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LAURA AND PETRARCA - TRUE EMOTIONS VS. MODELLED EMOTIONS

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Abstract. Laura, a very beautiful but also mysterious lady, inspired the famous poet Francesco Petrarca for poems, which express ecstatic love as well as deep despair. F. J. Jones – a scientist for literary - recognised in these changes between love and despair in the years 1328 to 1350 an oscillating behaviour, which he called Petrarch's emotional cycle. It is evident, that this cycle is based on the emotional relations between Laura and Petrarch and on Petrarch's inspiration.
This contribution introduces and reviews modelling approaches for emotional dynamics and inspiration by means of a fully mathematical approach using ODEs, by System Dynamics, and by transfer function modelling – with the aim for modelling and simulating Petrarch's emotional cycle, and with the aim to transfer to model to 'modern' emotional relations.
The mathematician S. Rinaldi investigated as first Petrarch's emotional cycle and established an ODE model, starting point for the investigations in two directions: mapping the mathematical model to a suitable modelling concept, and trying to extend the model for love dynamics in modern times.
The modelling concept of System Dynamics fits very well to model the qualitative behaviour. In principle, emotions and inspiration emerge from a source, and are fading into a sink. But the controlling parameters for increase and decrease of emotion create a broad variety of emotional behaviour and of degree of inspiration, because of the nonlinearities.
A control – oriented approach observes emotions and inspiration as states fading over time – behaving like a transfer function approaching a steady state. This observation suggests a modelling approach by transfer functions. Both model approaches allow an easy extension to modern times. Identification of genuine parameters – independent from the model approach – show interesting basic attitudes of Petrarch and Laura. Parameter studies allow mimicking daily life's behaviour of emotions.

1 Introduction

Dynamic phenomena in physics, biology, economics, and all other sciences have been extensively studied with differential equations, since Newton introduced the differential calculus. But the dynamics of love, perhaps the most important phenomenon concerning our lives, has been tackled only very rarely by this calculus. In literature two special contributions can be found:

- *Love Affairs and Differential Equations* by S.H. Strogatz ([5]), - harmonic oscillators making reference to Romeo and Juliet, and
- *Laura and Petrarch: an Intriguing Case of Cyclical Love Dynamics* by S. Rinaldi ([4]) – presenting a nonlinear ODE with cyclic solutions.

Both contributions start directly with nonlinear oscillations - observing a certain historic emotional behaviour of prominent couples. LAURA group at Vienna University of Technology tries to consider general modelling concepts for emotional relations, which cover or coincide with Petrarch's emotional cycle – in case of appropriate parameterisation. Recent work has investigated modelling concepts like transfer [1] and System Dynamics [2]. This paper compares both approaches and presents interesting case studies. Further work is intended with model other model approaches, e. g. cellular automata, agent-based modelling and Markov chains. But neither the authors will not forget nor the readers should not forget, that emotions are emotions – and not physics of special manner. Our formulas and model approaches for emotions are very simple sketches of a complex and beautiful phenomenon, which can be better described by poetry – as Petrarca did in *Sonnet CXLIX* (Listing 1).

Di tempo in tempo mi si fa men dura *l'angelica figura e'l dolce riso,* *et l'aria del bel viso* *e degli occhi leggiadri meno oscura*	*From time to time less reproachful seem to me* *her heavenly figure, and her charming face,* *and sweet smile's airy grace,* *while her dancing eyes grow far less dark I see*

Listing 1. Sonett CXLIX from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*) or *Rime in vita e morta di Madonna Laura* (after 1327) by Francesco Petrarca.

2 Historical facts.

Francis Petrarch (1304-1374), is the author of the *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*) or *Rime in vita e morta di Madonna Laura* (after 1327), a collection of 366 poems (sonnets, songs, sestinas, ballads, and madrigals). Petrarch studied first law in Bologna, before he changed to classics. By 1330 Petrarch finishes his Minor Orders of the church and enters the service of Cardinal Colonna. He spent the rest of his life in the service of the church under different cardinals and bishops. He undertook many diplomatic missions across Europe for various reasons. He was the first mountaineer, but he became famous as re-incarnation of a *Poeta Laureatus* and as father of humanism in Europe.

Figure 1. Portraits of Petrarch, from Internet resources.

But most famous Francesco Petrarca became for his *Sonnets to Laura*. In Avignon, at the age of 23, he met Laura, a beautiful but married lady. He immediately fell in love with her and, although his love was not reciprocated, he addressed more than 200 poems to her over the next 21 years. The poems express bouts of ardour and despair, snubs and reconciliations, making Petrarch the most lovesick poet of all time.

Laura was the love of Petrarch's life. For her he perfected the sonnet and wrote the *Canzoniere*. Who Laura was and even if she really existed is a bit of a mystery. It has often been believed that the name *Laura* was a play on the name *laurel* the leaves which Petrarch was honoured with for being the *poet laureate*. However, there is a lot of evidence to show that Laura really did exist and that she was Laure de Noves. Born six years after Petrarch in 1310 in Avignon she was the daughter of Audibert de Noves (a knight) and wife to Hugues II de Sade (and possibly the ancestor of the infamous Marquis de Sade). She married at the age of 15 (January 16th, 1325) and Petrarch saw her for the first time two years later on April 6th (Good Friday) in 1327 at Easter mass in the church of Sainte-Claire d'Avignon.

Figure 2. Portraits of Laura, from Internet resources.

Laura died at the age of 38 in the year 1348, on April 6th, Good Friday, exactly 21 years to the very hour that Petrarch first saw her (as Petrarch noted in his copy of a work by Virgil). Several years after her death, Maurice Sceve, a humanist, visiting Avignon had her tomb opened and discovered inside a lead box. Inside was a medal representing a woman ripping at her heart, and under that, a sonnet by Petrarch.

Figure 3. Portraits of Laura and Petrarch, from Internet resources.

The question if Laure de Noves was Petrarch's Laura, or even if there was a Laura is a question which may never be answered. Although he wrote the *Canzoniere*, a series of poems mostly about Laura and his love for her, she is absent from even being mentioned in his letters except for a few very rare cases where he talks about a past love he once had (*Letter to Posterity*) and once where he responds to an accusation that she is not real (*Familiares* II, IX). If she was real, it is unknown if they ever spoke, or if she ever knew of his feelings for her.

3 Classification of Petrarch's sonnets

Unfortunately, only a few lyrics of the Canzoniere are dated. The knowledge of the correct chronological order of the poems is a prerequisite for studying the lyrical, psychological, and stylistically development of Petrarch and his work. For this reason, the identification of the chronological order of the poems of the Canzoniere has been for centuries a problem of major concern for scholars.

In 1995, Frederic Jones presented an interesting approach and solution to the chronological ordering problem of Petrarch's poems in his book *The Structure of Petrarch's Canzoniere* ([3]).

Jones concentrated on Petrarch's poems written at lifetime of Laura (the first, *Sonnet X*, was written in 1330 and the last, *Sonnet CCXII*, in 1347). First, he analysed 23 poems with fairly secure date. After a careful linguistic and lyrical analysis, he assigned grades for the poems, ranging from -1 to +1, establishing *Petrarch's emotional cycle*: the maximum grade (+1) stands for ecstatic love, while very negative grades correspond to deep despair. Listing 2, Listing 3 and Listing 4 illustrate these grades by three sonnets: +0.6 ~ hope and love, -0.6 ~ despair, and -0.3 ~ 'melancholy' (the English version is taken from an English translation of the *Canzoniere* by Frederic Jones).

Amor con sue promesse lusingando, *mi ricondusse alla prigione antica*	*Love's promises so softly flattering me* *have led me back to my old prison's thrall*
Grading by Jones: + 0.6 'hope and love'	

Listing 2. Sonett LXXVI from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*), Jones' grading +0.6 'hope and love'.

Cosi mancando vo di giorno in giorno, *si chiusamente, ch'i' sol me ne accorgo* *et quella che guardando il cor mi strugge*	*Therefore my strength is ebbing day by day,* *which I alone can secretly survey,* *and she whose very glance will* *scourge my heart.*
Grading by Jones: - 0.6 'despair'	

Listing 3. Sonett LXXIX from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*), Jones' grading: –0.6 'despair'.

