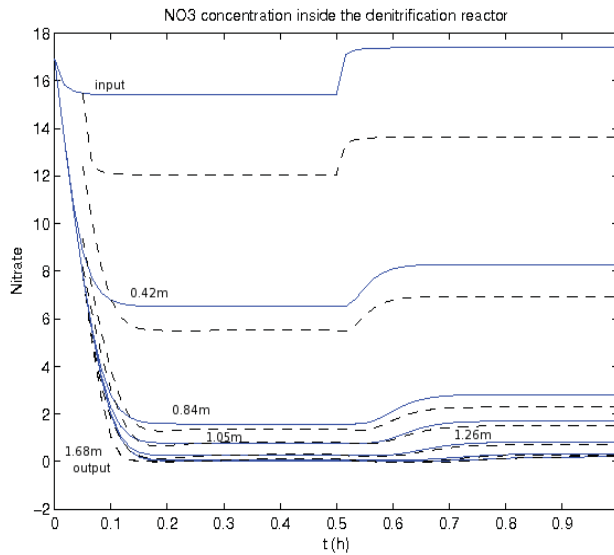
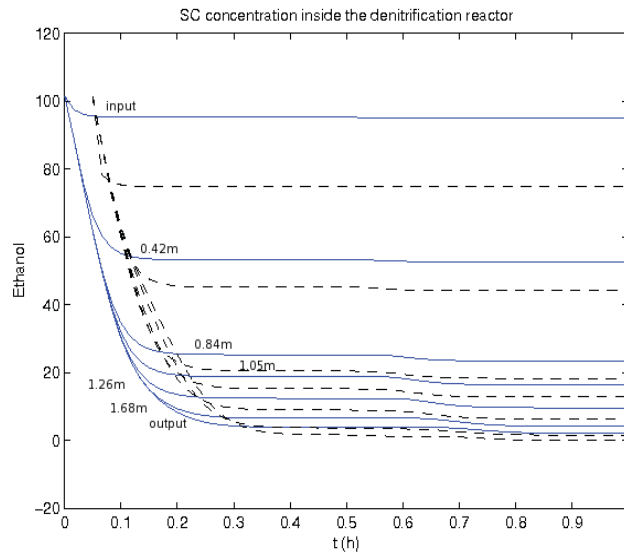**Figure 7:** Nitrate estimation with 40 modes.

Figure 8: Ethanol estimation with 40 modes.

6 Conclusion

A state observation strategy for a denitrification biofilter system using late lumping of the PDE equation has been proposed. It was demonstrated that such an approach may be positively applied to estimate the spatial profile of the process state variables while distributed nature of the system is preserved as long as possible. Furthermore, the observer PDEs were solved by FEM approximation and then, a modal analysis was made in order to determine the influence of the modes in the application of the observer. It is interesting to observe that such an influence is different from one state variable to estimate to another one. Therefore, we have used a powerful strategy to estimate distributed parameter systems and a method to solve the PDEs of the observer with the minimum of information.

7   References

[1] S. Bourrel. *Estimation et commande d'un procédé à paramètres répartis utilisé pour le traitement biologique de l'eau à potabiliser*. PhD thesis, LAAS-CNRS, Université de Toulouse, 1996.

[2] S. Bourrel, D. Dochain, J.P. Babary, and I. Queinnec. Modelling, identification and control of a denitrifying biofilter. *Journal of process Control*, 10:73–91, 2000.

[3] D. Dochain and P.A. Vanrolleghem. *Dynamical Modelling and Estimation in Wastewater Treatment Processes*. IWA Publishing, 2001.

[4] F. Fdz-Polanco, E. Méndez, M. A. Urueña, S. Villaverde, and P.A. García. Spatial distribution of heterotrophs and nitrifiers in a submerged biofilter for nitrification. *Water Research*, 34(16):4081–4089, 2000.

[5] K. Gaid, A. Y. Le Roux, and G. Martin. A nonlinear problem occuring in water purification in aerated activated carbon biofilters. *Mathematical modelling*, 3:293–303, 1982.

[6] J. Jacob. *Modelisation et simulation dynamique de procédés de traitement des eaux de type biofiltre: Traitement de systèmes d'équations differentielles partielles et algebraiques (EDPA)*. PhD thesis, INP de Toulouse, 1994.

[7] I. Queinnec, J.C. Ochoa, A. Vande Wouwer, and E. Paul. Development and calibration of a nitrification pde model based on experimental data issued from biofilter treating drinking water. *Biotechnology and Bioengineering*, 94(2):209–222, 2006.

[8] W. H. Ray. *Advances process control*. McGraw-Hill, 1981.

[9] E. Rother and P. Cornel. Optimising design, operation and energy consumption of biological aerated filters (baf) for nitrogen removal of municipal wastewater. *Water Science and Technology*, 50(6):131–139, 2004.

[10] V. Saravanan and T.R. Sreekrishnan. Modelling anaerobic biofilm reactors-a review. *Journal of Environmental Management*, 81:1–18, 2006.

[11] C. Vilas. *Modelling, simulation and robust control of distributed processes: Application to chemical and biological systems*. PhD thesis, Universidade de Vigo, 2008.

[12] W. Waldraff, D. Dochain, S. Bourrel, and A. Magnus. On the use of observability measures for sensor location in tubular reactor. *Journal of process Control*, 8(5-6):497–505, 1998.

[13] A. Vande Wouwer, N. Point, S. Porteman, and M. Remy. An approach to the selection of optimal sensor locations in distributed parameter systems. *Journal of Process Control*, 10:291–300, 2000.

[14] A. Vande Wouwer, C. Renotte, I. Queinnec, and Ph. Bogaerts. Transient analysis of a wastewater treatment biofilter - distributed parameter modeling and state estimation. *Mathematical and Computer Modelling of Dynamical Systems*, 12:423–440, 2006.

[15] A. Vande Wouwer and M. Zeitz. State estimation in distributed parameter systems. *Control Systems, Robotics and Automation, Theme in Encyclopedia of Life Support Systems*, 2003.

[16] D. Zwillinger. *Handbook of differential equations*. Academic Press, 3rd edition, 1998.