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THE UNSTEADY KUTTA CONDITION

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Abstract. A potential flow based Boundary Element Method was developed to estimate the hydrodynamic forces on flapping wing [1,2]. A new formulation of the unsteady Kutta condition, postulating a finite pressure difference at the trailing edge of the oscillating wing, was implemented in the numerical procedure: its theoretical and physical foundations are here discussed.
The trailing-edge condition, necessary to obtain a unique solution, is derived from the unsteady Bernoulli equation, that is, the conservation of momentum equation for incompressible fluid and irrotational flow.
More generally, such a condition implies that the energy supplied for the wing motion would generate trailing-edge vortices. Their overall effect, which depends on the motion initial parameters, would be a jet of fluid that propels the wing. As the kinetic energy is transferred from the jet back to the wing, the vortices would disappear. The postulated pressure difference at the trailing edge is then fundamental for such a model as it can justify the velocity difference that generates the thrust-producing jet.

1 Introduction

The idea of utilizing the forces generated by flapping wings for the propulsion of man-made objects emerged from the observations of fish and birds. Rozhdestvensky and Ryzhov [3] reviewed theoretical and experimental studies of flapping-wing propulsors and of vehicles equipped with them. The complex character of the problem was underlined and key areas of interest identified, e.g. the effects of flow unsteadiness, wing flexibility and three-dimensionality. Triantafyllou et al. [4] described the progress in understanding the mechanisms of force production and flow manipulation in oscillating wings. Their review, focused primarily on experimental studies, showed that there is a lot more to be learned about the functions and design of moving wings.

Triantafyllou et al. [5] proposed that the large-scale patterns observed in the wake of such oscillating propulsors are related with the production of a jet-like average flow. Jones et al. [6] investigated both experimentally and numerically the thrust produced by a sinusoidally heaving airfoil, known as the Knoller-Betz effect. Water-tunnel experiments were performed providing data about the unsteady wake formed by the moving wing: vortical structures and time-averaged velocity profiles in the wake were compared with computations from a code based on that developed by Basu and Hancock [7]. Qualitative and quantitative comparisons indicated that the formation and evolution of the wake structures are primarily inviscid phenomena.

Basu and Hancock [7] were among the first to devise a panel method for the calculation of the forces on a two-dimensional airfoil undergoing an unsteady motion in an inviscid and incompressible flow. An auxiliary condition, known as Kutta condition and related to assumptions on the flow characteristics at the trailing edge of the moving foil, was added to obtain a unique solution.

The steady trailing-edge condition was independently formulated by Chaplygin [8], Kutta [9] and Zhukovski [10] to avoid the mathematical difficulties of the conformal mapping method at the trailing-edge of the foil, that is, the infinite fluid velocity at this point. The condition ensures that the flow passes the trailing edge smoothly with a finite velocity.

Hess [11], in a broad review of various panel methods, asserted that the specification of a proper Kutta condition is more important than any other detail of the numerical implementation. Hess [12] also underlined that the mathematical description of the lifting problem is merely a model to describe by the means of a potential flow a phenomenon that is more complex and ultimately due to viscosity. In other words, a suitable condition has to account for the viscosity effects in a computational model that is essentially inviscid.

Poling and Telionis [13] examined a number of unsteady flowfields and their experimental results indicate that the classical Kutta condition, which states that the velocity at the trailing edge is finite and the pressure difference there is zero, is not valid in certain conditions, i.e. it is a function of the initial parameters. Strong viscous-inviscid interactions of the boundary layer can appear in the trailing-edge region of the moving wing and lead to large streamlines curvatures and not-negligible pressure gradients.

The numerical results of Young and Lai [14] show that flow separation occurs at the trailing edge of heaving foils creating an effective blunt-edge body. More precisely, the flow streamlines form a time-dependant trailing-edge vortex rather than smoothly leave the trailing edge on both sides. The edge flow mechanism was independently analysed by Liebe [15] who proposed to replace the classical Kutta condition with a more general condition based on the formation and periodic shedding of trailing-edge vortices. This led to the development of a novel approach (Finite Vortex Model) to compute the forces acting on fixed and moving wings.

There is not any experimental evidence supporting the notion that the pressure difference at the trailing edge ought to be equal to zero for unsteady motion, as it is usually assumed in the literature (e.g. see the Introduction in [1]). A logical first step in the formulation of a comprehensive unsteady Kutta condition could be then the relaxation of the postulated zero pressure difference at the trailing edge. This would allow considering the variation in direction and magnitude of the velocities in the vicinity of the trailing edge, that is, the formation and shedding of trailing-edge vortices.

