OPTIMAL CONTROL FOR GAS-LIFT WELL

Fikret Aliev, Institute for Applied Mathematics, Azerbaijan
Musa Ilyasov, Institute for Applied Mathematics, Azerbaijan
Mahammad Jamalbayov, Institute for Applied Mathematics, Azerbaijan

Corresponding Author: Mahammad Jamalbayov
Institute for Applied Mathematics
Baku University – AZ1065 Baku, Zahid Khalilov 125 – Azerbaijan
Email: mjamalbayov@ctnet.az

Summary. Natural oil flowing conditions finishes when energy of oil reservoir is insufficient to a lifting of oil to the ground surface. Since this moment it is required submissions of additional energy from a surface of the ground for a lifting of a liquid. Thus it is spoken about the mechanized methods of oil exploration, one of which is gas-lift. The oil and gas mixture lifting at gas-lift conditions is possible by means of the compressed gaseous working agent (natural gas or air) energy, submitted directly from a surface of the ground. The principle of action of gas-lift is based that at submission in a well of the working agent (gas) occurs decrease in density gas-liquid mixture in lift pipes and bottom-hole pressure. It provides the certain depression and flow of a liquid from a reservoir to the bottom-hole. Under influence of these and other factors the wellhead overflow condition is created.

The gas-lift is one of the most mechanized methods of oil well operations and high profitability. For gas-lift operation of wells at an optimal conditions in practice are guided by well flow rate (production) curves from the injected gas flow rate (fig.1, curve *Q (V)*) and the working agent specific flow rate from the injected gas flow rate (fig.1, curve *R (V)*). Thus it is considered to be an optimal mode the one which is characterized by the minimal value of the specific gas flow rate *(R)* or coordinate of a point *"A"* where the tangent through the beginnings of coordinate system adjoins to curve *Q=f (V)*. The point *"A"* corresponds to the maximal efficiency of gas injection work in a well of gas, though thus flow rate *(Q)* is not maximal. In practice it is accepted to maintain wells in conditions, which belong to piece *AB* on curve *Q (V)*.

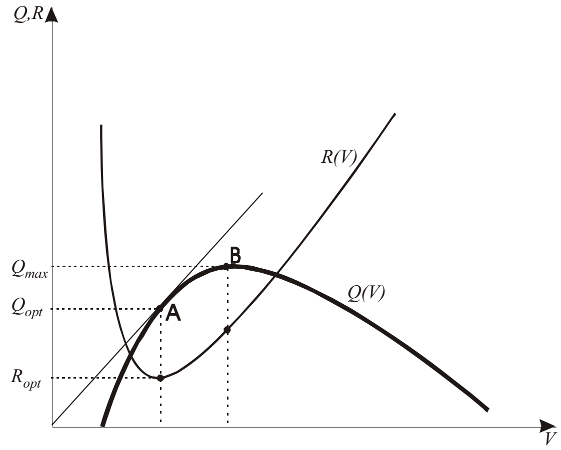

Fig. 1. Dependence of specific gas flow rate R on the total gas flow rate V for given curve Q (V.).

The process of optimal control of the gas-lift well is examined and has been solved in this work.

Introduction. The authors are based on the models of gas lift process that they have got with due regard for real processes in pump-compressor tubes and the reservoir (i.e. changes of physical parameters and reological features of reservoir-collector and filtering fluids of the reservoir pressure) and also the influence of the neighboring wells. The last means that the problem consists of two interrelated processes – the process of gas movement, gas oil mixture in pump-compressor tubes and filtering (flowing) processes in the reservoir (its permeability and porosity depend on importance of reservoir pressure). And flow of oil and gas to bottom-hole zone depends on it [2].

Movement of oil in this case is depicted in the next equations [2].