Parme d'udirla, udendo i rami et l'ore *et le frondi, et gli augei lagnarsi, et l'acque* *mormorando fuggir per l'erba verde*	*Her I seem to hear, hearing bough and wind's caress,* *as birds and leaves lament, as murmuring flees the* *streamlet coursing through the grasses green.*
Grading by Jones: - 0.4 'melancholy'	

Listing 4. Sonett CLXXVI from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*), Jones' grading: –0.4 'melancholy'.

Displaying the grades over time (Figure 4), F. Jones detected an oscillating behaviour, which he called *Petrarch's emotional cycle E*(*t*), with a period of about four years.

In a second step, F. Jones analysed and 'graded' all the other poems with unknown date and checked, in which part of the cycle they could fit. Taking into account additional historical information, he could date these poems by locating them in Petrarch's emotional cycle *E*(*t*) - Figure 4.

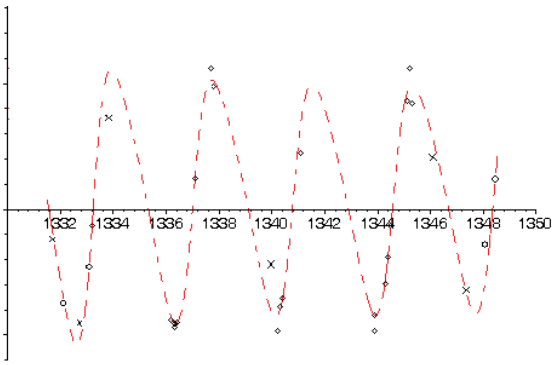

Figure 4. Petrarch's emotional cycle *E(t)* – dashed line, with 'graded' poems
(crosses for securely dated poems, and circles for poems dated based on the emotional cycle).

4 Linear model approach by System Dynamics

The challenge is to set up a dynamic model for Petrarch's emotional cycle $E(t)$. Obviously, Petrarch's emotions and love for Laura must be one of the characterising variable, denoted as variable $P(t)$. $P(t)$ describes Petrarch's love for Laura, whereby high values of P indicate ecstatic love, while negative values stand for despair. Laura is the counterpart in this emotional relation, with emotions and love for Petrarch – denoted as variable $L(t)$. There $L(t)$ represents Laura's love for the poet at time t; positive and high values of L mean warm friendship, while negative values should be associated with coldness and antagonism. Nobody would know about this love, if Petrarch would not have written his sonnets to Laura. This 'poetic output' is governed by inspiration, which he got from his emotions, and observes this historic love fiction - denoted as variable $I_P(t)$.

The three variables $P(t)$, $L(t)$ and $I_P(t)$, were the starting point for ODE model by Rinaldi, who introduced the Laura – Petrarch model in 1998 (see [2]), to be discussed in section 5.

System Dynamics (SD) is a well known modelling approach, introduced by J. Forrester. SD is a methodology for studying and managing complex feedback systems, such as one finds in business and other social systems. In fact it has been used to address practically every sort of feedback system. Feedback refers to the situation of X affecting Y and Y in turn affecting X perhaps through a chain of causes and effects. One cannot study the link between X and Y and, independently, the link between Y and X and predict how the system will behave. Only the study of the whole system as a feedback system will lead to correct results.

For modelling, SD starts at qualitative level with a causal loop diagram. A *causal loop diagram* (CLD) is a diagram that aids in visualizing how interrelated variables affect one another. The CLD consists of a set of nodes representing the variables connected together. The relationships between these variables, represented by arrows, can be labelled as positive or negative. There are two kinds of causal links, positive and negative. Positive causal links means that the two nodes move in the same direction, i.e. if the node in which the link start decreases, the other node also decreases. Similarly, if the node in which the link starts increases, the other node increases. Negative causal links are links in which the nodes changes in opposite directions (an increase cause a decrease in another node, or a decrease cause an increase in another node).

In case of Laura's and Petrarch's emotions $L(t)$ and $P(t)$, and Petrarch's inspiration $I_P(t)$, the causal relation is evident (Figure 5), but it is not possible to identify clearly positive and negative links.

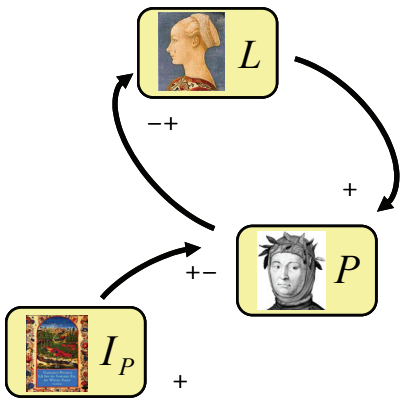

Figure 5. Causal loop diagram between Laura's and Petrarch's emotions and Petrarch's inspiration.

SD continues the modelling process now at the quantitative level by a *stock and flow diagram* – (SFD), sometimes also called *level and rate diagram*. A *stock* variable is measured at one specific time. It represents a quantity existing at a given point in time, which may have been accumulated in the past. A *flow* variable is measured over an interval of time. Therefore a flow would be measured *per unit of time*. The variables in the CLD must be identified either as stock (level) or flow (rate) – or as auxiliary, and each stock (level) is connected in the SFD with its inflow – coming from a source- and by its outflow to a sink; flows are represented by double arrows and flow-controlling valves. The causal links from the CLP are found in the SFD as characterising influences from stocks to flows (or from parameters and auxiliaries to flows).

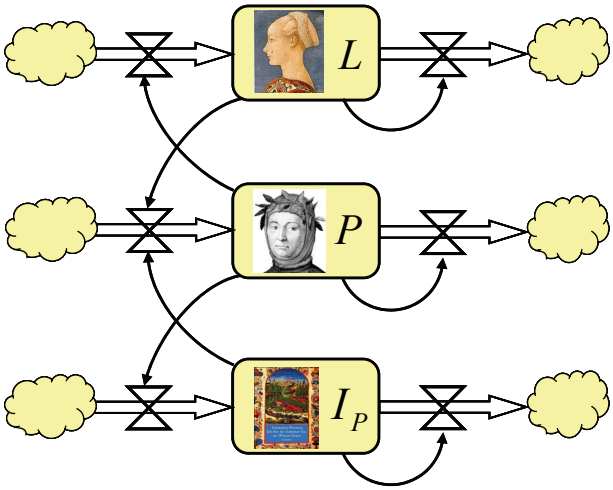

Figure 6. Qualitative stock and flow diagram for Laura's and Petrarch's emotions and for Petrarch's inspiration.

For the dynamics of emotion and inspiration under investigation, all $L(t)$, $P(t)$, and $I_P(t)$, are considered as stocks, whereby the adjoin flows are the changes (derivatives). A first simple SFD (Figure 6) for these dynamics shows similar structures for all three variables $L(t)$, $P(t)$, and $I_P(t)$. They are emerging from a source flow, influenced by other stocks, and they are fading to a sink (outflow) controlled by the stock itself. The stocks' feedback to the outflow let them 'converge' to zero (stabilising feedback), if no inflow is driving emotions and intuition.

SD's modelling procedures now quantifies the SFD by introducing parameters and auxiliaries for the causal links and for the influences for the flows. Laura's and Petrarch's emotions and Petrarch's intuition are fading with certain celerity, characterised by the parameters α_L, α_P, and α_{IP} in the direct feedbacks. And in principal, also the feedback from Laura's emotion $L(t)$ to Petrarch's emotion $P(t)$ and vice versa is given by the parameters β_P and β_L, and the feedback from Petrarch's emotion $P(t)$ to Petrarch's inspiration $I_P(t)$ by the parameter β_{IP}. The appeal parameters A_L and A_P influence directly the inflow for the stocks $P(t)$ and $L(t)$ – indicated by additional control inputs for the respective inflows (summarised with the feedbacks from other stocks).

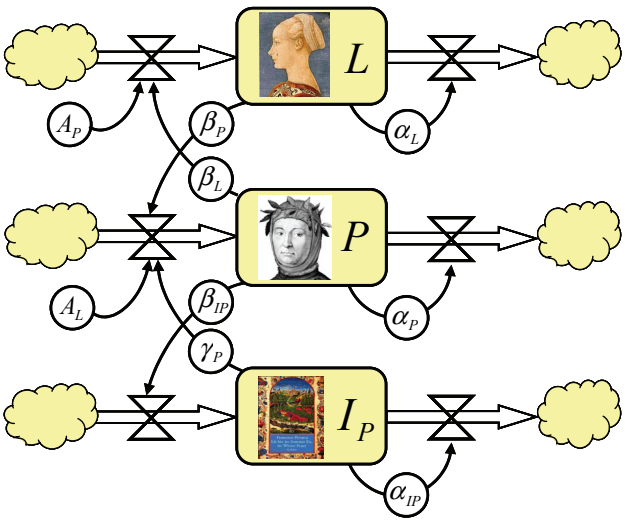

Figure 7. Stock and flow diagram for Laura's and Petrarch's emotions and for Petrarch's inspiration with simple linear influences.