Such modified trailing-edge condition, postulating a finite pressure difference at the trailing edge of the oscillating wing, was implemented by the authors in the potential panel method presented in [1,2]: its theoretical and physical foundations are here discussed.

2 Discussion

An unsteady Boundary Element Method computer program was developed to estimate the hydrodynamic forces on oscillating wing: comparisons with published experimental data showed very good agreement with the computational results [1,2]. Following Katz and Plotkin [16], the flow is assumed incompressible and irrotational. Each wing section is represented by a finite number N of linear panels. The wing is divided into N_S strips, i.e. the wing is geometrically approximate by using N_S+1 sections perpendicular to the span. The wing is then modelled by using N*N_S quadrilateral panels. Constant strength distributions of source σ and doublet μ are situated on each panel, the midpoint of which is called collocation point. The flow potential function φ^* at each collocation point is defined as the sum of a local (perturbation) potential φ, related to the unknown doublet strength, and a free-stream potential φ_∞, linked to the fluid kinematic velocity. An internal Dirichlet boundary condition is imposed, that is, an inner potential function is specified on the internal wing surface in order to meet the non-penetration condition. At each wing collocation point the source strength is known, $\sigma = q_k\, \boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector normal to the wing surface pointing into the body and q_k the fluid kinematic velocity due to the motion of the wing. The governing integral equation is derived by using the Laplace's equation and Green's third identity. In the body-fixed coordinate system, at time t and for each wing collocation point it can be written as

$$\frac{1}{4\pi}\int_S [\,\sigma\,(\frac{1}{r}) - \mu\,\frac{\partial}{\partial n}(\frac{1}{r})\,]\,\mathrm{d}S - \frac{1}{4\pi}\int_{S_w} \mu_w\,\frac{\partial}{\partial n}(\frac{1}{r})\,\mathrm{d}S = 0 \tag{1}$$

where S and S_w indicate the wing and wake surface, respectively, and μ_w is the strength of the wake doublet distribution. To define uniquely the problem μ_w has to be known or related to the unknown doublets on S by the means of a suitable condition, that is, the Kutta condition.

It is important to note that S_w changes with time, that is, new portions of the wake surface are added as the time advances. Besides, the wake shape has to be properly modelled: this can be seen as an additional assumption to in effect represent initial conditions for the dynamic problem.

The discretized form of equation (1) can be written as

$$\sum_{j=1}^{N*N_S} C_j\,\mu_j \;+\; \sum_{j=1}^{N*N_S} B_j\,\sigma_j \;+\; \sum_{l=1}^{N_t}\sum_{k=1}^{N_S} C_{w\,k\,l}\,\mu_{w\,k\,l} = 0 \tag{2}$$

where N_t is the number of time steps. At each time step, N_S wake panels are shed, one for each wing strip. The number of wake panels is then equal to N_t*N_S. Each quadrilateral wake panel has an assigned chordwise length l_w and a constant strength doublet distribution μ_w on it. Moreover, C_j and B_j are the appropriate three-dimensional doublet and source influence coefficients of panel j at the considered wing collocation point, respectively. They are only dependent on the wing geometry, where r is the distance between the panel j and the respective collocation point and S the surface of panel j. C_{wkl} is defined as C_j, that is, r is the distance between the wake panel l and the respective wing collocation point and S the surface of the wake panel l, where the subscript k indicates the wake panel position along the span.

As previously stated, the wake has to be modelled. In the three-dimensional code [2] it was assumed that the wake panels remain where shed in the inertia coordinate system, that is, the wake follows the wing path. It has to be noted that, since the wing is moving, the position of the wake collocation points in the body-fixed coordinate system has to be calculated at each time step starting from their position in the inertia frame of reference. The body-fixed position of the wake panels closest to the trailing edge, i.e. those ones that are added at each time step, was set parallel to the chord. Besides, as discussed by Katz and Plotkin [15], their length l_w was set proportional to the time step length.

Equation (2) represents an algebraic system of $N*N_S$ equations but the unknowns are $N*N_S+N_S$ since at time t the doublet strengths of the previously shed wake panels are already derived. This means that, at each time step, N_S unsteady Kutta conditions at the wing trailing edge are needed to solve the system of equations.

The trailing-edge condition, necessary to obtain a unique solution, is derived from the unsteady Bernoulli equation, that is, the conservation of momentum equation for incompressible fluid and irrotational flow. It implies that the second derivative of every involved function, such as the fluid velocity and pressure, exists and is continuous (C^2 space). This is not the case everywhere in the considered domain as the airfoil is a C^0 profile with one singularity at the trailing edge.

