ALGORITHMS, MATHEMATICS AND ART

V. Ceric
Faculty of Economics & Business, University of Zagreb, Zagreb, Croatia
Trg J.F. Kennedya 6, 10000 Zagreb, Croatia; vceric@efzg.hr

Abstract. Algorithms, mathematics and art are interrelated in an art form called algorithmic art. Algorithmic art is visual art generated by algorithms that completely describe creation of images. This kind of art is strongly related with contemporary computer technology, and especially computer programming, as well as with mathematics used in algorithms for image generation. We first discuss two aspects of algorithmic art: the fact that it is based on rational approach of constructing algorithms, and that it involves a strong constraint that image has to be created by an algorithm. We are then describing mathematical influences to art during history, and especially in Renaissance and during the 20th century. Besides we present works of several artists influenced by mathematics and most notably work of M. C. Escher who attempted to visually express various mathematical concepts like infinity, recursion and self-similarity and studied the use of regular periodic divisions of a plane, convergence to a limit and various transformations of shapes. Then we are describing the main characteristics of algorithmic art as well as its most important proponents and their works, and make distinction between visualization of mathematical objects and algorithmic art. Besides we discuss algorithmic art as the form of visual art notation and compare it with musical notation.

1 Introduction

Mathematics and science had influenced arts from ancient times, and this influence became intensive in Renaissance with artists like Leonardo da Vinci and Albrecht Dürer who were studying and using knowledge on perspective, human body proportions, optics and science of colours. Still the most intensive influence of mathematics and science on arts began in recent times, in the 20th century with artists like M. C. Escher, Max Bill and many others. M. C. Escher, a leading figure among artists that was influenced by mathematics, was developing strange perspectives and perspectives on the sphere, and studied the use of tessellations of a plane, convergence to a limit, as well as various transformations of shapes.

Best known mathematical computing approach to art in 20th century is probably due to mathematician Benoît Mandelbrot who invented fractals, a family of self-similar and scale invariant objects that have a simple recursive definition. He illustrated fractals with computer-based visualizations, and these images were so interesting that they soon became extremely popular and stay like this to the present days.

Algorithmic approach to art existed in classical visual arts, and probably the best known example of an artist using algorithmic approach is again M. C. Escher. Many of his artworks are structured in such a way that it is possible to write algorithms that generate them and indeed a number of algorithms and computer programs that generate Escher's work were developed. It is interesting too that works of an op art pioneer Victor Vasarely were created on the basis of a deliberate plan of their structure and colouring, and it can be said that he was using a kind of algorithmic approach to his work.

Algorithms, mathematics and art are interrelated in an art form called algorithmic art. Algorithmic art is visual art generated by algorithms that completely describe creation of images. This kind of art is strongly related with mathematics used in algorithms for image generation as well as with contemporary computer technology, and especially computer programming.

The rest of this paper is organized as follows. In Section 2 we describe influence of constraints on art, while Section 3 discusses rational approach to art by presenting approaches of several well known artists. Section 4 presents rich relations of mathematics and art. Section 5 presents use of computers and mathematics in art that gave rise to such visual structures as fractals. Section 6 presents the role of algorithms in generation of visual art pieces from the technical side, while Section 7 gives an overview of algorithmic art, its history and main proponents. Section 8 illustrates some algorithmic art methods and techniques, while Section 9 gives conclusions.

Author of this paper used part of the text from his earlier paper [2].

2 Art and constraints

Constraints are present in all aspects in life, and they usually limit humans in accomplishing their goals. Art itself is usually related with freedom, or lack of constraints. However, is it really true that constraints complicate or even make impossible artistic creation? Famous Russian composer Igor Stravinsky wrote following about constraints in art in his essay "The Poetics of Music» [12]: «My freedom thus consists in my moving about within the narrow frame that I have assigned myself for each one of my undertakings". And: "I shall go even further: my freedom will be so much the greater and more meaningful the more narrowly I limit my field of action and the more I surround myself with obstacles».

Decades later another artist, British painter and computer artist James Faure Walker in his book “Painting the Digital River” [14] wrote about algorithmic art: “Its strength lie in its limitations.”, and continues about comparison of work of algorithmic artists with other digital artists: “When shown alongside the photo collages and “processed” images in digital exhibitions, the work of these artists does look more assured”.

3 Art and rationality

Creating of artworks is not (only) an intuitive activity related mostly with emotions and inspiration. During the history of art numerous techniques were devised in order to make the faithful representation of objects or to obtain particular artistic effects.

Perspective is example of the analytic achievement that enabled projection of three-dimensional objects onto a two-dimensional surface. Linear perspective was used by Islamic artists in the Middle Ages and was rediscovered in Italy in the 15th century, during Renaissance. Golden ratio, so popular in visual arts and architecture, is known from ancient times and was largely employed in Renaissance.