The SFD for the dynamics of emotion and inspiration in Figure 7 shows all basic feedbacks and direct inputs for the flows, weighted with parameters, but neglecting the nonlinear reaction of Laura, and the inspiration's reciprocal feedback to Petrarch's emotion (only 'approximated' by a classic feedback with parameter nature γ_L, which may be seen as linearization of the reciprocal feedback).

The model presented in the SFD in Figure 7 is still a simplified (linear) model, but the SFD can directly generate the system governing ODEs, which might be by used by any simulator. This automatic generation of the ODE model is the last step in the SD modelling procedure. In principal, the SFD balances stocks and flows, so that for the simplified model the following ODEs are generated:

$$\frac{dL(t)}{dt} = -\alpha_L L(t) + \beta_L P(t) + \beta_L A_P$$
$$\frac{dP(t)}{dt} = -\alpha_P P(t) + \beta_P L(t) + \gamma_P I_P(t)$$
$$\frac{dI_P(t)}{dt} = -\alpha_{IP} I_P(t) + \beta_{IP} P(t)$$

Of course, the above equations are linear ODEs, and they are not able to produce oscillations with nonlinear effects, as the emotional cycle represents. Furthermore, emotions almost never act and react in a linear way. For incorporation of nonlinear effects, SD makes use of *auxiliaries,* which define the nonlinear nature of a feedback. Auxiliaries may have more than one input, and the output may be fed back into many different flows, or – if necessary, into further auxiliaries, and auxiliaries may obey more or less complex nonlinear relations. But SD gives no answer how to choose the nonlinear auxiliaries – their choice depends on the investigated dynamics.

5 Linear model approach by transfer functions

Transfer functions are a natural way to describe instationary changes between stationary levels. It may be a good idea, to use transfer functions to describe the instationary change of emotion and inspiration.

The three variables $P(t)$, $L(t)$ and $I_P(t)$ again are starting points. One knows and observes a sad but essential fact: love and emotion is *fading, fleeting, and fugacious.* Driven first by external and internal stimulation, emotion reaches a certain level, before emotion again goes back to zero. For a control engineer a clear situation: at a first glance, supposing only linear relations, a model for Laura's and Petrarch's love dynamics $L(t)$ and $P(t)$ consist of first order transfer functions for the dynamic behaviour, with gain β_L and time constant α_L for Laura, and with gain β_P and time constant α_P for Petrarch. The love of both is driven by the appeal A_P and A_L to each other, being set point and input to the respective transfer function – Figure 8. The time constants represent the ability for forgetting or remembering, resp., and the gain factors the sensitivity for the input and stimulus for emotion.

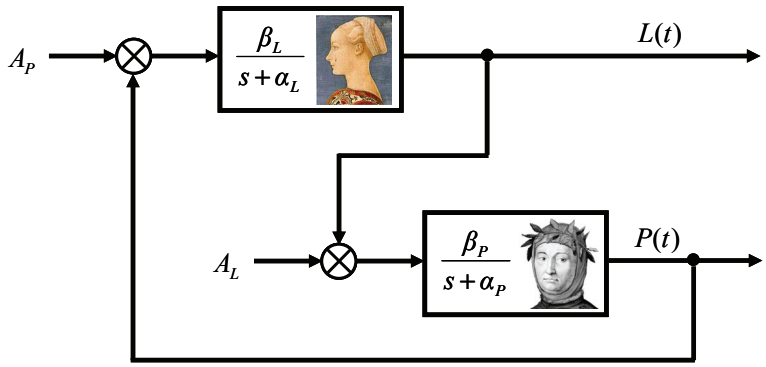

Figure 8. Linear transfer function model for Laura's and Petrarch's love dynamics $L(t)$ and $P(t)$.

The second driving power for the love is clearly the reaction of Laura to Petrarch's love, and the reaction of Petrarch to Laura's emotions, summed with the appeal. Taking this linear approach, the reaction functions are simple the grade of love of the partner, being the second.

His emotion $P(t)$ inspires Petrarch for poems, so that the inspiration $I_P(t)$ is – at first glance – a direct consequence of the emotion $P(t)$, with some time delay. For a control engineer again a clear situation: inspiration is output of a first order lag with input emotion – Figure 9. A time constant α_{IP} controls the tracking, and a gain factor β_{IP} scales the power of emotion.

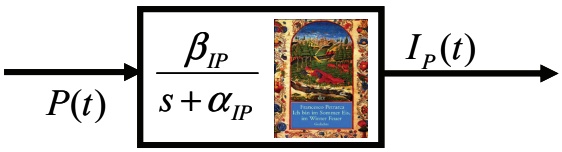

Figure 9. Linear transfer function model for Petrarch's inspiration $I_P(t)$.

But Petrarch feels and lives with and in his poems – so the inspiration is fed back as driving force for the emotion. Three forces are diving Petrarch's emotion: Laura's appeal A_L to him, his inspiration $I_P(t)$, and Laura's reaction (Figure 10).

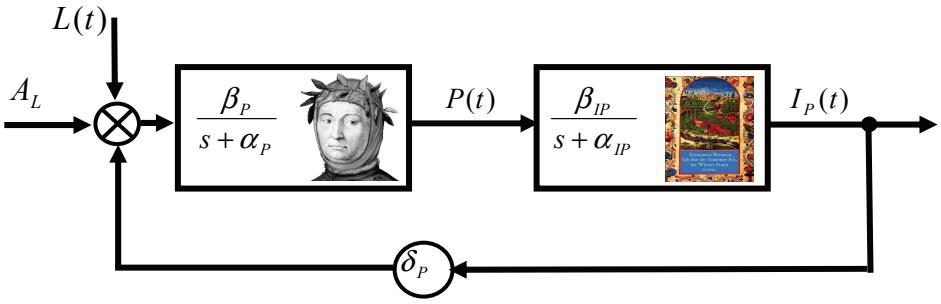

Figure 10. Linear transfer function models for Petrarch's inspiration $I_P(t)$ with feedback to emotion $P(t)$.

Also in this linear approach with driving force appeal and driving force inspiration an ambiguity appears: high moral tensions, like those associated with artistic inspiration, may also attenuate the role of the most basic instincts. And there is no doubt that the tensions between Petrarch and Laura are of a passionate nature, as can be read in *Sonnet XXII* (Listing 5).

Con lei foss'io da che si parte il sole, *et non ci vedess' altri che le stelle,* *sol una nocte, et mai non fosse l'alba.*	*Would I were with her when first sets the sun, and* *no one else could see us but the stars,* *one night alone, and it were never dawn.*

Listing 5. Sonett XXII from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

And in his *Posteritati*, Petrarch confesses the ambiguity of his love to Laura:

Libidem me prorsus expertem dicere *posse optarem quidem,* *sed si dicat mentiar.*	*I would truly like to say absolutely that* *I was without libidinousness, but if I said* *so I would be lying.*

Listing 6. From *Posteritati,* F.- Petrarca.

