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CAS - BASED E-LEARNING FOR THE IMPROVEMENT OF REFRESHER COURSES IN MATHEMATICS

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Abstract. The sheer increase in scientific output the information age created brought a number of serious challenges to university lecturers. Most scientific fields get more and more diverted. The usual five years of an engineering program are therefore barely enough to teach the basic knowledge plus advanced knowledge for a very small part of a certain engineering discipline. This leads to an even steeper learning curve, and often to a reduction in the basic subjects which are no direct part of the discipline, especially mathematics.

A new approach was therefore devised at the Vienna University of Technology for students enrolling in the bachelor program for electrical engineering. The Maple T.A. software package was introduced into the courses in two steps.

First, a refresher course in mathematics is offered. The content of this course is a balance between the level of the general qualification for university entrance, allowing students to fill the gaps in their knowledge that would hamper their understanding of the electrotechnical lectures. Afterwards the Maple T.A. system is used in the exercises accompanying the mathematics lectures.

This paper focuses on the voluntary refresher course, which had a very wide acceptance, and Maple T.A. proved to be a very versatile tool, but showed a few shortcomings when trying to teach very basic subjects.

1 Introduction

In the last years, the mathematical knowledge of the students enrolling in the electrical engineering program at the Vienna University of Technology (TU Vienna) dropped significantly. While this was no concern whatsoever to the lecturers in mathematics, it affected the ability of the students to follow the basic lectures in electrotechnology and physics. Students had to concentrate their attention on the underlying mathematics and not the respective subject, resulting in content being learned by rote instead of understood.

The reasons for this trend where identified as following:

- The amount of mathematics, along with other natural sciences, was reduced in the curricula of Austrian university-preparatory schools over last ten years. On the other hand, rapid development in the field of electrical engineering made it impossible to adjust the bachelor and master program accordingly.
- Calculators with inbuilt computer algebra systems (CAS) became cheap and widely available. This lead to whole classes learning higher mathematical subjects only using these calculators. Apart from the inability to perform calculations manually, students graduating from such classes or schools have often a very high disregard for mathematics, as they believe that everything can be calculated using the CAS tools used in their education. This trend is further emphasized by the companies producing these calculators, which of course do not point out the weaker points of their products and the software used.
- The advances in information technology also feeds this notion as the amount of knowledge needed for using personal computers dropped significantly up to the point where they became 'black boxes' for the average user.
- Not all students enrolling in the bachelor program graduated from university-preparatory schools or secondary engineering colleges. This horizontal mobility, which grows higher over the years and is in fact encouraged, leads to students entering the program missing several important branches of mathematics.
- Military or civil service for the male population was made compulsory after the secondary education instead after the tertiary education. This leads to a year of lowbrow activity for the over ninety percent male beginners of the electrical engineering program.

Additionally the advanced technical colleges, which were not present in Austria until 1994, have lost their novelty by now and are competing aggressively for students, often offering special courses for beginners.

To keep the bachelor program in electrotechnology attractive and to reduce the dropout rate due to the lower mathematical knowledge of the beginners, the vice rector for academic affairs, former dean of the faculty for electrotechnology, approved and financed a one year trial of a beginners course in mathematics for the electrical engineering program. The Institute for Analysis and Scientific Computing held this course, called 'AKMATH - Auffrischungskurs Mathematik für Elektrotechnik' (ger.: 'Refresher Course in mathematics for electrotechnology').

2 Design of the course

Several requirements were defined for the course at the start of the project, in January 2008:

- The content should encompass several topics from the curriculum of university-preparatory schools which were reckoned to be most important in the first term of the bachelor program were chosen. For those schools, learning outcomes are already defined and will become compulsory in a few years time (for details on the content see section 2.3).
- All parts of the course which need attendance are to be held in the first two weeks of the term.
- The has to be augmented with E-learning, allowing the students to train their skill before the start of the term and after the first two weeks.
- The course has to be held in the afternoons and evenings so it does not collide with the compulsory lectures.
- A voluntary knowledge test should be available before the start of the course, allowing possible students to decide whether they need the course, or even if they are suited for this bachelor program at all.

These requirement lead to the AKMATH being devised as a blended learning course.

2.1 E-learning Platform

The first and most important choice was therefore the choice of the the e-learning platform used for the course. Programming of a custom e-learning system meeting all the requirements was excluded from the start, as there were neither the time nor the resources available. Considered were the TUWEL (TU Wien E-Learning platform, a localization of the open source Moodle platform), a semi-experimental e-learning server based on MATLAB which is already used by the working group for mathematical modeling and simulation, and the commercial software Maple T.A.

