A HYPERELASTIC REISSNER-TYPE MODEL FOR NON-LINEAR SHEAR DEFORMABLE BEAMS

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Abstract. The present paper is concerned with the non-linear modelling of a practically important distributed parameter system of mechanics, namely with the deformation of beams being bent, sheared and stretched by external forces and moments. We restrict to plane deformations and static conditions, in which framework we intend to solve an open question in the modelling of this type of distributed parameter systems. Our task is to present a continuum mechanics based extension of the celebrated large displacement finite deformation structural mechanics theory presented by Eric Reissner in [1] for shear deformable beams. A main advantage of Reissner's structural mechanics theory [1] is that it is variationally consistent, due to a special construction of generalized beam strains based on the principle of virtual work. Reissner himself however indicated a conceptual problem associated with this formulation, namely that the constitutive relations, which are needed in the structural theory to be established between generalized static entities (bending moment, shear and normal forces) and the generalized strains, must be evaluated from appropriate physical experiments to be performed for the beam under consideration as a whole. This drawback can be attributed to the fact that Reissner in his structural mechanics derivations [1] did not use the notions of stress and strain, which are fundamental for continuum mechanics. Thus, the common constitutive modelling at the stress-strain level could not be used in connection with Reissner's theory in a rational manner so far. In order to overcome this problem, we present a continuum mechanics based converse to Reissner's structural mechanics modelling. We show that substituting the kinematical assumptions of Reissner's shear-deformable beam theory directly into the continuum mechanics version of the principle of virtual work is equivalent to the structural mechanics virtual work relation between the static structural entities and the generalized strains, which was postulated by Reissner in [1]. This automatically attaches a proper continuum mechanics based meaning to both, the generalized static entities and the generalized strains used in Reissner's theory [1]. Consequently, these generalized static entities can be rationally related to the generalized strains on the basis of a constitutive modelling on the stress-strain level. We show this in some detail in the present contribution for a hyperelastic stress-strain model suggested by Simo and Hughes in [2]. It turns out that the resulting constitutive relations can be represented in a series expansion, where the first terms coincide with the expressions that are well known from the linear structural mechanics theory of bending and stretching of beams. Since we present an analytic expression for the sum of this series expansion, the error of restricting oneself to few terms in the series expansion can be easily estimated in a post-processing routine. The proposed constitutive model can be directly used in a dynamic model for studying, e.g., the control of beam deformations.

1 Introduction.

The present paper is concerned with the non-linear modelling of plane deformations of beams that are bent, sheared and stretched by external forces and moments. Our task is to present a continuum mechanics based extension of the celebrated large displacement finite deformation structural mechanics theory for plane shear deformable beams, which was laid down by Eric Reissner in [1].

The principal result derived by Reissner in [1] was a system of non-linear strain displacement relations being consistent with exact one-dimensional equilibrium equations for forces and moments via an appropriate structural mechanics version of the principle of virtual work. Reissner first derived the local structural mechanics relations of beam equilibrium by directly studying the equilibrium of a deformed beam element of differential length. In the plane case, this involves normal force, shear force and bending moment as generalized static entities. Reissner then assumed constitutive relations at the structural mechanics level to exist in the form of functional dependencies between these static entities and a set of generalized strains, namely a bending strain, an axial force strain and a shear force strain. In order to find out the correct kinematic meaning of the latter generalized strains, Reissner in an a-priori manner postulated a virtual work expression that connects the generalized static entities and the generalized strains in a particularly simple manner. He then required that this virtual work relation must lead to the local structural mechanics relations for beam equilibrium that he had derived before, from which he in turn found the required kinematical meaning of the generalized strains.

A main advantage of Reissner's structural theory [1] is that it is variationally consistent, due to his special construction of the generalized strains based on the principle of virtual work. Reissner himself however indicated a conceptual problem associated with this formulation, namely that the constitutive relations between the generalized static entities and the generalized strains must be evaluated from appropriate physical experiments to be performed for the beam under consideration as a whole. This severe drawback can be attributed to the fact that the notions of stress and strain, which are basic for continuum mechanics, were not used by Reissner in the above sketched structural mechanics derivations, such that the usual constitutive modelling at the stress-strain level could not be used in connection with Reissner's theory so far in a rational manner.