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\frac{p\beta\gamma_г f_г(1-\rho_н)}{Z(p)p_{am}\mu_г(p)}+\frac{S(p)f_н(\rho_н)}{a(p)\mu_н(p)}\right]k(p)\frac{\partial p}{\partial r}\right\}=-\frac{\partial}{\partial t}\left\{\left[\frac{p\beta\gamma_г(1-\rho_н)}{Z(p)p_{am}}+\frac{S(p)}{a(p)}\rho_н\right]m(p)\right\} \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\frac{f_н(\rho_н)}{a(p)\mu_н(p)}\right]k(p)\frac{\partial p}{\partial r}\right\}=-\frac{\partial}{\partial t}\left\{\left[\frac{\rho_н}{a(p)}\right]m(p)\right\}, \quad (2)$$

where p- is a pressure; $\rho_н$ - oil saturation; k – absolute permeability of reservoir rock; $f_н, f_г$ - relative permeability of oil and gas phases; $\mu_H, \mu_г$ – dynamic viscosity of oil and gas; S(p)- solubility of gas in oil; a(p)- volume coefficient of oil; γ(p)- specific gravity of gas by pressure p; β- temperature correction; m- porosity of reservoir; Z(p)- coefficient of condensability of gas; p_{am} – atmospheric pressure.

In [2] to solve the equation (2) was inserted the function like Khristianovich function

$$H=\int\varphi(p,\rho_н)dp+C, \quad (3)$$

where C is a permanent of the integration; under integral function looks like:

$$\varphi=\frac{k(p)f_н(\rho)}{a(p)\mu_н(p)}.$$

Here, using the middle [3], the equation (2) may be written in the next form:

$$\nabla^2 H=-\Phi(t).$$

Using border conditions for the given problems, from this equation we get expression for definition of instantaneous value of oil flow (and free gas) to the bottom-hole:

$$q_н=\frac{2\pi h(H_k-H_з)}{\ell n\frac{r_к}{r_з}-\frac{1}{2}}, \quad (4)$$

where $q_н$ - is an instantaneous value of oil flow rate; $r_к$, $r_з$ - radiuses of boundary and bottom-hole zone.

In [3] is shown that $\varphi(p,\rho_н)$ in practically interesting intervals of pressure is exactly approximated by polynomial of the second degree in the pressure function, i.e.:

$$\varphi=Ap^2+Bp+C \quad (5)$$

or, as in [2] (for oil) this curve may be changed into the curve, every rectilinear part of which is depicted by the next binomial:

$$\varphi=A+Bp, \quad (6)$$

where B is a corner coefficient of the rectilinear part which belongs to pressure p.

This approximation allows to determine the meaning of the real depression of $p_k-p_з$.

In the solution of gas-lift problems determination of flow or displacement of oil into the reservoir, which is possible with the above given method, has an important meaning. But, if it is necessary, the given method may also be used for determination of bottom-hole pressure by the known value of $q_н$. In this case expression (3) with (4) rectilinear approximation is written as the next form:

$$q_{н} = \frac{2\pi h}{\ell n\frac{r_{к}}{r_{з}} - \frac{1}{2}}\left(\frac{B}{2}p_{к}^{2} - \frac{B}{2}p_{з}^{2} + Cp_{к} + Cp_{з}\right), \tag{7}$$

where coefficients B, C of approximations are determined by the next expressions:

$$B = \frac{\varphi_k - \varphi_з}{p_k - p_з}, \; C = \varphi_k - Bp_k .$$

According to the equality of the well flow rate and oil flow rate to the bottom-hole, we determine value of the bottom-hole pressure:

$$p_з = -\frac{C}{B} + \frac{C}{B}\sqrt{1 - \frac{B^2}{C^2}\left(q_{н}\frac{\ell n\frac{r_k}{r3} - \frac{1}{2}}{2\pi h} - \frac{B}{2}p_k^2 + Cp_k\right)} \tag{8}$$

where B and C are determined by the next expressions:

$B = \frac{\varphi_k - \varphi_3}{p_k - p_3}$, $C = \varphi_k - Bp_k$; $q_{н}$ – in this case is a oil flow rate of well.

The described solution of (1)-(2) let to determine a number of importance parameters of the well-reservoir system (spread of the reservoir pressure and its mean value, instantaneous value of the reservoir flow rate etc.) through which it is possible to determine static level of oil in lifting tube and total time, necessary for its formation.

Besides, together examination of well-reservoir let for every concrete value of the formed bottom-hole pressure to prognoses oil flow rate from the reservoir to the bottom-hole zone and to consider influence of neighboring working wells through changes of the boundary pressure value.