The TUWEL system was considered first. On the one hand, this system already had all the interfaces needed, especially regarding user administration, administration of tests and results and even featured randomized tests. But it lacked the capability to handle mathematical input at a higher level. Therefore either highly specified input would have to be demanded from the students, or a multitude of input fields would have to be provided. Both methods have huge implications on e-learning, as each variant gives valuable hints about the solution to the student, and both do not encourage creative thinking. Programming a parser to solve this problem was out of the scope of this project. Furthermore, the usage of this system was next to impossible to non-students, limiting the capabilities for the voluntary knowledge test.

The MATLAB server already used by the institute for e-learning in modeling and simulation addressed the main problem, a certain freedom in mathematical input, by using the symbolic calculation toolbox included in MALAB. On the other hand, this server lacked the capability for user administration and especially for recording test results. While these features could have been programmed, the license provided by The Mathworks does not cover the usage of the system by people not enrolled in or working for the TU Vienna.

The Maple T.A. 3.2 (Testing and Assessment) software package was considered last, because it was not used by the TU Vienna up to this point. As a professional product especially designed for the purpose in mind, it easily fulfilled all the requirements. Its license includes the usage for non-students and additionally it is backed by the Maple CAS system. (During July 2008, Maple T.A. was upgrade from version 3.2 to version 4.0 without any problems.)

2.2 Additional Software

Although Maple T.A. features the capabilities for self enrollment of users, those were disabled. This was mainly due to security reasons, as they did not feature a CAPTCHA (Completely Automated Public Turing test to tell Computers and Humans Apart) system to fend of automated software.

Therefore a simple PHP script attached to a MySQL database was implemented for the website of the institute, requiring a possible user to possess a valid email address at least. A CSV file was send daily to an administrator, who checked it for irregularities before registering the user for the Maple T.A. server. This PHP script was also used to send prepared combinations of user and password to people interested in the public knowledge test. These users were disabled after the start of the AKMATH.

2.3 Content of the course

The content of the AKMATH was implemented on the Maple T.A. server, taking suggestions from books like [1] and [2]. The topics finally were divided into eight modules, in accordance with the deanery of electrotechnology, and the course was integrated in the timetable of the first term. For each module, between thirteen and forty questions were available.

1. fractions and powers: definition; prime decomposition, greatest common divisor and least common multiple; basic operations with fractions and powers, fractions and decimal numbers, floating point notation,
2. terms: basic operations with terms; conversion; polynomials; binomials; factoring
3. basic functions I: definition; linear function and equation; absolute value; power functions; rational functions; square root; exponential function; logarithm;
4. basic functions II: sine, cosine and tangent function; geometrical interpretation; goniometric equations;
5. derivatives: definition and geometric interpretation; rules; derivatives of basic functions; extreme points and inflection points
6. integration: definition and geometric interpretation; antiderivatives of basic functions;
7. vectors and coordinates: definition; length, basic operations, angles and inner product in two and three dimensions, cross product
8. complex numbers: definition; complex plane; basic operation; conjugate complex numbers; solution of the quadratic equation

Apart form the on line questions, handouts ranging between eight and fifteen pages were prepared for each module. For the motivation of the students, each handout started with an application of the content in electrical engineering, from Ohm's law to linear equation systems using Kirchhoff's circuit laws.

2.4 Experiences with Maple T.A.

Maple T.A. proved to be a very versatile tool. Apart a font problem discussed in the results section, no further critical problems were found during the course. But the mathematical capabilities of Maple T.A., especially the parser, are sometimes to extensive for simple problems. As the parser for example knows that the inputs

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4x x     4*x*x     (2x)^2     2^2x^2
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etc. actually are the same mathematical expression, in case of very simple examples entering the problem will already give the student full marks. The only way around this problem is to forbid every simplification of a formula. This leads to the same problem mentioned in section 2.1. But as this was a voluntary refresher course, this was left to the discretion of the students, and the parser was enabled.

It should be mentioned though that there are still some problems with the creation of proper MATHML code, especially in case of the interaction with random generated values, leading to formulas where a plus sign and a minus sign are next to each other. This is mathematically sound, but proved to be a obstruction to the participants of the course which are not used to more complex mathematical notation.

2.5 Adjusting the knowledge test

To test whether the difficulty of the voluntary knowledge test was appropriate, three classes at the start of the last year of university-preparatory schools could be won over for trying the test at the beginning of September. They were presented with an easier version and the intended version of the knowledge test (omitting integration, as this is only content of the last year). Results showed that the intended version was slightly more difficult then their capabilities right after the summer holidays, so it was assumed that level of the intended test was achievable in the course of the AKMATH, and the more complex test was chosen as the knowledge test.