Various optical advices like *camera obscura* and *camera lucida* helped artists to make accurate representation of objects they draw or paint. A recent study of British painter David Hockney [6] showed that these kinds of tools were used earlier and in much higher degree than it was previously thought. He found that some of the world's most famous painters like Ingres, Velázquez or Caravaggio used optics and lenses in creating their masterpieces. It is probably needles to say that Hockney considers that the use of optics does not diminish the value of artistic achievement since it is the artist's hand and creative vision that produce a work of art.

Pointillism and cubism are also based on rational approach. Thus Georges-Pierre Seurat in late 19th century, using knowledge of colour theorists on scientific approach to painting, developed pointillism as an analytical technique in which painting is done using discrete colour dots with complementary colours being next to each other. Cubists like Pablo Picasso and Georgeos Braque devised in the beginning of 20th century technique of painting objects from several points of view that provided a novel view on portrayed objects.

A remarkable evidence of rational method of creation of an artwork is given in Edgar Alan Poe’s essay “The Philosophy of Composition” [11]. In this text E. A. Poe gives reconstruction of steps of development of his famous poem “The Raven”. He literary wrote that “… no one point in its composition is referable either to accident or to intuition“, and that “the work proceeded step by step, to its completion, with the precision and rigid consequence of a mathematical problem”. In this text Poe described how he decided about the poem length, the effect to be expressed by the poem, the refrain and its sound (“Nevermore”), a creature repeating his sound (Raven), etc.

It is time now to concentrate on relation of art with mathematics, one of strongest foundations of human rational thought.

4 Mathematics and art

Mathematics and science had influenced arts from ancient times: e.g. façade of Parthenon built in the fifth century B.C. contains a number of proportions such as the golden ratio and the square root of 2, while periodic patterns frequently occur in Islamic and Moorish ornaments. However, it was only in Renaissance that artists began using science and mathematics more intensively. They were influenced by the rediscovery of Greek philosophy and were convinced that mathematics was the true essence of the physical world and that the universe was ordered and explainable in geometric terms [8]. Thus for example two famous Renaissance artists, Leonardo da Vinci and Albrecht Dürer were studying and using knowledge on perspective, human body proportions, optics and science of colours in creation of their works.

However, the most intensive influence of mathematics and science on arts began in recent times, in the 20th century. Importance of mathematics for artists is emphasized by Swiss artist, architect and graphical designer Max Bill in his well-known text on the mathematical approach in contemporary art [1]. He wrote that he is convinced that “it is possible to evolve a new form of art in which the artist’s work could be founded to quite a substantial degree on a mathematical line of approach to its content”.

The best known 20th century artist that was influenced by mathematics was by no means Dutch graphic artist M. C. Escher. Although he was not mathematically trained he was interested in mathematics. In mid 1930th he started to develop mathematical approach to structure his prints, and in construction of his famous “impossible objects” prints he was influenced by mathematician Roger Penrose. In his works Escher developed strange perspectives and perspectives on the sphere and studied the use of tessellations of a plane (i.e. regular periodic divisions of a plane), convergence to a limit, various transformations of shapes, mathematical concepts of infinity, recursion and self-similarity, and he also used polyhedra and Möbius strip in his works. The first stage in process of creation of his works Escher devoted to developing a geometric model of image.

In his work with tessellations Escher used both regular and irregular tessellations and he especially liked "metamorphoses" in which the shapes changed. He exploited basic patterns, triangles, squares and hexagons, applying reflections, translations, and rotations to obtain a greater variety of patterns and also changes the basic shapes into animals or birds that sometimes even left the plane and go to the third dimension. Escher wrote about his work in this field in the lecture about the "Regular division of a plane" [4].

The shape of space fascinated Escher and he created many representations of a non Euclidean hyperbolic space (like in *Circle Limit III*). Escher also had interest in visual aspects of topology, a branch of mathematics working with properties of space that remain unaffected by distortions (like stretching or bending), and was making various graphical representations of Möbius strip. Several of his works present intermingling of inner and outer spaces that sometimes include singularities (like in *Print Gallery*), again revealing his interest in the topology of space.

One of Escher's favourite topics was visualisation of paradoxes of space logic and geometry that was often displayed in his perspective drawings. In perspective drawings one chooses vanishing points which represent for the eye the points at infinity. Escher introduced unusual vanishing points and was using them in positioning of objects of the scene. This gave the effect that up-down and left-right orientation change with the way the viewer's eye look at them (like in *High and Low*). Another type of his paradoxical drawings relies on the brain's persistence in using visual clues to construct a three-dimensional object from a two-dimensional representation (like *Waterfall*).