In order to overcome this problem, we present a continuum mechanics based converse to Reissner's structural mechanics modelling. We show that substituting the kinematical assumptions of Reissner's shear-deformable beam theory directly into the continuum mechanics version of the principle of virtual work is equivalent to the virtual work relation postulated by Reissner in [1] between the static structural entities and the generalized strains. This derivation then automatically attaches a continuum mechanics meaning to the generalized static entities and the generalized strains introduced in [1] by relating them, e.g., to the work-conjugate notions of second Piola-Kirchoff stress and Green strain. This general formulation can be used to explain problems with the correctness and accuracy of certain formulations in the literature, which have been encountered and resolved by Gerstmayr and co-authors in [3] and [4]. In the light of the present solution we may say that the drawback of needing physical experiments for the beam under consideration as a whole in order to obtain constitutive relations between the generalized static entities and the generalized strains has led authors to suggest proposing such constitutive relations by analogy to related, often linear, theories. If, however, the exact kinematical meaning of the generalized strains is not captured in a correct manner by such a proposed constitutive relation, the latter is not variationally admissible, since it will not lead to the correct local structural mechanics relations for beam equilibrium stated by Reissner in [1]. This problem can not arise when starting from a constitutive modelling at the stress-strain level of continuum mechanics and using the relations of second Piola-Kirchoff stress and Green strain to the generalized static entities and the generalized strain, which are presented subsequently.

In the present contribution, we particularly derive relations between generalized static entities and generalized strains that are valid for a simple hyperelastic constitutive relation proposed by Simo and Hughes [2]. This model allows writing the constitutive relations in a series representation, the first terms of which do coincide with representations well-known from the linear theory of beams. In principle, however, the presented methodology should enable to introduce any suitable continuum mechanics based constitutive modelling at the stress-strain level, be it elastic or inelastic, into the structural theory laid down by Reissner in [1]. The present work represents an extension of a previous paper by the present authors devoted to the case of beams rigid in shear, so-called Bernoulli-Euler beams, [5]. The relations derived in the latter paper turn out to be compatible with the present results, when shear deformation is neglected.

2 Kinematics of shear-deformable beams.

In the present Section, we present a kinematical model for a beam. A beam is a solid body occupying a certain region that extends in three-dimensional space, where one extension of this region, the axial length of the beam, is much larger than the other two extensions, which are called cross-sectional extensions. We make use of the material or Lagrangian description of continuum mechanics, in which a certain reference configuration is selected in order to describe the deformed configuration of the beam in space. For the sake of brevity, in the following we adopt the kinematical situation studied in [1], namely that the deformation is plane, i.e. that the beam axes in the actual deformed and in the reference configurations are situated in a common plane, and that no twist of the cross-sections about the beam axis takes place. We furthermore restrict to the practical important case of beams having a straight axis in the reference configuration.

In terms of notions that are fundamental for the kinematics of the material description of continuum mechanics, namely position vector r, deformation gradient F, Jacobian of the deformation gradient J and Green strain G, see e.g. Simo and Hughes [2] and Ziegler [6], we subsequently present the consequences of a frequent kinematical approximation for the plane beam deformation, which is usually named after Stephen Timoshenko, and which was addressed in the structural mechanics formulations by Reissner in [1] also, see e.g. Ziegler [6] for the linear theory. This kinematical assumption approximates the beam deformation by assuming that that a cross section of the beam, which was plane and normal to the beam axis in the reference configuration, remains plane and undistorted in the actual configuration, and that its unit outer normal vector in the deformed configuration encloses a certain (generally non-vanishing) shear angle with the tangent to the deformed axis. Compared to the reference configuration, the axis in the deformed configuration in general however will be stretched and curved in comparison to the reference configuration.

In order to allow direct transition to notions of structural mechanics, we try to remain as elementary as possible in our following derivations, without violating the fundamental spirit of the material description of continuum mechanics. We therefore refer to a common Cartesian x-y-z coordinate system in order to describe the plane beam deformation in connection with the Timoshenko assumption. This common Cartesian frame is fixed in the reference configuration, and it is oriented there such that *x* represents the coordinate of the axis in the reference configuration. The *y*-axis is selected such that the deformed axis remains situated in the *x*-*z* plane. Plane cross-sections of the beam in the reference configuration therefore are spanned by the unit vectors e_y and e_z. The position vector of a particle, which was located at the place $x\,e_x + y\,e_y + z\,e_z$ in the reference configuration, in the transverse distance $|z|$ and out-of-plane distance $|z|$ away from the straight reference axis, in the deformed configuration can be written in the form

$$r(x,y,z) = r_0(x) + y\,e_y + z\,e(x) \tag{1}$$

This is due to the Timoshenko assumption, since the deformed cross-section is assumed to remain plane and undistorted in comparison to the reference configuration. The position vector of an axis point in the deformed configuration is denoted by r_0, which describes the position of the deformed axis as a function of the reference coordinate *x*. A cross-section in the deformed configuration is spanned by the unit vectors e_y and e, the latter vector being situated in the *x*-*z* plane. In Eq.(1), furthermore note the spatial functional dependencies that result from the Timoshenko assumption. In the framework of the latter, Eq.(1) represents the field of position vectors of the particles of the beam in the deformed configuration, as described by their coordinates *x*, *y*, *z* in the reference configuration.