2.6 The control group

For comparison, the lecturer of 'Mathematik 0 für Vermessungswesen' (ger.: mathematics 0 for surveying and mapping') volunteered to use the test devised for the AKMATH to test her students as well after the first and the last lecture of this course. The course has a similar content, but is held on a weekly basis in the first 9 weeks of the term.

3 The Course

The students were divided into six groups according to an arrangement by the deanery of electrotechnology which enables them to attend all first year practices and laboratory tutorials without interference. Each module was held twice, each time for three groups, at different times. The division into six groups was only informal. This, together with the two lectures, allowed also students which had other appointments to take part in the whole course. Each module consisted of

- A 45 minute lecture. In this conventional lectures, the theoretical basics of the modules content were explained and a few simple examples were shown.
- A 90 minute exercise. Depending on availability of resources, this exercise was either held in a PC laboratory, where the students practiced directly on the Maple T.A. Software, with a tutor supervising them an helping them along. Or if there was no laboratory available, the tutor printed hard copies of the on line examples and then practiced with students conventionally using chalk and blackboard.
- After the exercise, the content of the on line system was available to the student to practice on their own.

The exact content of every module was published before the course, allowing students with deficits in just a few modules to focus on these, instead of having to sit through the whole course up to the point the content they are interested in is presented.

In the second half of the first and last practice module, a test was taken by the students. The first test was an exact copy of the knowledge test (of course with randomized numbers for each student) while the last one was a slight variance, enabling the assessment of progress for each student.

These test were held in PC laboratories under the supervision of one or several proctors who made sure that the student really engaged in test. This was necessary because one ECTS credit point was awarded to each student taking part in both tests, an as the results were to be analyzed. So it was vital to prohibit students from just clicking though the test in a few minutes to get the the credit, but spoiling the statistical results at the end, as no passing score was set to encourage students unsure of their knowledge to take participate in the final test, too. This should assure that the fraction of the students not improving although participating also contributed to the evaluation.

Shortly after the test, the content of the AKMATH was moved to the Maple T.A. course for the regular mathematics lecture in the first term.

The control group passed the tests on the 10th of October and the 3rd of December under the same conditions.

4 Results and Outlook

4.1 Attendance

Over 200 students used the anonymous knowledge test in the September and the shortly before the course in October. About 180 students requested a regular login for the Maple T.A. server to be able to access the e-learning content. 150 students, or roughly half of the beginners, took part in some or all of the modules, and 88 students participated in both the knowledge test and the final test. As can be seen in table 1, the practices were slightly less attended then the lectures, which implicates that student get more confident in their capabilities during the lectures and do not attend the accompanying practice any more.

When comparing the numbers for lectures and practices at different times, the late lectures were attended by fewer students. Therefore the late time alienated more students and appealed to fewer working students then expected. It also has to be noted that the time slots where switched back and forth between the two groups, assigning each group four times to the earlier slot and four times to the later one, which proved this conclusion, as a significant part of students which are only able to take the later time slot would have improved the attendance at the cost of the earlier lectures and practices.

4.2 Test Results

The results are very promising. Among those 88 students, the mean score was 3.89 out of 10 points, with a variance of 4.19. After the course, the mean value improved to 5.96 out of 10. The variance nearly stayed the same, slightly falling to 4.15. This indicates, that the improvement was not due to just a few students. Figure 1 shows histograms of results both before (in red) and after the AKMATH (red).

Table 2 shows detailed results for the ten questions asked, with the better result being emphasized.

In eight out of ten cases the students got better, often by significant amounts. In case of the trigonometric function, the decrease of less then eight percent is not statistically significant. At last, in case of the manipulation of powers, an error occurred while synchronizing databases. An old instance of the question was used, which led to problems with the certain pairs of letters. In the font used by Maple T.A., the combinations (g,9),(l,1),(x,X) and (O,0) are not easily distinguished. This led to a significant amount of answers Maple T.A. graded as wrong, while basically the student had understood the problem and solved it correctly.

It is especially noteworthy that the students improved from 0.456 percent to 0.818 percent on the subject of complex numbers. As briefly mentioned in section 3, the final test took place after the first halve of the final exercise. This shows that students clearly benefit from such an intensive course a start of their bachelor program.

subject	time	attendance lecture	attendance practice
fractions and powers	15-16.30	82	70
	17-19.30	55	55
terms	15-16.30	75	67
	17-19.30	32	27
basic functions I	15-16.30	49	55
	17-19.30	49	39
basic functions II	15-16.30	55	55
	17-19.30	48	44
derivatives	15-16.30	65	54
	17-19.30	30	23
integration	15-16.30	55	55
	17-19.30	45	34
vectors and coordinates	15-16.30	54	30
	17-19.30	65	53
complex numbers	15-16.30	30	37
	17-19.30	75	64
daily average		107,875	95,25