All these by together examination of the gas-lift process model, afterwards let to create principally new method for the optimal control of gas-lift well, by which all hydro-gas dynamic researches of well become possible in a machine model of the common system (well-reservoir). The last means that “construction” of the characteristic curve (fig. 1), let to determine optimal conditions for gas-lift process in the computer for any moment of the time and without change of technological regime of the real well.

Except the noted, the offered approach let to prognoses need of the pass to periodical gas-lift and determine parameters of the new regime of the well.

By this, gas injection and movement of gas-liquid mixture in lifting tubes may be described by the next system of equations [1].

$$-\frac{\partial P}{\partial x} = \frac{1}{F}\frac{\partial Q}{\partial t} + \frac{2a}{F}Q, \tag{9}$$

$$-\frac{\partial P}{\partial t} = \frac{c^2}{F}\frac{\partial Q}{\partial x}, \tag{10}$$

where $Q = F\rho w$, F - area of horizontal section of pump-compressor tubes and is permanent on the axis x, P- is a pressure, t, x- time and coordination, ρ ,Q – density and volume flow rate of gas, oil and gas-liquid mixture depending on the coordinate; $2a = \frac{g}{w_c} + \frac{\lambda_c w_c}{2D}$.

Equation system of the movement (9)-(10) are partial differential equations, where raising of problems of optimization in traditional form [4, 5] face problems. To linearization the system (9)-(10) let’s imagine, that pump-compressor tubes consists from n – number of parts, every one of which has length $l_k = L_k - L_{k-1}$ where $k=1,2,3,...n$. If consider, that change of pressure

$\frac{\partial P}{\partial x} \approx \frac{P_k - P_{k-1}}{l_k}$ in every part , so it is possible to except from the system the depending P from x (in the examined part) and system (9) and (10) to write in the form of the next ordinary differential equation:

$$\frac{dP_k}{dt} = -\frac{c^2}{Fl_k}(Q_k - Q_{k-1}) \tag{11}$$

$$\frac{dQ_k}{dt} = -\frac{F}{l_k}(P_k - P_{k-1}) - 2aQ_k \tag{12}$$

The system of equations (11) and (12) describes movement of the object (gas, oil and mixture). For the optimization problem, in this case, pressure $P_0, P_1, \ldots P_n$ and $Q_1, Q_2, \ldots Q_n$ are the controlled objects, but Q_0 is the controlling parameter, where Q_0 is a temp of gas injection.

Then, the demanded functional may be taken in the next form:

$$J = \int_0^T [(Q_n(t) - \overline{Q})^2 + Q_0(t)]dt, \tag{13}$$

where T is the value of duration of process; $\overline{Q}$ is demanded value of well flow rate; begining conditions for (11)-(12) $P_0(0), P_1(0), \ldots P_n(0)$ and $Q_1(0), Q_2(0), \ldots Q_n(0)$ are the known values. So, it is demanded to find such control $Q_0(t)$, which by the conditions (11)-(12) with the corresponding beginning conditions minimized to the square quality criterion (13).
The analogical sum may be given also for the pressure spread, whose answer may be got by using results [4].

References.

[1] Алиев Ф.А., Ильясов М.Х., Джамалбеков М.А. *Моделирование работы газлифтной скважины.* Доклады НАН Азерб. Респ, № 6, с, 15-28, Баку, 2008.

[2] Абасов М.Т., Дадашзаде Х.И., Джамалбеков М.А. *Фильтрация летучих нефтей в деформируемых коллекторах*. Известия АН Азерб. ССР, сер. “Науки о Земле”, № 5, 1990.

[3] Абасов М.Т., Гасанов Ф.Г., Оруджалиев Ф.Г. *О фильтрации газоконденсатной смеси*. ДАН Азерб. ССР, 1966, №4.

[4] Алиев Ф.А. *Методы решения прикладных задач оптимизации динамических систем*. Баку, "Элм", 1989, 320 с.

[5] Алиев F.A., Larin V.B. *Optimization of Linear Control Systems. Analytical Methods and Computational Algorithms*. Gordon and Breach Publ, Amsterdam, 1998, 272 p.