Hence, the deformation gradient tensor can be obtained by partial spatial differentiation with respect to the reference coordinates, noting that the common Cartesian frame is fixed in the reference configuration:

$$F = \mathrm{Grad}\, r = r' \otimes e_x + e_y \otimes e_y + e \otimes e_z = \left(r_0{}' + z\,e'\right) \otimes e_x + e_y \otimes e_y + e \otimes e_z \tag{2}$$

where a prime denotes the derivative with respect to the axial co-ordinate *x*, and the symbol $\otimes$ stands for the dyadic vector product.

In order to obtain a suitable kinematical description for the entity e', which also is a function of *x*, we denote the angle, by which a reference cross-section with axial coordinate *x* is rotated about the *y*-axis into the deformed configuration, as $\varphi(x)$. Hence, the unit vector *e* can be considered as a rotation of the unit vector e_z by the angle φ about the *y*-axis also. We thus may write

$$e = R\,e_z = \sin\varphi\, e_x + \cos\varphi\, e_z \tag{3}$$

from which we obtain

$$e' = \varphi'\left(\cos\varphi\, e_x - \sin\varphi\, e_z\right) \tag{4}$$

The deformation gradient in Eq.(2) now becomes

$$\begin{aligned} F = &\left(r_0{}' \cdot e_x + z\,\varphi' \cos\varphi\right) e_x \otimes e_x + \sin\varphi\, e_x \otimes e_z \\ &+ e_y \otimes e_y \\ &+ \left(r_0{}' \cdot e_z - z\,\varphi' \sin\varphi\right) e_z \otimes e_x + \cos\varphi\, e_z \otimes e_z \end{aligned} \tag{5}$$

where we have projected $r_0{}'$, which is tangent to the deformed axis, and thus is situated in the *x*-*z* plane, onto the fixed *x* and *z* directions in order to obtain a formulation which can be directly assigned with a matrix in the common frame, if desired.

A suitable kinematical description can be obtained for the latter projections by noting that $r_0{}'$ in general will not be perpendicular to the deformed cross-section, and hence will not be perpendicular to the vector e, since the tangent to the deformed axis generally encloses the shear angle χ with the normal to the cross section in the deformed configuration. When the unit vector e is perpendicular to the deformed axis, such that no shear deformation is present, one has $\chi = 0$, which is commonly denoted as Bernoulli-Euler assumption. In the following, however, we study the Timoshenko case of generally non-vanishing shear-angles χ. The tangent angle to the deformed axis, and hence to $r_0{}'$, measured from the *x*-direction about the *y*-axis, then becomes $\varphi - \chi$. Denoting moreover the stretch of an element of the beam axis, i.e. the ratio of the length of a differential axis element in the deformed and in the undeformed configuration, by Λ_{x0}, where

$$\Lambda_{x0} = \left\| r_0' \right\| \tag{6}$$

the projections of r_0' in Eq.(5) can be written as

$$r_0' \cdot e_x = \Lambda_{x0} \cos(\varphi - \chi) \tag{7}$$

$$r_0' \cdot e_z = -\Lambda_{x0} \sin(\varphi - \chi) \tag{8}$$

From this, we obtain the following matrix representation for F in the common frame

$$[F] = \begin{bmatrix} \Lambda_{x0} \cos(\varphi - \chi) + z\varphi' \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\left(\Lambda_{x0} \sin(\varphi - \chi) + z\varphi' \sin\varphi\right) & 0 & \cos\varphi \end{bmatrix} \tag{9}$$

The Jacobian determinant J of F in the framework of the Timoshenko assumption follows to

$$J = \Lambda_{x0} \cos\chi + z\varphi' \tag{10}$$

The symmetric right Cauchy-Green tensor, generally defined as

$$C = F^T F \tag{11}$$

within the Timoshenko assumption has the matrix representation

$$[C] = \begin{bmatrix} \left(\Lambda_{x0}{}^2 + 2z\varphi' \Lambda_{x0} \cos\chi + (z\varphi')^2\right) & 0 & \left(\Lambda_{x0} \sin\chi\right) \\ 0 & 1 & 0 \\ \left(\Lambda_{x0} \sin\chi\right) & 0 & 1 \end{bmatrix} \tag{12}$$