Following Cebeci et al. [17] the unsteady Bernoulli equation can be written for a generic point on the wing as

$$p_\infty + \frac{1}{2}\rho\, q_k^2 = p + \frac{1}{2}\rho\, q^2 + \rho\frac{\partial\varphi^*}{\partial t} \tag{3}$$

where p_∞ is the fluid pressure far from the oscillating wing, p the pressure, q the velocity, q_k the kinematic velocity due to the motion of the wing and φ^* the potential function. The fluid velocity is the sum of q_k and the perturbation velocity, which is estimated by the means of the derivative of the perturbation potential φ over the wing. Besides, ρ is the fluid density, which is assumed constant and uniform. At the trailing edge the unsteady Bernoulli equation can be written as

$$p_l + \frac{1}{2}\rho\,(q_l^2 - q_{k\,l}^2) + \rho\frac{\partial\varphi_l^*}{\partial t} = p_u + \frac{1}{2}\rho\,(q_u^2 - q_{k\,u}^2) + \rho\frac{\partial\varphi_u^*}{\partial t} \tag{4}$$

where the subscripts l and u indicate the collocation points of the lower and upper panels that meet at the trailing edge, respectively. It can be rearranged as

$$\frac{p_u - p_l}{\rho} = \frac{1}{2}(q_l^2 - q_{k\,l}^2) + \frac{1}{2}(q_{k\,u}^2 - q_u^2) - \frac{\partial(\varphi_u^* - \varphi_l^*)}{\partial t} \tag{5}$$

to highlight the pressure difference in the vicinity of the trailing edge.

It is important to underline that, as the trailing edge is the domain singularity point, the velocity and pressure can there assume more than one value, that is, a pressure difference can there exist. Besides, equation (5) shows that the unsteady Bernoulli equation alone is not sufficient to obtain the solution: it is necessary to specify the relevant boundary conditions, e.g. the values of velocity or pressure.

It can be noted that the jump in the potential at the trailing edge, whose time derivative is the third term on the right hand side of equation (5), is usually assumed equivalent to the bound circulation Γ, i.e. the line integral of the fluid velocity around the relevant wing section; this is also represented by $-\mu_w$, that is, the constant strength of the wake doublet distribution shed at the considered time step [16].

The existence of a singularity at the wing trailing edge (C^0 space) allows such a result: the jump in the potential at the trailing edge can be generally different from zero and a component of the fluid velocity perpendicular to the wake panel at the trailing edge can then exist.

Besides, as already anticipated, it is here postulated that the pressure difference at the trailing edge could be finite rather than zero for unsteady motion. For example, the flow velocity and pressure difference can be assumed finite at the trailing edge by imposing that the third term on the right hand side of equation (5) there is also finite [1] or by using a linearized expression of the same equation [2]. This would allow considering the variation of the velocities in the vicinity of the trailing edge.

More generally, such an assumption implies that the energy supplied for the wing motion would generate time-dependant trailing-edge vortices. Their overall effect, which depends on the motion initial parameters, would be a jet of fluid that propels the wing. As the kinetic energy is transferred from the jet back to the wing, the vortices would disappear and it is consequently not necessary to assume the fluid viscous.

The postulated pressure difference at the trailing edge is then fundamental for such a model as it can justify the velocity difference that generates the thrust-producing jet. It has to be noted that the trailing-edge vortices could also rotate in the opposite direction, that is, depending on the initial conditions the generated force can propel the wing or oppose its forward motion.

The mentioned pressure gradient (perpendicular to the wake panel at the trailing edge) can be viewed mathematically as a δ function applied at the domain singularity point, that is, the trailing edge. It could be linked to the energy supply by the means of a suitable weight factor that depends on the motion initial parameters. The integral of such a function over the domain would then represent the pressure difference at the trailing edge.

The thrust production can consequently be modelled by taking into account the singularity at the wing trailing edge (that exists as the domain is a C^0 space) and the Boundary Element Method allows such an approach. Moreover, the unsteady motion of the wing causes the pressure difference at the trailing edge and the resulting thrust-producing jet.

In other words, imposing a null pressure difference at the domain singularity point leads to computational results that agree well with experimental data (e.g. see the Introduction in [1]) but do not actually account for the physics of the flapping wing problem.

3 Conclusion

A novel approach to study the unsteady flow around oscillating wing has been proposed. It takes into account an existing domain singularity at the wing trailing edge (generally neglected in the literature) to model closely the physics of the problem, that is, the trailing-edge pressure difference and the resulting thrust-producing jet. The computational results obtained by methods that do not account for such physical features agree well with experimental data but do not explain the thrust-generation mechanisms that are ultimately due to the trailing-edge pressure difference.

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