Combining the transfer function models for Laura's and Petrarch's emotions with the transfer function model for Petrarch's inspiration, results in three coupled first-order lags (Figure 11). Transforming the transfer function model into time space, we get the same basic linear ODEs then we got with the SD approach:

$$\frac{dL(t)}{dt} = -\alpha_L L(t) + \beta_L P(t) + \beta_L A_P$$
$$\frac{dP(t)}{dt} = -\alpha_P P(t) + \beta_P L(t) + \gamma_P I_P(t)$$
$$\frac{dI_P(t)}{dt} = -\alpha_{IP} I_P(t) + \beta_{IP} P(t)$$

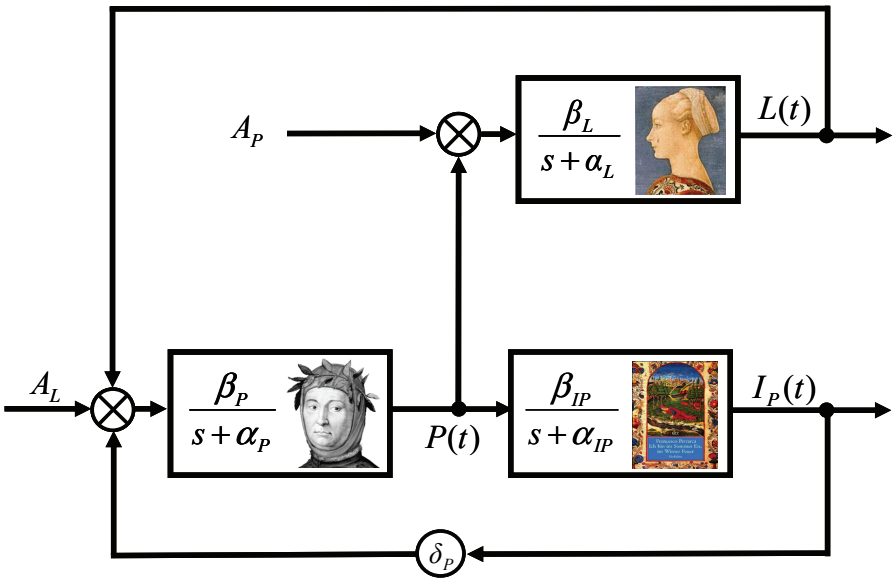

Figure 11. Coupled Linear transfer function model Laura's emotion Petrarch's emotion $L(t)$, for Petrarch's emotion $P(t)$ and Petrarch's inspiration $I_P(t)$.

Of course, the above equations are linear ODEs, and they are not able to produce oscillations with nonlinear effects, as the emotional cycle represents. Furthermore, emotions almost never act and react in a linear way. For incorporation of nonlinear effects, in transfer functions gain factors and time constants are made functions dependent on other state variables in a nonlinear way. But control technique does not offer a general approach for suitable nonlinear dependencies.

6 Model approach by nonlinear ODEs

The big challenge for S. Rinaldi was to set up an ODE model for the cyclic love dynamics and emotional dynamics of Petrarch and Laura, which would fit the experimentally founded emotional cycle of Petrarch, incorporating nonlinear effects from the beginning on.

The three variables $P(t)$, $L(t)$ and $I_P(t)$ again are starting points. Emotions between a couple may be seen as predator – prey system, where it is not really known, who is prey, and who is predator. Denoting $L(t)$ Laura's grade of feelings and emotions for Petrarch - positive and high values of L mean warm friendship, while negative values should be associated with coldness and antagonism, and denoting $P(t)$ the grade of Petrarch's love and emotions for Laura - whereby high values of P indicate ecstatic love, while negative values stand for despair, a special predator- prey model can be set up:

$$\frac{dL(t)}{dt} = -\alpha_L L(t) + R_L(P(t)) + \beta_L(A_P, Z_L(t))$$
$$\frac{dP(t)}{dt} = -\alpha_P P(t) + R_P(L(t)) + \beta_P(A_L, Z_P(t))$$

The speciality here is the fact, that the change of emotions in both cases in negative proportional to itself, in contrary to a classical predator-prey model. The feeding rates $R_L(P)$ and $R_P(L)$ – here called the reaction rates - are suitable nonlinear functions of the partner's emotion, and the external feeding input – here the appeal functions β_L and β_P – depend in a nonlinear way on a constant appeal of the partner, A_L or A_P resp., and on some additional emotional input $Z_L(t)$ or $Z_P(t)$, resp.

Next, it is necessary to model in more detail the complex personality Petrarch. Petrarch observes, in his poems, he lives in his poems, and he feels in his poems – so his inspiration $I_P(t)$ must be the additional emotional input for his emotions, on the other side the Petrarch's inspiration follows more or less directly Petrarch's emotion. Here the inspiration as third state variable comes up, Petrarch's inspiration $I_P(t)$, expressing his productivity for poems (in general modelling the inspiration for work related to the love) – here a linear approach may be sufficient:

$$\frac{dI_P(t)}{dt} = -\alpha_{IP} I_P(t) + \beta_{IP} P(t)$$

Women at that time were not allowed to express their feelings in public, so for Laura the appeal function is more or less constant, a grade for Petrarch's appeal on her. Combining the emotions and the inspiration results in the following nonlinear systems:

$$\frac{dL(t)}{dt} = -\alpha_L L(t) + R_L(P(t)) + \beta_L A_P$$
$$\frac{dP(t)}{dt} = -\alpha_P P(t) + R_P(L(t)) + \beta_P(A_L, I_P(t))$$
$$\frac{dI_P(t)}{dt} = -\alpha_{IP} I_P(t) + \beta_{IP} P(t)$$

The rate of change of the emotions of Laura $L(t)$ is the sum of three terms. The first describes the forgetting process characterising each individual. The second term $R_L(P)$ is the reaction of Laura to the love of Petrarch – a nonlinear function), and the third is her response to his appeal (linear). The rate of change of the love of Petrarch $P(t)$ is of similar structure, with an extension: the response of Petrarch to the appeal of Laura depends also upon his inspiration $I_P(t)$.

Already in the linear approach using transfer functions, the appeal function β_L - there driving force appeal and driving force inspiration – tried to mimic the ambiguity between physical instincts and ideal feelings – as simple weighted sum. And there is no doubt that the tensions between Petrarch and Laura are also of a passionate nature, as can be read in Sonnet XXII (Listing 5) and as Petrarch himself confesses in his *Posteriati* (Listing 6). The idea is, to make use of o driving force, which keeps emotions running and oscillating on:

$$\beta_P(A_L, I_P(t)) = \beta_P \frac{A_L}{1 + \delta_P I_P(t)}$$

This hyperbolic appeal functions indeed prevents from stationary behaviour in many cases (Figure 12). The equation for the inspiration $I_P(t)$ says that the love of Petrarch sustains his inspiration which, otherwise, would exponentially decay.

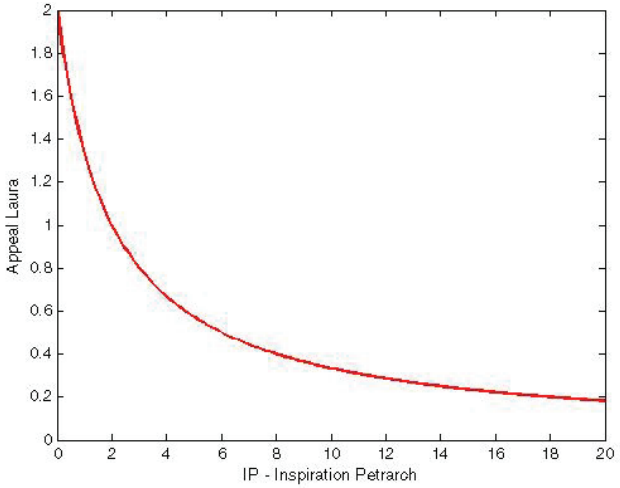

Figure 12. Nonlinear hyperbolic appeal function β_L for Petrarch.

The reaction functions $R_L(P)$ and $R_P(L)$ are partly nonlinear. A linear approach would simply say that individuals love to be loved and hate to be hated. The linearity of $R_P(L)$ is more or less obvious, since in his poems the poet has very intense reactions to the most relevant signs of antagonism from Laura: $R_P(L(t)) = \beta_P \cdot L(t)$.

But a linear reaction function is not appropriate for Laura. Only around $R_L(P) = 0$ it can be assumed to be linear, thus interpreting the natural inclination of a beautiful high-society lady to stimulate harmless flirtations. But Laura never goes too far beyond gestures of pure courtesy: she smiles and glances. However, when Petrarch becomes more demanding and puts pressure on her, even indirectly when his poems are sung in public, she reacts very promptly and rebuffs him, as described explicitly in a number of poems, as in *Sonnet XXI* (Listing 7).

Mille fiate, o dolce mia guerrera, *per aver co' begli occhi vostri pace* *v'aggio proferto il cor; ma voi non piace* *mirar si basso colla mente altera.*	*A thousand times, o my sweet enemy,* *to come to terms with your enchanting eyes* *I've offered you my heart, yet you despise* *aiming so low with mind both proud and free.*

Listing 7. Sonett XXI from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

Consequently, the reaction function $R_L(P)$ should, for $P > 0$, first increase, and then decrease. But the behaviour of Laura' reaction is also nonlinear for negative values of P. In fact, when $P < 0$ (when the poet despairs), Laura feels very sorry for him. Following her genuine Catholic ethic she arrives at the point of overcoming her antagonism by strong feelings of pity, thus reversing her reaction to the passion of the poet. This behavioural characteristic of Laura is repeatedly described in the *Sonett LXIII* (Listing 8).