Table 1: attendance of lectures and practices of the different modules

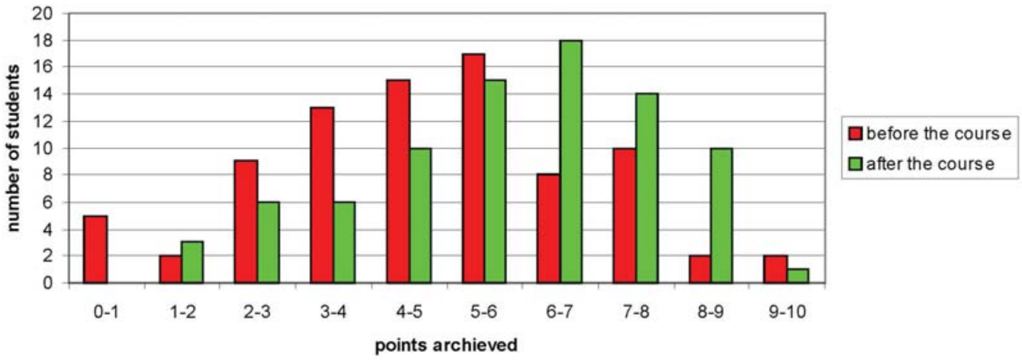

Figure 1: Histogram of the points achieved the knowledge test (red) and the final test (green).

question content	percentage of correct answers knowledge test	percentage of correct answers final test
polynomial division	0.288	**0.579**
manipulation of terms	0.496	**0.806**
powers	**0.664**	0.443
basic functions	**0.406**	0.329
powers and fractions	0.488	**0.681**
cross product	0.12	**0.386**
differentiation	0.184	**0.477**
integration	0.008	**0.193**
complex numbers	0.192	**0.556**
nested fractions	0.456	**0.818**

Table 2: percentage of fully correct answers in the two tests

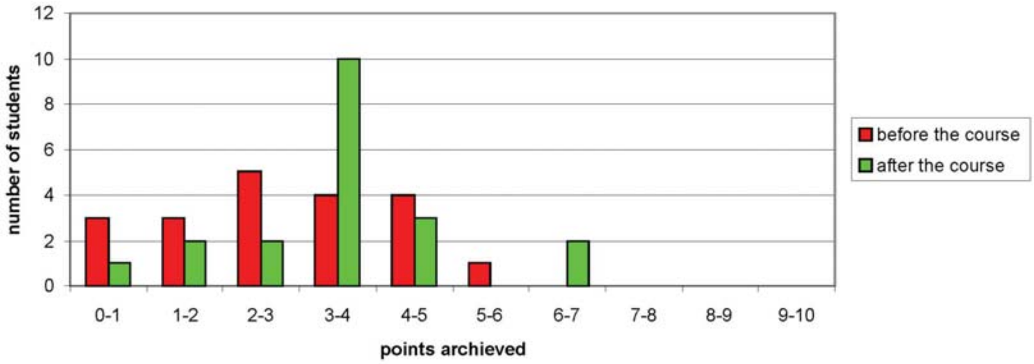

Figure 2: histogram of the points achieved the knowledge test (red) and the final test (green) by the control group

4.3 Results of the control group

Although the control group had much more time to memorize the content, the results were not better. Due thr to small size (only 20 participants, or about two-thirds of the beginners of this bachelor program), the statistical significance is of course much smaller. They increased from an average of 2.8 points to an average of 3.56 points. The Variance slightly lowered from 1.96 to 1.77. But as seen in figure 2, half of the students had between three and four points at the end of the course, while at the start the knowledge was quite distributed.

Analysis of the individual questions showed that in neither test a single student was able to integrate, and not more then one was able to differentiate or work with complex fractions. This is explained easily with a look a the curricula of this bachelor program, where neither of these fields is applied in the first term, leading to the conclusion that the students readily learn only those things in these voluntary courses which are needed during the course of the first term.

4.4 Outlook

As a next step, the results of the AKMATH on the performance of students in the exams at the end of the first term will be investigated. The results with respect to the dropout rate are also reported to be promising, although the exact numbers will not be available until march 2009. Also, due to the success of the course, the vice rector for academic affairs approved a repetition of the course and an extension to the students in mechanical engineering, a bachelor program which also has to compete with the advanced technical colleges. Until next fall, the modules will be revised and extended to the needs of this group. Furthermore, the questions will be categorized with respect to the taxonomy of educational objectives, allowing even more insights. For better comparison, another control group has to be chosen in the next year.

5 References

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