The Green strain tensor, which has the general definition

$$G = \frac{1}{2}(C - I) \tag{13}$$

is thus found to have a matrix representation that reads for the Timoshenko assumption

$$[G] = \frac{1}{2} \begin{bmatrix} \left(\Lambda_{x0}{}^2 + 2z\varphi' \Lambda_{x0} \cos\chi + (z\varphi')^2 - 1\right) & 0 & \left(\Lambda_{x0} \sin\chi\right) \\ 0 & 0 & 0 \\ \left(\Lambda_{x0} \sin\chi\right) & 0 & 0 \end{bmatrix} \tag{14}$$

Note that $[G]$ in Eq.(14) indeed reflects the Timoshenko kinematical assumption, in the framework of which only axial strains G_{xx} and shear strains $G_{zx} = G_{xz}$ should be present.

We are now in the position to relate the above continuum mechanics based results to the generalized strains that are fundamental in Reissner's structural mechanics formulation [1]. Reissner introduced a bending strain as

$$\kappa = \varphi' \tag{15}$$

and generalized normal and shear strains in the form

$$\varepsilon = \Lambda_{x0} \cos\chi - 1 \tag{16}$$

$$\gamma = \Lambda_{x0} \sin\chi \tag{17}$$

It is to be emphasized that the expressions for J, C and G given in Eqs.(10), (12) and (14) can be completely expressed by the generalized strains presented in Eqs.(15)-(17) and the transverse coordinate z. This proves that Reissner's generalized strains are proper strain measures also in the sense of continuum mechanics, since J, C and G are known to be material frame indifferent deformation measures, for which the components of proper matrix representations must not change when the reference or the deformed configurations are subjected to rigid body rotations. In the present case, this becomes evident when kinematically interpreting the term $\Lambda_{x0} \cos\chi$ as projection of the axial stretch into the direction normal to the deformed cross-section, and the term $\Lambda_{x0} \sin\chi$ as projection onto the deformed cross-section. Moreover, the expression φ' represents a proper curvature measure, known from the geometric analysis of curves, see e.g. [7]. These kinematical entities, which describe the deformation of the actual with respect to the reference configuration only, of course do not change when rigid body rotations of the respective configurations are superimposed. Last but not least, the transverse coordinate z remains fixed in the framework of the Timoshenko assumption.

3 Virtual work considerations.

The application of the principle of virtual work is basic for Reissner's structural mechanics theory presented in [1], in which he postulated a particular simple expression for the virtual work of the internal forces per unit length of the undeformed axis as

$$\delta W_{\text{int}} = N\,\delta\varepsilon + Q\,\delta\gamma + M\,\delta\kappa \tag{18}$$

where generalized static entities were introduced as normal force N, shear force Q and bending moment M. Reissner in [1] denoted these generalized static entities as stress resultants; the exact relations to stress definitions from continuum mechanics however were not addressed in [1]. In Eq.(18) and in the following, δ denotes a variation, a virtual change in deformation.

In the present Section, we show that, when using the above Timoshenko-type expressions for the Green strain, Eq.(14), and defining the generalized static entities properly, the virtual work expression, which is known to be generally valid for the material description of continuum mechanics, exactly does lead to Reissner's structural mechanics postulate for the virtual work, Eq.(18).

We introduce the symmetric second Piola Kirchhoff stress S by writing its matrix representation in the common frame as

$$[S] = \begin{bmatrix} S_{xx} & S_{yx} & S_{zx} \\ S_{yx} & S_{yy} & S_{zy} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \tag{19}$$

The virtual work of S done upon a virtual change of the Green strain G is known to define the virtual work of the internal forces in the framework of the material description of continuum mechanics in the form:

$$\delta W_{\text{int}} = \int_A S \cdot \delta G \, dA \tag{20}$$

where the dot indicates the scalar or double-contracted tensor product, and the integration is to be performed over the cross-sectional area in the reference configuration. Using the matrix representations in Eqs. (14) and (19), the Timoshenko assumption yields

$$\delta W_{\text{int}} = \int_A \left(S_{xx} \frac{1}{2} \delta\left(\Lambda_{x0}{}^2 + 2z\,\varphi'\,\Lambda_{x0} \cos\chi + (z\,\varphi')^2 - 1 \right) + S_{zx}\,\delta\left(\Lambda_{x0} \sin\chi \right) \right) dA \tag{21}$$