Douglas R. Hofstadter in his famous book *Gödel, Escher, Bach: An Eternal Golden Braid* [7] draw attention to Escher's use of self-reference, the concept that seems to be the key for understanding intelligence and consciousness. Thus in Escher's work *Drawing Hands* each of the two hands draw another one, while in other works self-reference is not so direct or appears as mutual mirroring of several worlds (like in *Three Spheres II* where three spheres reflects one another and the world around them in their spherical mirrors).

Among interesting mathematically based artistic objects are sculptures based on mathematical objects, as well as different type of perspectives painted on the sphere. Helaman Ferguson is well known mathematically oriented sculptor who is using mathematics as a design language for his sculptures [3]. His sculptures in stone and bronze have been named "theorems in bronze and stone". They can be found on many public spaces and were shown in a number of exhibitions. It is interesting that Ferguson is also an internationally known mathematician whose algorithm has been listed as one of the top ten in the twentieth century.

One of the leading artists working with perspectives painted on a sphere is Dick Termes. He developed his own six-point perspective system that he calls Termespheres, being "visual environments painted on the surface of spheres that hang and rotate from ceiling motors" [3]. These perspectives show everything that one can see from one point looking in all directions. It is well known that geometries that fit on the sphere are totally different from geometries that fit on the plane, and they are related to the study of polyhedra. Six-point perspective is related to the cubical polyhedron that extends in all six directions of the space.

5 Mathematics and computer technology in art

Development of computer technology and particularly graphic cards and software in the second part of the 20th century made an enormous influence on the ability of visualizing mathematics and using these visualizations in design and art.

One of the best known mathematical approaches created in the era of computer graphic and exceptionally suited for generating attractive visual structures are fractals, a family of self-similar (they appear similar at all levels of magnification) and scale invariant objects that have a simple recursive definition. Fractals were invented by a mathematician Benoît Mandelbrot who illustrated them with computer-based visualizations, and these images were so interesting that they soon became extremely popular.

Some interesting examples of fractals are Mandelbrot set and Julia set. Mandelbrot set is defined by a family of complex quadratic polynomials. It is a subset of a complex plane, i.e. a set of complex numbers or points in the complex plane the boundary of which does not simplify at any magnification. A picture of the Mandelbrot set can be made by different type of colouring, e.g. by colouring all the points which belong to the Mandelbrot set black, and all other points white. In the centre of the Mandelbrot set image is the characteristic large cardioid-shaped region. Various algorithms are used for producing computer drawings of the Mandelbrot set. They are quite simple, and e.g. the so called "escape time algorithm" has only about a dozen of steps. This is a good example of a simple mathematical model generating complex results.

Some of the other mathematically based methods used for generation of attractive images are genetic algorithms and cellular automata. A number of interesting examples of cellular automata visualization are presented in the Stephen Wolfram's book "A New Kind of Science" [15]. Cellular automata are also used for generation of random numbers for the Mathematica software, which is again an excellent example of complex structures generated by a simple algorithm.

As can be seen, a lot of work in this field has been done with the goal of visualizing mathematical objects. This approach is certainly valuable for the sake of research in mathematics, but it doesn't necessarily lead to interesting visual solutions to the so called fine art or 'high art' [3]. Too much precision, symmetry or repetition doesn't have much chance to be regarded as art. However, mathematical approach can inspire artists to use their imagination and creativity and to use it in the process of creation artworks.

Quite a different approach is to use mathematics as a tool in developing artists' ideas. Following this approach artists typically use rather simple mathematics that enable accomplishment of their idea but is of no interest from the mathematical point of view. In such case a lot of experimenting is typically needed in order to achieve visually attractive result. It is this kind of approach that is mostly used in algorithmic art.

6 Algorithms in art

Algorithmic art is based on generation of visual artworks using algorithms, i.e. precise defined procedures that computer executes step by step. Algorithms are implemented in computer programs coded in some programming language. Algorithm, or computer program implementing it, describe the process of generation of image that we can see either on the screen or as printed. Program for image generation contains the author's idea about the image as well as the technique by which this idea is transformed into an image. It also has to define which graphical elements and their structures should be generated (straight or curved lines, shapes, a group of elements with a specified structure, etc.), what are values of their parameters (e.g. position of a rectangle, its dimensions and its elevation toward axes), colours of lines and shapes, etc. Algorithms use computational structures such as loops, subprograms and recursion, as well as various mathematical expressions.

Algorithmic artists (also called "algorists") as a rule don't use special graphical software but rather general program languages that enable drawing basic graphical elements like line, circle or rectangle. Such approach requires more work but offers much more flexibility and freedom in expression. Using general program languages algorists develop their own software in which they embed their artistic ideas. After the image is generated no further intervention with image processing tools like Photoshop is done. Interventions for changing image are done merely by changing the algorithm (i.e. the corresponding computer program).