Volgendo gli occhi al mio novo colore *che fa di morte rimembrar la gente,* *pieta vi mosse; onde, benignamente* *salutando, teneste in vita il core.*	*Casting your eyes upon my pallor new,* *which thoughts of death recalls to all mankind,* *pity in you I've stirred; whence, by your kind* *greetings, my heart to life's kept true.*

Listing 8. Sonett LXIII from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

Consequently, a good choice for Laura's reaction function $R_L(P)$ would be a cubic function of the following type, displayed in Figure 13.

$$R_L(P) = \beta_L P \cdot \left(1 - \left(\frac{P}{\gamma_L}\right)^2\right)$$

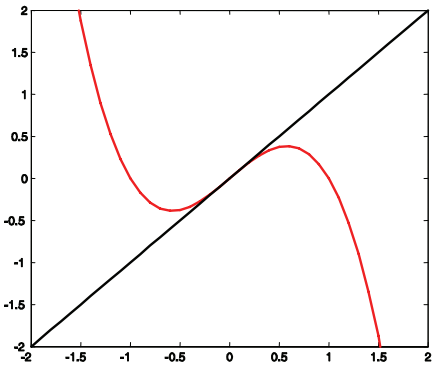

Figure 13: Nonlinear reaction function $R_L(P)$ of Laura - red line, and linear reaction function $R_P(L)$ of Petrarch – straight black line.

With these approaches, the full *Laura-Petrarch model* is given by the following equations:

$$\frac{dL(t)}{dt} = -\alpha_L L(t) + \beta_L P\left(1 - \left(\frac{P}{\gamma_L}\right)^2\right) + \beta_L A_P$$

$$\frac{dP(t)}{dt} = -\alpha_P P(t) + \beta_P L(t) + \beta_P \frac{A_L}{1 + \delta_P I_P(t)}$$

$$\frac{dI_P(t)}{dt} = -\alpha_{IP} I_P(t) + \beta_{IP} P(t)$$

7 Nonlinear relations in transfer function model and System Dynamics model

It is a common practice in control modelling, to incorporate nonlinearities by making gain factors and time constants nonlinear functions depending on other states. The nonlinear reaction function $R_L(P)$ of Laura can be interpreted as additional additive nonlinear gain $R_L(P)$ (modified cubic gain with parameter γ_L) as input to Laura's transfer function, see Figure 14.

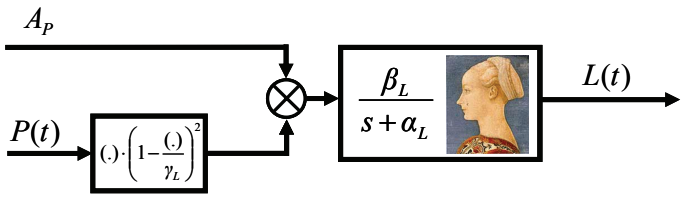

Figure14. Transfer function model for Laura's love $L(t)$ with nonlinear gain for Petrarch's love $P(t)$ as reaction input.

For Petrarch's nonlinear dynamics first an additional transfer function for his poetic inspiration $I_P(t)$ is introduced, with gain β_{IP} and time constant α_{IP} . This variable acts as input for a nonlinear gain for Laura's appeal A_L with parameter δ_P; both additional blocks are summarised in Figure 15.

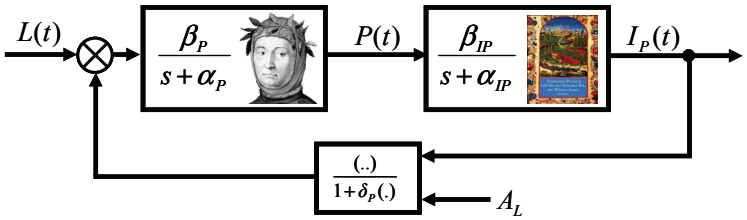

Figure15. Transfer function model for Petrarch's love $P(t)$, with additional transfer function for poetic inspiration $I_P(t)$, setting up the nonlinear gain for Laura's appeal A_L

The nonlinearities are of different quality. Choosing in the nonlinear cubic-like gain for Laura's reaction $R_L(P)$ a big value for the parameter γ_L, the nonlinear gain becomes almost linear (the nominator is bounded, usually less than 1). The nonlinear gain for Laura's appeal A_L becomes linear, if the parameter δ_P is set to zero, letting the influence of Petrarch's poetic inspiration vanish. Now the nonlinear transfer function models for Laura and Petrarch can be combined to a nonlinear transfer function model for the love dynamics of Laura and Petrarch $L(t)$ and $P(t)$, and the poetic inspiration dynamics of Petrarch $I_P(t)$, presented in Figure 16.

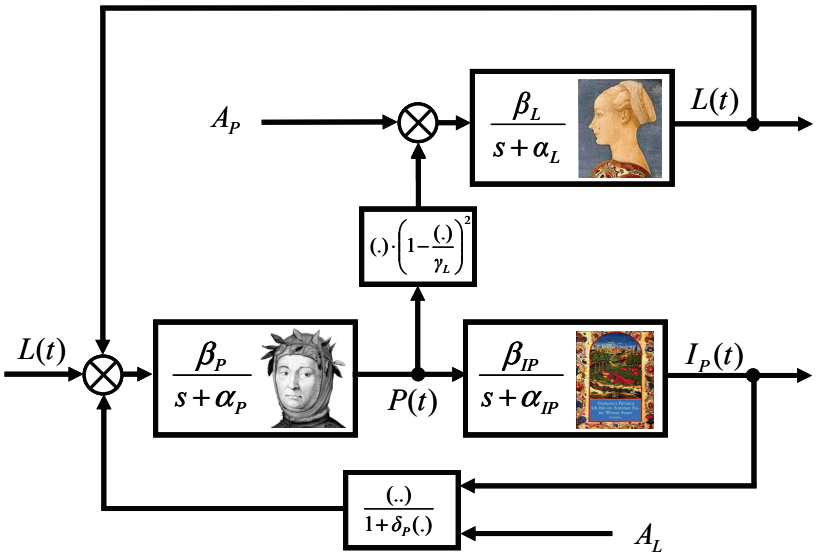

Figure16. Nonlinear transfer function model for the love dynamics of Laura and Petrarch, $L(t)$ and $P(t)$, and for the poetic inspiration dynamics of Petrarch $I_P(t)$; model equivalent to ODE model,

In System Dynamics, nonlinear elations are 'hidden' in auxiliaries. The parameters on the feedback arcs (Figure 7) become nonlinear functions of arbitrary complexity and with arbitrary dependencies from other levels and auxiliaries. Figure 17 shows the FSD for the full nonlinear model - to be used by a simulator, to be used directly in the SD modelling and simulation environment – but with nonlinear functions derived heuristically by mathematical formula. It can be concluded, that System Dynamics is a very appropriate tool for modelling the emotional dynamics under investigation – again it is underlined, that SD offers big benefits for modelling social systems, because relations and influences can be modelled step by step, from qualitative causal links to quantitative stock and flow diagrams, which can be simulated directly.

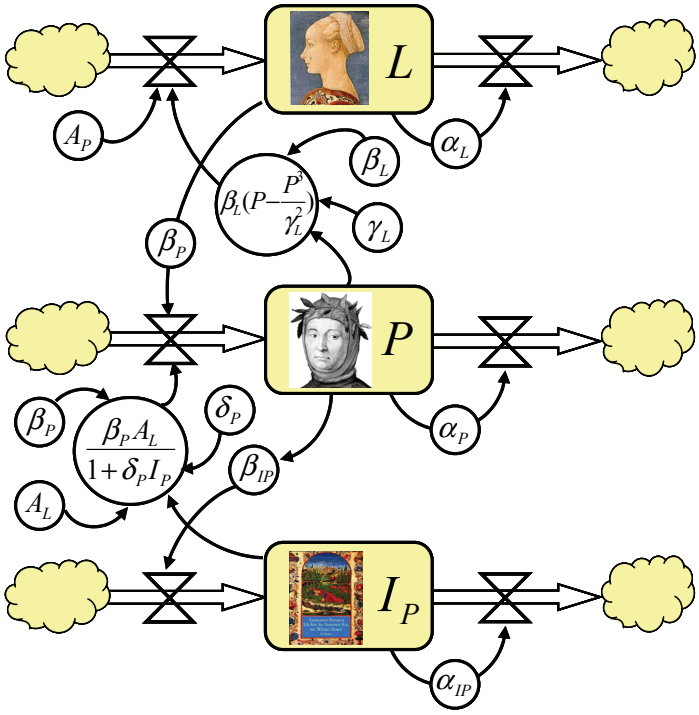

Figure17. Nonlinear SD model for the love dynamics of Laura and Petrarch, $L(t)$ and $P(t)$, and for the poetic inspiration dynamics of Petrarch $I_P(t)$; model equivalent to ODE model

Finally it should be remarked, that SFDs can be directly mapped to block diagrams with transfer functions: stock and output flow with direct feedback of stock correspond to a first-order transfer function, with input represented by the input flow (mapped on a sum block); parameters in feedback links are mapped to gain blocks, auxiliaries nonlinear blocks. And vice versa transfer functions can be expresses by SD diagrams.