Motivated by the fact that Eqs. (15)-(17) yield the variations

$$\delta\kappa = \delta(\varphi') \tag{22}$$

$$\delta\varepsilon = \delta\Lambda_{x0} \cos\chi - \delta\chi\,\Lambda_{x0} \sin\chi \tag{23}$$

$$\delta\gamma = \delta\Lambda_{x0} \sin\chi + \delta\chi\,\Lambda_{x0} \cos\chi \tag{24}$$

the virtual work expression in Eq.(21) is identically expanded to the form

$$\begin{aligned} \delta W_{\text{int}} = \int_A & (S_{xx} \left(\Lambda_{x0} \cos\chi + z\,\varphi' \right) \left(\delta\Lambda_{x0} \cos\chi - \delta\chi\,\Lambda_{x0} \sin\chi \right) \\ & + \left(S_{xx}\,\Lambda_{x0} \sin\chi + S_{xz} \right) \left(\delta\Lambda_{x0} \sin\chi + \delta\chi\,\Lambda_{x0} \cos\chi \right) \\ & + S_{xx}\,z \left(\Lambda_{x0} \cos\chi + z\,\varphi' \right) \delta\varphi')\, dA \end{aligned} \tag{26}$$

Hence, defining the generalized static entities as

$$N = \int_A S_{xx} \left(\Lambda_{x0} \cos\chi + z\,\varphi' \right) dA = \int_A S_{xx}\,J\,dA \tag{27}$$

$$Q = \int_A \left(S_{xx}\,\Lambda_{x0} \sin\chi + S_{xz} \right) dA = \int_A \left(S_{xx}\,\gamma + S_{xz} \right) dA \tag{28}$$

$$M = \int_A S_{xx}\, z \left(\Lambda_{x0} \cos\chi + z\varphi'\right) dA = \int_A S_{xx}\, z\, J\, dA \tag{29}$$

it follows that the continuum mechanics expression for the virtual work, Eq.(20), in the framework of the Timoshenko assumption indeed yields Reissner's relation, Eq.(18). As is seen from Eqs.(27)-(29), the static entities *N*, *Q* and *M* represent generalized stress resultants. Using other stress measures than the second Piola Kirchhoff stress would yield a more direct relation to the usual notion of stress resultants, which will be reported in a further contribution. However, the second Piola Kirchhoff stress tensor is generally considered as a proper stress measure to be related to the Green strain tensor in the form of constitutive stress-strain relations.

That the continuum mechanics expression for the virtual work, Eq.(20), in the framework of the Timoshenko assumption, yields Reissner's structural mechanics relation, Eq.(18), proves that the definitions for the generalized static entities given in Eqs.(27)-(29) are consistent with the local structural mechanics relations of beam equilibrium that were stated by Reissner in [1]. Moreover, we now have at our disposal relations between the static entities *N*, *Q* and *M* and components of the second Piola Kirchhoff stress, which can be used to consistently introduce constitutive models at the stress strain level of continuum mechanics into the boundary value problems for the generalized static entities and the generalized strains that were stated by Reissner in [1]. For the sake of brevity, we do not repeat these boundary-value problems, which would involve several additional displacement relations. Rather, we show an example for producing proper relations for *N*, *Q* and *M*.

4 A special hyperelastic material law.

The following constitutive stress-strain relation was suggested in the book by Simo and Huges [2] for hyperelastic isotropic materials:

$$S = \frac{\lambda}{2}\left(J^2 - 1\right)C^{-1} + \mu\left(1 - C^{-1}\right) \tag{30}$$

where λ and μ denote the Lame parameters. Using the expressions for J , C and γ stated in Eqs.(10), (12) and (17) in the framework of the Timoshenko assumption, the matrix representation of the inverse of *C* in the common frame can be written as

$$\left[C^{-1}\right] = \begin{bmatrix} \frac{1}{J^2} & 0 & -\frac{\gamma}{J^2} \\ 0 & 1 & 0 \\ -\frac{\gamma}{J^2} & 0 & 1 + \frac{\gamma^2}{J^2} \end{bmatrix} \tag{31}$$