The algorithms could also be executed by human hand, and they have been executed by humans before computers came into arena (and sometimes even after that). However, the evident advantage of computer and its output units is in enormous processing power of contemporary computers and precision of its output units. Because of that they can execute very complex algorithms that include a huge number of graphical operations in a rather short time and without mistake. This enables routine execution of algorithms that would require years to be executed by hand. All this gives computer technology a role of enabler of a real revolution in algorithmic art.

Another particularly important fact is that analysis of consequences of changes in the algorithm, i.e. in the corresponding computer program, is considerably simpler and faster in comparison with repeating of manual construction of somewhat changed image. This enables intensive experimentation needed for discovering the most appropriate artistic result.

Algorithmic approach to art existed in classical visual arts. Probably the best known example is M. C. Escher whose mathematical inclination was already mentioned here. Many of his artworks are structured in such a way that it is possible to write algorithms (computer programs) that generate them. This was indeed done by several authors from the computer science field. One example is Michael Trott who in his book "Mathematica Guidebook for Graphics" [13] constructed a computer program that generates image of the Escher's well-known lithograph Reptiles.

Works of op art pioneer Victor Vasarely were created on the basis of a deliberate plan of their structure and colouring, as can be seen from Vasarely's blueprints. Robert C. Morgan in his book «Vasarely» [10] wrote: «He created algorithmic systems that in many ways parallel the development of the computer», and also: «Vasarely made his own visual software in the form of mathematics, referring to the blueprints of his works as programs».

Since algorithmic art consists of generation of images on the basis of algorithms, algorithms can be viewed as a notation, and notation is something that music has but visual arts in general miss. There are further parallels of algorithmic notation with music. First, is not the author but the computer that executes the algorithm. Moreover, algorithm can be executed by different computers, using different operation systems, programmed in different software and presented by diverse output devices with various resolutions and other characteristics.

7 Algorithmic art

We review here some of the key events in the history of computer generated art and present some of the best known authors from the field of algorithmic art.

First experiments with usage of computers in visual art were performed in late 1950th and early 1960th. More intensive development of algorithmic art began in early 1960th, and pioneers were two researchers in the area of computer graphics, Dr. A. Michael Noll from USA and Prof. Frieder Nake from Germany. Dr. Noll also wrote the first computer generated ballet in 1965.

In 1963 first computer art competition was held, sponsored by the U.S. journal "Computers and Automation". In 1965 first computer art exhibitions were held at Technische Hochschule in Stuttgart and at Howard Wise Gallery in New York. In 1971 Herbert Franke published the book "Computer Graphics - Computer Art" [5] in which he described the tools and principles of computer generated graphics, its artistic applications, history of computer art and its theoretical foundations. In 1976 Ruth Leavitt edited the book "The Computer in the Visual Arts" [9] which presented texts of three dozen of early computer artists, and among them algorithmic artists like Vera Molnar, Manfred Mohr, Edward Zajec and Charles Csuri.

Several authors proposed the name "algorithmic art", and Benois Mandelbrot wrote that "… astonishingly complex and beautiful graphics can be generated by surprisingly plain algorithms. Hence the term algorithmic art, which I use at present" [3].

Some of the best known algorithmic artists we present here are Vera Molnar, Jean-Pierre Hébert, Roman Verostko, Manfred Mohr and Charles Csuri. What is particularly interesting is their background and how they approach algorithmic art.

Vera Molnar was born in Budapest, Hungary in 1924. She was a classical artist using geometric themes, and her aim was to create valuable works of art in a conscious way. In 1968 she began to use a computer to assist her. She started to create a series of abstract works generated from a procedure in which simple geometrical shapes and their combination were successively altered in small steps, and she wanted to find which alteration lead to the aesthetically most appealing result.

Jean-Pierre Hébert was born in Calais, France, in 1939. He is educated as a classic artist and he started to create digital conceptual algorithmic art in 1974, while in 1979 he engaged in intensive experimentations in creating plotter drawings with computers. His professional background in engineering and computer programming languages originally led to a consulting career, though he maintained his interests in arts and mathematics. In 1983 he moved to United States and ceased consulting to devote himself fully to art. Hébert is experimenting with various media from paper, glass and mirror to wood, steel, and sand. He is also working on digital video animations, computerized kinetic sculptures, and organic algorithmic drawing devices such as paint-dripping pendulums. In 2003 Hébert was appointed Artist in Residence at the Kavli Institute for Theoretical Physics at University of California at Santa Barbara.