The structure of the transfer function model and of the SD model (Figure 16, Figure 17) suggest a natural extension: also Laura writes poems, so that Petrarch's appeal is influenced by her poetic inspiration, and Petrarch shows more sensibility in his reaction to Laura. The model would become 'symmetric'; some aspects of an extended model will be discussed in Section 10 and Section 11.

8 5. Identification of the Laura–Petrarch Model

The big challenge is to identify the model parameters in the nonlinear Laura-Petrarch model, with two appeal parameters, with three gains, with three time constants, and with two parameters for the nonlinearity – in sum ten parameters. A brute-force identification starting with arbitrary values for these parameters is not successful, especially as the appeals may also be negative.

Consequently first the size of the parameters and relations between them should be qualitatively analysed, following S. Rinaldi ([4]). The time constants α_L, α_P, and α_{IP} describe the forgetting processes. For Laura and Petrarch obviously $\alpha_L > \alpha_P$ holds, because Laura never appears to be strongly involved, while the poet definitely has a tenacious attachment, documented in *Sonnet XXXV* (Listing 9).

Solo et pensoso i piu deserti campi *vo mesurando a passi tardi e lenti,* *.....* *Ma pur si aspre vie ne' si selvage* *cercar non so ch' Amor non venga sempre* *ragionando con meco, et io col'lui.*	*Alone and lost in thought, each lonely strand* *I measure out with slow and laggard step,* *.....* *Yet I cannot find such harsh and savage trails* *where love does not pursue me as I go,* *with me communing, as with him do I.*

Listing 9. Sonett XXXV from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

The inspiration of the poet wanes very slowly, because Petrarch continues to write (over one hundred poems) for more than ten years after the death of Laura. The main theme of these lyrics is not his passion for Laura, which has long since faded, but the memory for her and the invocation of death, Sonnet CCLXVIII (Listing 10). Consequently between the time constants α_{IP} and α_p the relation $\alpha_{IP} < \alpha_p$ must hold.

Tempo e ben di morire, *et o tardato piu ch'i non vorrei.*	*It's time indeed to die,* *and I have lingered more than I desire.*
Madonna e morta, et a *seco il mio core; e volendol seguire,* *interromper conven quest'anni rei,* *perche mai veder lei* *di qua non spero, et l'aspettar m'e noia.*	*My lady's dead, and with her my heart lies;* *and, keen with her to fly,* *I now would from this wicked world retire,* *since I can no more aspire* *on earth to see her, and delay will me destroy.*

Listing 10. Sonett XXXV from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

As Petrarch's inspiration holds about ten years, whereas Laura forgets Petrarch in about four months, and Petrarch's passion fades in one year, suitable relations and values are

$$\alpha_L \sim 3 \cdot \alpha_P, \alpha_P \sim 10 \cdot \alpha_{IP}, \alpha_P \sim 1$$

The gains or reaction parameters β_L, β_p, and β_{IP} also can be estimated qualitatively, with respect to the time constants:

$$\beta_L \sim \alpha_P, \beta_P \sim 5 \cdot \alpha_P, \beta_{IP} \sim 10 \cdot \alpha_P$$

Here the assumption is that Laura's reaction equals the forgetting time of Petrarch, and Petrarch reacts five times stronger. For simplicity, the parameters γ_L and δ_P are normalised to one, since it is always possible to scale $P(t)$ and $I_P(t)$ suitably.

The choice of the appeal parameters A_L and A_P is crucial, because these parameters determine the qualitative behaviour of the love dynamics – cyclic nonlinear behaviour, or damped oscillation toward an equilibrium. In case of Laura and Petrarch, cyclic love dynamics are expected in order to meet the experimentally founded emotional cycle $E(t)$ of Petrarch. There exist no real pictures of Laura, and pictures of Petrarch are caressing – also in Figure 18.

Figure 18. Portraits of Laura and Petrarch, from Biblioteca Medicea Laurenziana, ms. Plut. cc. VIIIv- IX, Florence, Italy (courtesy of Ministero per i Bene Culturali e Ambientali).

Clearly, Petrarch loves Laura, so $A_L > 0$ must hold. By contrast, Petrarch is a cold scholar interested in history and letters. He is appointed a *cappellanus continuus commensalis* by Cardinal Giovanni Colonna, and this ecclesiastic appointment brings him frequently to Avignon, where Laura lives. Consequently Petrarch's appeal A_P is assumed to be negative. Appropriate choices for the appeals A_L and A_P are:

$$A_L \sim 2, \;\; A_P \sim -1$$

The negativity of the appeal of Petrarch for Laura is somehow recognized by the poet himself, as the poet says in *Sonnet XLV* (Listing 11), while talking about Laura's mirror.

Il mio adversario in cui veder solete *gli occhi vostri ch'Amore* *e'l ciel honora, ...*	*My rival in whose depths* *you're wont to see* *your own dear eyes which* *Love and heaven apprize, ...*

Listing 11. Sonett XXXV from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

The above estimated ten parameter values, together with zero initial values for the love dynamics and for the poetic inspiration, are a good choice for identification. The ODE model has been implemented in Maple, the transfer function model in MATLAB / Simulink. For identification, a least squares method can be used, available in both systems:

$$\sum (P(t_k) - E_k)^2 \quad \rightarrow \quad \min$$

There, it makes sense to use relations between parameters, so that the number of parameters to be identified is reduced. As the quality of data is relatively poor, also different sets of parameter values may be seen as good approximation. Figure 19 shows an identification result for $P(t)$, with data E_k ('graded' poems).

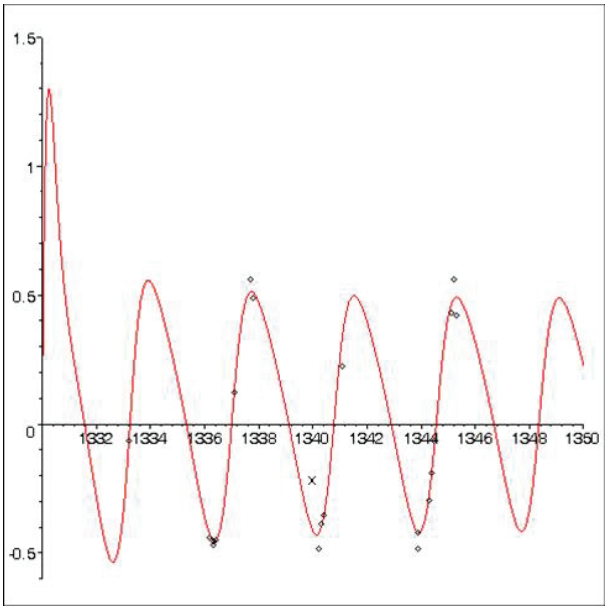

Figure 19. Result of model identification: love dynamics $P(t)$ for Petrarch coinciding with data from Petrarch's emotional cycle $E(t_k)$, with data E_k (crosses and circles).