Substituting this into Eq. (30) and using the resulting respective components of *S*, the expressions given in Eqs.(27)-(29) can be evaluated in the form

$$N = \int_A \left(\mu + \frac{\lambda}{2}\right)\left(J - \frac{1}{J}\right) dA \tag{32}$$

$$Q = \gamma \int_A \mu\, dA \tag{33}$$

$$M = \int_A \left(\mu + \frac{\lambda}{2}\right)\left(J - \frac{1}{J}\right) z\, dA \tag{34}$$

Note that the shear force depends on the shear strain only linearly in the present case. The integrals in Eqs. (32)-(34) can be easily evaluated for a given distribution of elastic parameters and a given form of the cross-section. In many practical situations, elementary integration will suffice. This then allows to formulate the constitutive relations of the present theory in a closed form. Assuming the material to be homogeneous, the Lame constants can be put in front of the integrals. Substituting *J* from Eq.(10), using the generalized bending and normal strains stated in Eqs.(15) and (16), expanding first into a series in the transverse coordinate *z* and afterwards into a series in the normal strain ε , we obtain the following series representation for the normal force:

$$\begin{aligned} N = (2\mu + \lambda) A \varepsilon + \left(\mu + \frac{\lambda}{2}\right) A \sum_{n=2,3,5,}^{\infty} (-1)^{n-1} \varepsilon^n \\ - \left(\mu + \frac{\lambda}{2}\right) \sum_{n=2,4,6,}^{\infty} K^{(n)} \frac{\kappa^n}{(1+\varepsilon)^{n+1}} \end{aligned} \tag{35}$$

with cross-sectional integrals in the form

$$K^{(n)} = \int_A z^n dA \tag{36}$$

where it has been assumed that the beam axis is chosen such that cross-sectional integrals with an odd exponent n do vanish. In case of a material with vanishing Poisson ratio, there is

$$\nu = 0: \quad (2\mu + \lambda) = Y \tag{37}$$

where Y denotes Young's modulus of elasticity. The first term in Eq.(35) then becomes equal to the well-known expression for the normal force that is used in the linear structural mechanics theory of beams, see e.g. Ziegler [6]. An analogous series expansion can be derived for the bending moment M in Eq.(34). Since Eqs. (32) and (34) represent analytic expressions, the numeric errors of limiting these series expansions to the few first terms can be easily estimated in a post-processing procedure, after having solved the non-linear boundary value problem following from Reissner's theory [1] in connection with the approximate series expansion.

5 Conclusion.

In the present paper, we have shown, that and how a continuum mechanics based meaning can be attached in a rational manner to the generalized static entities and the generalized strains in Reissner's structural mechanics theory for large displacements and finite strains of beams, [1]. This enables to bring into the play a constitutive modelling at the stress-strain level of continuum mechanics, which we have demonstrated for a special isotropic hyperelastic model proposed by Simo and Hughes in [2]. In principle, the presented methodology should enable to introduce any suitable continuum mechanics based constitutive modelling at the stress-strain level. The present work considers shear deformation in the framework of the Timoshenko assumption. It represents an extension of our previous paper [5] for beams rigid in shear in the framework of the so-called Bernoulli-Euler assumption. The proposed formulation can be directly used in a dynamic model for studying, e.g., the control of beam vibrations.

6 References.

[1] Reissner, E.: *On one-dimensional finite-strain beam theory: the plane problem*, J. Appl. Math. Phys. 23 (1972), 795 - 804.

[2] Simo, J.C., Hughes, T.J.R.: *Computational Inelasticity*, Springer-Verlag New York, 1998.

[3] Gerstmayr, J., Irschik, H.: *On the correct representation of bending and axial deformation in the absolute nodal coordinate formulat ion with an elastic line approach*, Journal of Sound and Vibration, 318 (2008), 461-487.

[4] Gerstmayr, J., Matikainen, M.K., Mikkola, A.M.: *A geometrically exact beam element based on the absolute nodal coordinate formulation*, Journal of Multibody System Dynamics, 20 (2008), 359-384.

[5] Irschik, H. and Gerstmayr, J.: A *continuum mechanics based derivation of Reissner's large-displacement finite-strain beam theory: The case of plane deformations of originally straight Bernoulli–Euler beams*. Acta Mechanica (2008), online first.

[6] Ziegler, F.: *Mechanics of Solids and Fluids,* 2nd ed., 2nd corr. print. Springer-Verlag New York, 1998.

[7] Meyberg, K., Vachenauer, P.: *Höhere Mathematik 1*, 6th edn, 1st corr. print. Springer, Berlin, 2003.

Acknowledgement: Support of the present work in the framework of the Comet-K2 Austrian Center of Competence in Mechatronics (ACCM) is gratefully acknowledged.