Roman Verostko was born in Tarrs, Pennsylvania, in 1929. He was educated as artist and historian, but spent 16 years as a Benedictine monk. In the beginning of 1980th, after 30 years of work as a traditional visual artist, he started working with algorithmic graphics using classical pen plotter. He is one of the most original algorithmic artists, and besides a voluminous art production he also wrote a number of essays on algorithmic art and on his own methods of work. He is using the term epigenetic to describe the algorithmic procedures in his work, identifying the biological analogues for art works executed with algorithms created by artists. In his own words: "The epigenesis of organisms is the process whereby a mature life form grows from its seed. In this analogy the software may be viewed as genotype or the seed that contains all the information necessary for growing the mature form."

Manfred Mohr was born in Pforzheim, Germany, in 1938, He was educated as an artist, but he also used to play tenor-saxophone and oboe in a jazz group. In 1969 he made his first drawings with a computer. From early 1970ties he began his artistic work on a cube structure, and in 1977 he begins to work with the 4-D hypercube and graph theory. In late 1980ths he extends work to the 5-D and 6-D hypercube. In 1998, after using exclusively black and white colours for three decades, he starts using colours to show the complexity of the work through differentiation. In his own words: "What interests me, are the two-dimensional signs (graphics) and their visual ambiguity resulting from the projection of the lines of the cubes from higher dimensions into two-dimensions". Manfred Mohr received a number of international prizes for his work. Since 1981 he lives and works in New York.

Charles Csuri is an artist and computer graphics pioneer. He joined the faculty at the Ohio State University and also exhibited his paintings in New York City from 1955-1965. In 1964 Csuri became interested in the digital computer and experimented with computer graphics technology, while in 1965 he began creating computer animated films. His research activity in computer animation and graphics has received international recognition and he directed basic research in computer graphics for over twenty years. The results of the research have been applied to flight simulators, computer-aided design, visualization of scientific phenomena, magnetic resonance imaging, education for the deaf, architecture, and special effects for television and films. He co-founded Cranston/Csuri Productions which produced animation for all three major U.S. television networks. Csuri exhibited at the 42nd Biennale de Venezia, Italy, 1986. *Ars Electronica*, a major international competition on computers and the arts held in Austria each year, awarded him prizes in 1989 and 1990.

8 Some algorithmic art techniques

In order to provide better understanding about generation of algorithmic art we will illustrate some algorithmic art methods and techniques, and these will be techniques used by the author of this text since it is rather difficult to describe approaches and techniques used by other artists because of the lack of appropriate information.

The author of this text uses several methods for generation of algorithmic artworks: let us roughly call them constructivist approach, mathematical modelling approach, and digital manipulation of photographs (or in fact any kind of images). There are no strict boundaries between these methods, and especially use of mathematics is definitely not limited to mathematical modelling approach but appears in two other categories too. Here is a description of these methods along with a few examples of images generated with each method.

We will start with a *constructivist approach* as the technically simplest one. This approach is based on rather simple preconceived structures that can be rather straight described by algorithms. Example of such structures is a network of regularly positioned squares (or some other geometric figures) that can have different sizes and colours. Algorithms for generation of such structures are in principle easy to formulate and programs developed on basis of these algorithms are primarily useful in fast analysis of numerous possible variations of image form, thus providing a rich source of variants for a selection of visually most interesting images. Besides, algorithmic approach power also lies in enabling generation of extremely precise images.

Let us demonstrate this approach by example of a graphics from the series "Cartesian rhythm" based on a matrix of squares, as can be seen on Figure 1.

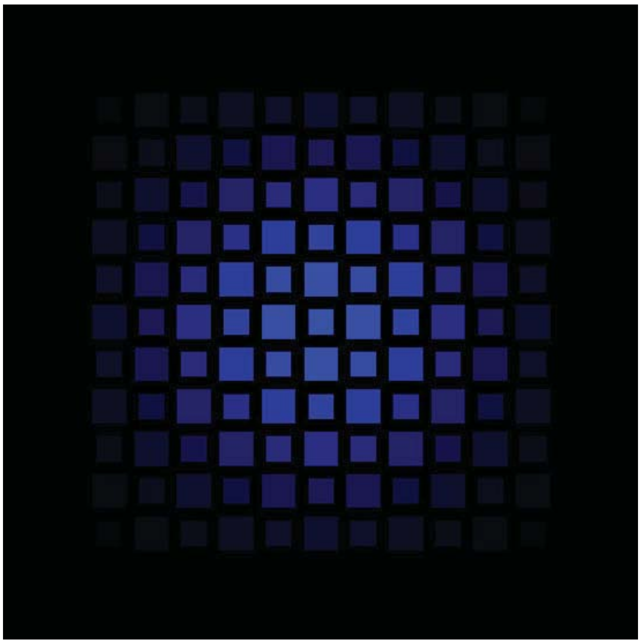

Figure 1. Example of a constructivist approach – graphics "Cartesian rhythm 11" by V. Ceric

Algorithmic approach enables one to study influence of various parameters on generated image. One can e.g. analyse how many groups of squares of different sizes give the best visual result. In this case it was found that the best visual rhythm is obtained when only two groups of squares of different sizes are used. After that one would like to know in which manner these two types of squares should be spatially intermixed, what is the optimum ratio of sizes of two squares and what is the optimum distance between squares. After resolving these questions follows the study of colours to be used for a pleasing aesthetical result: should all squares be of the same colour or should e.g. smaller squares have different colour than bigger ones.