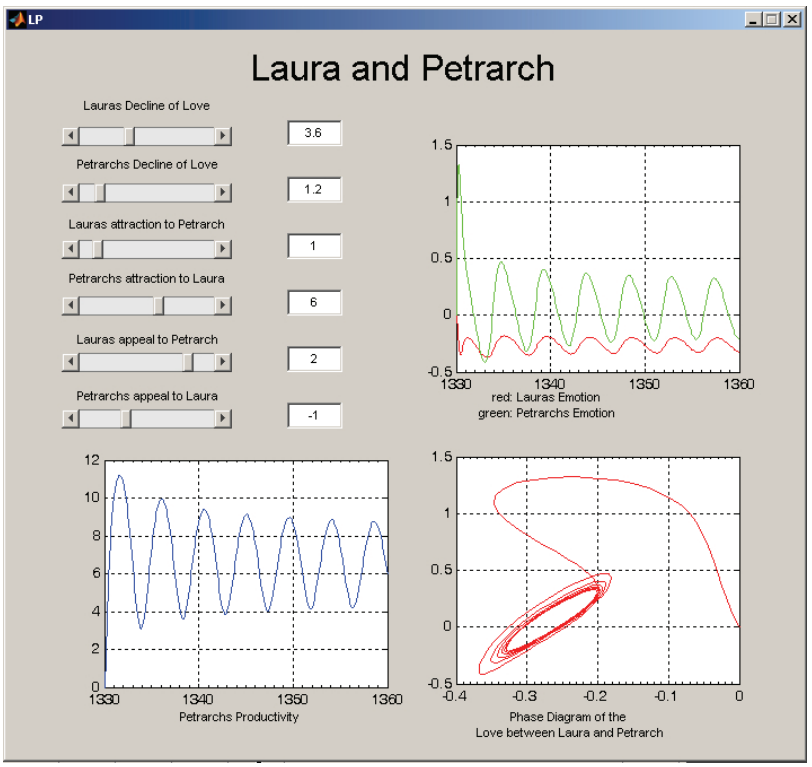

Figure 20. Maple Maplet / MATLAB GUI for experimenting with the Laura- Petrarch model:
i) above left: sliders for gain β_L, time constant α_L, gain β_P, time constant α_P, and appeals A_L and A_P;
ii) upper right: love dynamics for Laura - $L(t)$, green - and Petrarch - $P(t)$, red – over time period 1130 - 1360;
iii) lower left: poetic inspiration of Petrarch - $I_P(t)$, blue - over time period 1130 – 1360;
iv) lower right: phase portrait $P(L)$ of love dynamics of Petrarch and Laura – P over.

Of interest are of course also the love dynamics $L(t)$ for Laura and Petrarch's poetic inspiration $I_P(t)$. All variables are presented in a MATLAB GUI and in a Maple Maplet, which drive experiments with the Laura-Petrarch model. The user interface (Figure 20) offers parameter input (sliders) and displays time courses for $P(t)$ and $L(t)$ (together), the time course for $I_P(t)$, and a phase portrait $P(L)$.

Figure 20 shows all results for the identified parameters. The results of the numerical solution are qualitatively in full agreement with the *Canzoniere* and with the analysis of Frederic Jones. After a first high peak, Petrarch's love $P(t)$ tends toward a regular cycle characterised by alternate positive and negative peaks. Also, Laura's love $L(t)$ and Petrarch's poetic inspiration $I_P(t)$ tend towards a cyclic pattern. At the beginning, Petrarch's inspiration $I_P(t)$ rises much more slowly than his love and then remains positive during the entire period. This might explain why Petrarch writes his first poem more than three years after he has met Laura, but then continues to produce lyrics without any significant interruption. By contrast, Laura's love is always negative. This is in perfect agreement with the *Canzoniere*, where Laura is repeatedly described as adverse, e.g. in *Sonnet XXI* and *Sonnet XLIV* (Listing 12).

dolce mia guerrera ...	*my sweet enemy ...*
ne lagrima pero discese anchora *da' be' vostr'occhi, ma disdegno et ira.*	*and still no tears your lovely eyes assail,* *nothing as yet, but anger and disdain.*

Listing 12. Sonetts XXI and XLIV from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

The fit between $P(t)$ for Petrarch's love and $E(t_k)$ for Petrarch's emotional cycle is actually very good. It is of similar quality which is usually obtained when calibrating models of electrical and mechanical systems. Moreover, the fit could be further improved by slightly modifying the parameter values and by loose some parameter relations. But improvement might be skipped, citing and agreeing with Rinaldi: we do not want to give the impression that I believe that Petrarch had been producing his lyrics like a rigid, deterministic machine. Nevertheless, one can conclude that the Laura-Petrarch Model strongly supports Frederic Jones's conjecture on Petrarch's emotional cycle.

9 Experiments and Analysis with the Laura–Petrarch model

Experiments with the parameters show, that the cyclic love dynamics may change to a damped oscillation converging to equilibrium. It is difficult to find out which parameter quality causes a cyclic behaviour, and which the damped oscillations. Starting with the classic Laura–Petrarch parameters, for instance an increase of only one parameter α_L by a factor of 2.5 changes the qualitative behaviour essentially (Figure 21) – this parameter change means, that Laura forgets Petrarch in about half time then before.

In this case all variables for $P(t)$, $L(t)$, and $I_P(t)$, show strongly damped oscillation, reaching equilibrium almost in ten years. The phase portrait $P(L)$ underlines this convergence in relatively fast time to an equilibrium of about $P =$ 0.05 and L = -0.0105. This equilibrium has still a negative value for Laura's love emotion, but it is very small – almost as small as the positive value for Petrarch's emotion (see Fig. 21).

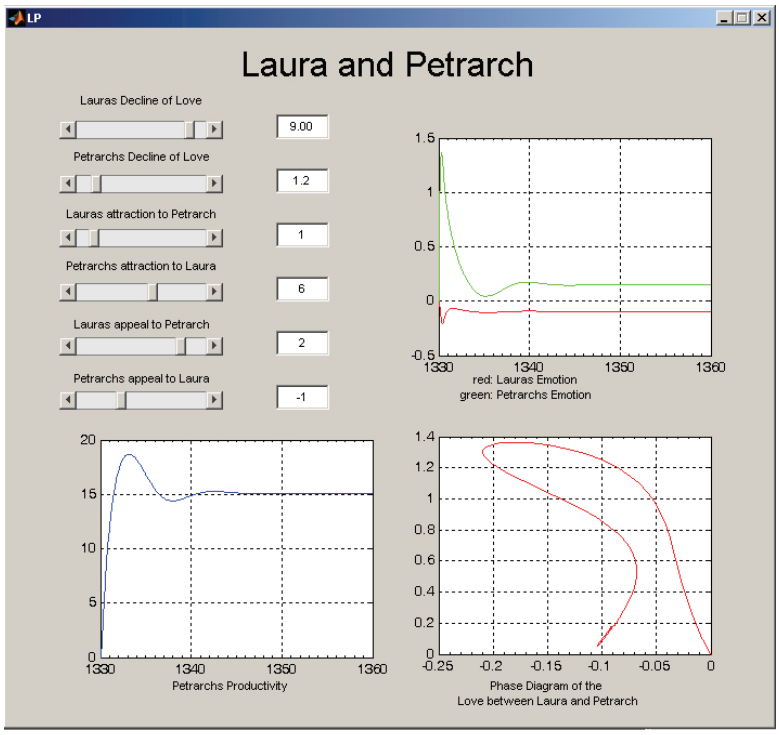

Figure 21. Experiment with Laura–Petrarch parameters and changed parameter α_L by a factor of 2.5: strongly damped variables converging to a an equilibrium with very small positive and very small negative value for P and L, resp.

In principle, the existence of an equilibrium is tied to the existence of steady state solutions for the ODE. Setting all derivatives to zero, and substituting I_P by the corresponding steady state for P, i.e.

$$I_P(t \to \infty) = \frac{\beta_{IP}}{\alpha_{IP}} P(t \to \infty),$$

results in two coupled nonlinear algebraic equations for P and L at steady state:

$$0 = -\alpha_L L + \beta_L P - \beta_L P^3 + \beta_L A_P$$
$$0 = -\alpha_P(\alpha_{IP} + \beta_{IP} P)P + \beta_P(\alpha_{IP} + \beta_{IP} P)L + \alpha_{IP}\beta_P A_L$$

Rinaldi investigates in detail by means of analytical methods the case $A_L < 0$ and $A_P > 0$. First he tested the robustness of the Laura – Petrarch cyclic love dynamics with respect to perturbations of the parameters.

For this, the package LOCBIF, a professional software package for the analysis of the bifurcations of continuous-time dynamical systems, has been used. By varying only one parameter at a time, Rinaldi detected a supercritical Hopf bifurcation, by which the cycle eventually disappears.