As can be seen on Figure 1 for graphics "Cartesian rhythm 11" it was decided that the colour of both groups of squares should vary in such a way that a particular variant of blue colour from the central square looses its colour in a centrally symmetric manner and becomes darker and darker toward edges, finally disappearing on a black background. This radial fall of colour intensity of squares leads to an interesting circular effect obtained by using squares.

Somewhat more complicated image also based on a matrix of squares is graphics "Order 1" from the series "Order and Chaos" shown in Figure 2.

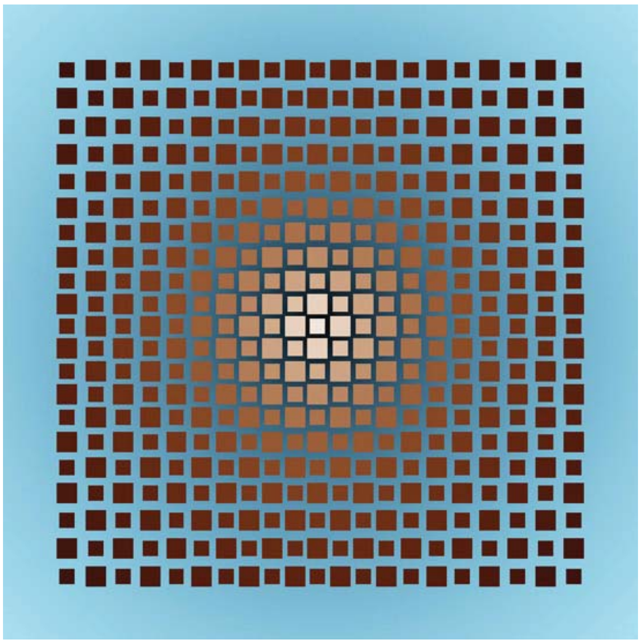

Figure 2. Example of a mixture of constructivist and mathematical modelling approach – graphics “Order 1” by V. Ceric

This image can be regarded as a mixture of constructivist and mathematical modelling approach. Two sets of squares of different sizes are used again, but now the colour of both sets of squares is from almost white in the centre to dark brown on the periphery. Colour of the background is changing in opposite direction, from dark blue in the centre to light blue on the periphery. As a consequence of such behaviour of colours the distance between squares in the centre should be smaller than distance between squares on the periphery of the plane (and this distance should behave as a centrally symmetric function) because in that case more densely populated brighter centrally positioned squares would together give stronger shining effect. Such arrangement of squares also gives a more dynamical appearance of the whole image. So we see that algorithm that generates this image requires somewhat more mathematics than one for generation of “Cartesian rhythm 11”.

Mathematical modelling approach starts from the rough idea about how the image should look like and requires intensive experimenting in order to find an appropriate mathematical model that gives interesting visual result. So it should be clear that in this approach mathematical modelling is not used in order to visualize mathematical objects but rather to obtain interesting images by more intuitive approach.

We will demonstrate this approach by example of the graphics from the series “Nexus” shown on Figure 3.

The technique used in generation of works from this series was often used in 20th century by geometrically inclined artists. This technique is based on the division of squares on two right triangles with two equal sides. There are just two ways this division can be done, by dividing triangle with a diagonal going from lower left vertex to upper right vertex or from lower right vertex to upper left vertex. If we use just two colours for colouring triangles (e.g. black and white) then there are evidently just four different divisions of a square. Although such small number of possibilities may seem insufficient for producing interesting images, it has a potential for complexity if image contains enough square divisions. However, if one works in a traditional manner then one can either use a small number of square divisions in order to be able to construct a pleasing image. If larger number of square divisions were used then traditional artists were only able to employ simple rules governing which of four divisions will be used for a square on a particular location in the network - otherwise it would be an enormous job to calculate results of a more complex formula by hand and apply it.

Now, a power of mathematical and algorithmic approach married with the use of high-speed processors enables one to work with a large number of square divisions and simultaneously use as complex mathematical relations as one wish in order to obtain interesting visual structures. The basic idea behind Nexus series was that the type of square division would depend on the distance of the particular square from one or more points on the plane. So the whole complexity of Nexus graphics structure was translated into the selection of the mathematical form of distance of the square, and this distance was defined as some function of distances of the square in some location from the two selected points in a plane. Exact form of this function was determined after analysing visual structures obtained in numerous experiments. As can be seen, the particular selection of the distance function in Nexus 1 is giving interesting and nontrivial graphics giving the impression of some kind of circular flow of triangles on the plane.

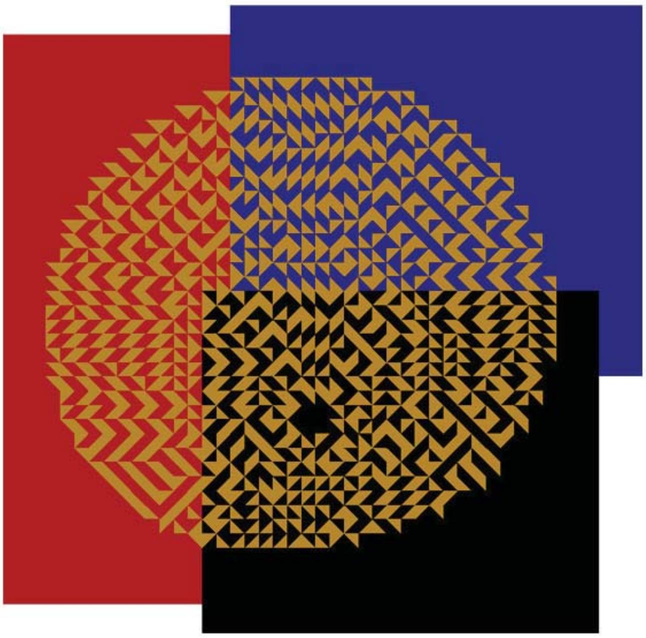

Figure 3. Example of a mathematical modelling approach – graphics “Nexus 1” by V. Ceric

Another quite different type of mathematical modelling approach was used in generation of graphics from the “Unclassified objects” series, one example of which is shown in Figure 4. These geometrical structures in strong black and gray transitional tones are based on a relatively simple idea of successive covering of a plane with rectangles, where positions of rectangles in the plane, their shapes and sizes as well as level of gray color applied to them are governed by mathematical models based on parametric equations that use periodic functions. Interplay of sharp geometrical structures and black and gray tones leads to intensive feeling of volume of these works.

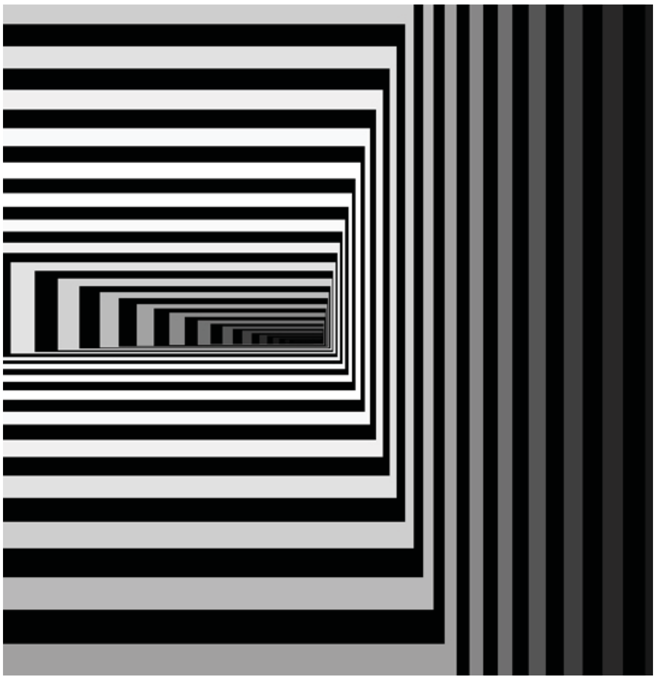

Figure 4. Another example of a mathematical modelling approach – graphics “Unclassified objects 14” by V. Ceric

For generation of some images in this series only a few dozen of iterations of rectangle generation were needed, while for others hundreds of iterations of rectangle generation were performed. Careful selection of initial and final iteration number is required in order to produce a clean, closed and interesting image. However, plentiful experiments have to be carried out in order to achieve this goal.

Digital manipulation of photographs (or in fact any other images) is using algorithmic approach for manipulation of pixels of a photograph. Because of an enormous richness of colour spectrum and structure of the world surrounding us, richness that surpasses almost anything that can be created artificially, the images obtained in such a way can be quite complex and interesting. Besides that, the author discovered that this approach enables transformation of figurative input photograph into an abstract generated image.

We will illustrate one specific digital manipulation approach by example of the graphics from the series “Spectral variations” shown on Figure 5.

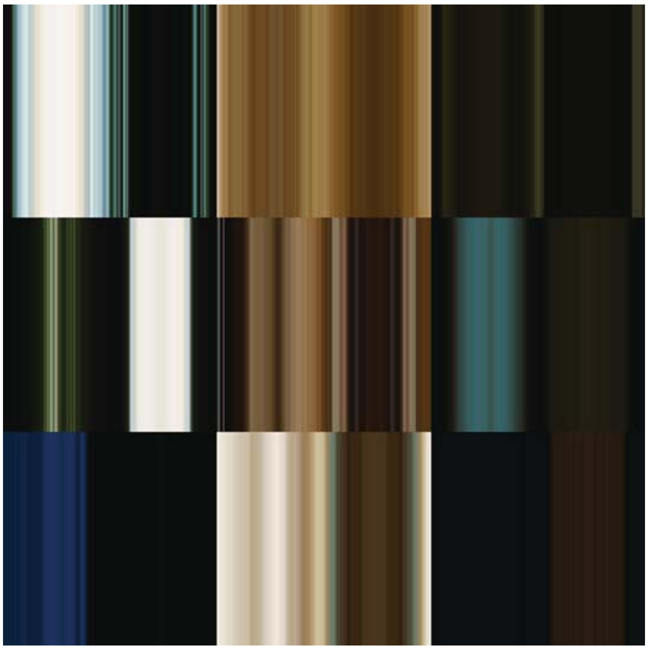

Figure 5. Example of a mathematical modelling approach – graphics “Spectral variations 10” by V. Ceric

In the first pre-processing stage we transform input photograph (or any kind of input image) into a two-dimensional pixel matrix using the *Digital Image Processing* add-on to the *Mathematica* software. After that we divide this matrix into *n x n* smaller pixel sub-matrices made from the corresponding parts of the input photograph. Then we use *Mathematica* to transform pixels of all created sub-matrices using algorithms based on some mathematical procedures that enable displacement of sub-matrices pixels to different locations (coordinates) in the corresponding sub-matrix. In the final post-processing stage we again use *Digital Image Processing* add-on in order to compose a final image from the *n x n* transformed parts of the original photograph. For the “Spectral variations” series input photograph was a small 420 x 420 pixels photograph of a face of the author, so we can say that graphics from this series present the very author of the work.

9 Conclusions

We have seen that not only mathematics and algorithms have influenced art for a long period of time, and particularly in the 20th century, but also that constraints in general can be fruitful in forcing artist to concentrate his creative forces and search for the artistic possibilities hidden in particular constraint. We also realized that rational approach is not so rare in art and that it has the power to open doors of art.

Algorithmic art itself, a discipline that unites mathematics, computing and art, has a rather specific character since the author has to possess rational abilities required to compose the algorithm and write the corresponding computer code correctly from both the syntactic and semantic point of view, but he also needs intuitive and aesthetic abilities required to select visually promising alternatives. The author of algorithmic artworks constantly evaluates visual outputs obtained during experimentation with the program and on the basis of these evaluations makes changes to the program until satisfactory visual results are obtained. Undoubtedly, experimentation is also present and important in a traditional art, but algorithmic art is using its full potential.

10 References

[1] Bill, M.: *The Mathematical Way of Thinking in the Visual Art of Our Time*. Werk, No. 3 (1949).

[2] Ceric, V.: *Algorithmic Art: Technology, Mathematics and Art*, In: Proc. 30th International Conference Information Technology Interfaces ITI’2008, Cavtat, 2008, 75-81.

[3] Emmer M., editor: *The Visual Mind: Art and Mathematics*, The MIT Press, 1995.
[4] Escher, M. C.: *Escher on Escher: Exploring the Infinite*, New York: Harry N. Abrams, Inc., 1989.
[5] Franke, H. W.: *Computer Graphics –Computer Art*. Phaidon Press Ltd., 1971.
[6] Hockney, D.: *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters*, Thames & Hudson, 2001.
[7] Hofstadter, D. R.: *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, 1979.
[8] Kline, M.: *Mathematics in Western Culture*, Oxford University Press, 1964.
[9] Leavitt, R.: *The Computer in the Visual Arts*, Harmony Books, 1976.
[10] Morgan, R. C.: *Vasarely*, New York: George Braziller, Inc., 2004.
[11] Poe, E. A.: *The Philosophy of a Composition*, Graham's Magazine, April 1846.
[12] Stravinsky, I.: *The Poetics of Music – in the form of six lessons*. Oxford University Press, 1947.
[13] Trott, M.: *Mathematica Guidebook for Graphics*, Springer, 2004.
[14] Walker, J. F.: *Painting the Digital River*, Upper Saddle River, NJ: Prentice Hall, 2006
[15] Wolfram, S.: *A New Kind of Science*, Wolfram Media; 2002.