Rinaldi continued with a detailed analysis of the limit cycle using substitution of the variable $L(t)$, transformation of parameters, and singular perturbation for $P(t)$ and $I_P(t)$. Results derive that indeed a supercritical Hopf bifurcation causes a change of the qualitative behaviour. One detailed result is a stability chart (Figure 19) for new parameters ε and μ :

$$\varepsilon = \alpha_{IP},\ \mu = \frac{\beta_{IP}}{\alpha_{IP}},\ \ \beta_P \cdot \beta_L > \alpha_P \cdot \alpha_L$$

In Figure 22, the border between cyclic solutions and solutions converging to equilibrium can be seen, represented as graph $\mu(\varepsilon)$. In general, cyclic solutions exist only in case of nonsymmetric reactions, or as in simple case for appeal parameters with different signs. More details about these analytical investigations can be found in Rinaldi's work ([4]).

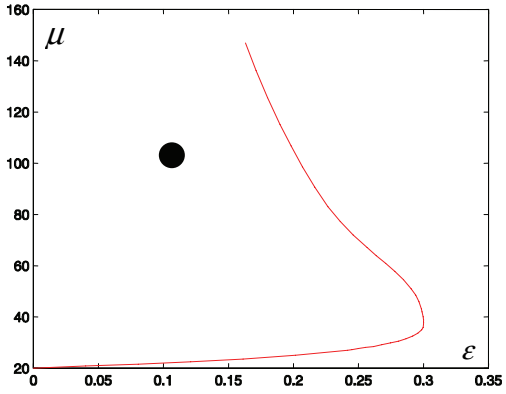

Figure 22. Stability chart $\mu(\varepsilon)$ showing the border (red line; Hopf bifurcation) between cyclic solutions (left) and solutions converging to equilibrium (right); the circle indicates the classical cyclic Laura–Petrarch solution

Usually, if people fall in love, both are attractive for each other, that means that A_L and A_P must be positive. In this case we meet only damped oscillations converging to a stable steady state:

$$A_L > 0, A_P > 0 \rightarrow \exists\,(P > 0, L > 0)$$

An interesting experiment is the case of an attractive Petrarch. Supposing e.g. that Petrarch is a young beautiful men, almost like Apollo, he may have the appeal $A_L \sim 6$ to Laura, three times the appeal of Laura to him (all other parameters unchanged). Figure 23 shows the MATLAB GUI / Maple Maplet with the results of this experiment. All variables $P(t)$, $L(t)$, and $I_P(t)$, are very strongly damped and converge to steady states with relative high positive values. The poetic inspiration $I_P(t)$ shows almost (negative) exponential behaviour.

Nevertheless $L(t)$ becomes negative for a short time period in the first three years. The phase diagram shows two crossings and it seems that the love dynamics first cycles two times, before it decides to converge to a stable positive feeling of both lovers. The MATLAB GUI allows experiments with a broad variety of parameter sets. Of interest are for instance also cases, which cycle more then hundred times, before they decide to converge to an equilibrium. For 'extreme' parameter values the numerical solution may cause problems, or at least may take a long time; as ODE solver a stiff solver has been chosen, having the best relative performance.

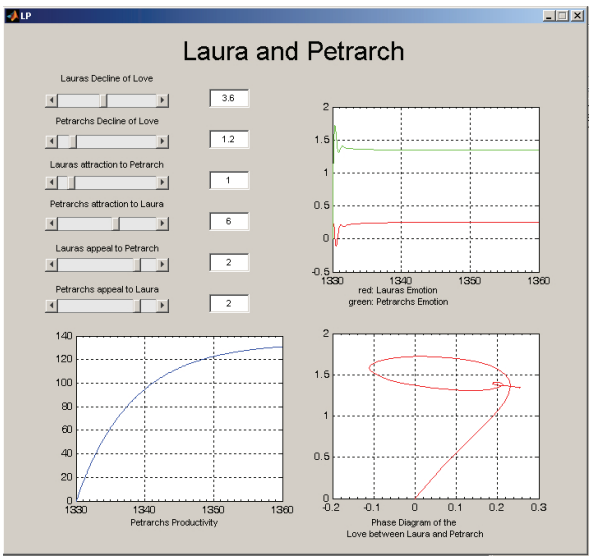

Figure 23. Experiment with Laura–Petrarch parameters and positive appeal $A_L \sim 6$ of Petrarch to Laura: very strongly damped variables converging to a an equilibrium with positive steady states for P and L, resp.

10 From Laura-Petrarch model to Woman-Man - model

In times of gender equality woman as well as men may play an active part in a love affair. Consequently also women express their love by poems or other media, and they confess their love to public. By this, an additional stock with flow for the woman's inspiration can be introduced easily. For Laura and Petrarch this would mean, that also Laura writes poems, that Petrarch's appeal is influenced by Laura's poetic inspiration, and that Petrarch shows more sensibility in his reaction to Laura.

Consequently, the structure of the System Dynamics model (Figure 6) suggests a genuine and natural extension: symmetric stocks, flows, and feedbacks gains as well for 'Petrarch' and for 'Laura', which should now generally represent a man and a women who fall in love.

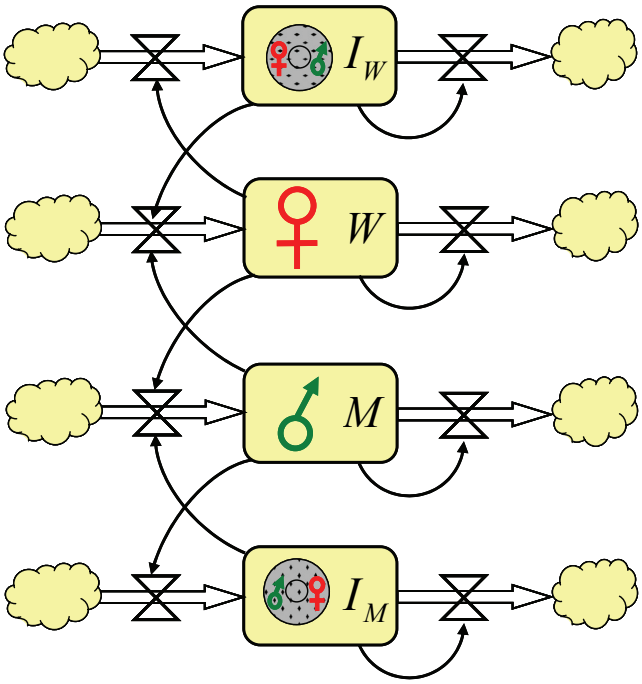

Figure 24. Qualitative System Dynamics Model for Woman's and Man's emotions and inspirations.

The *Woman–Man Model* presented in Figure 24 describes the love dynamics $W(t)$ for a woman, and $M(t)$ for a man both falling in love to each other; love inspires both the communicate their love to public, in letters, in videos, with CDs and DVDs, etc. – represented by the inspiration variables $I_W(t)$ and $I_M(t)$. A quantification of the qualitative SD model (Figure 7) with linear causal links and influences yields a SD – representation of *Man-Woman Model* with full symmetry (Figure 25).

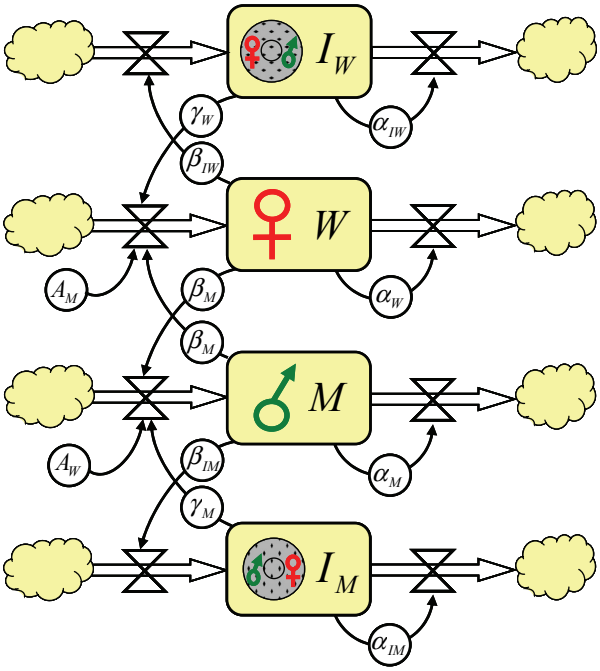

Figure 25: Simplified System Dynamics model for *Woman-Man Model* with linear influences.

The linear transfer function model (Figure 11) ca also be used as basis for a 'modern times' – extension. By adding state and first order lag for woman's inspiration, a symmetric model is defined (Figure 26).

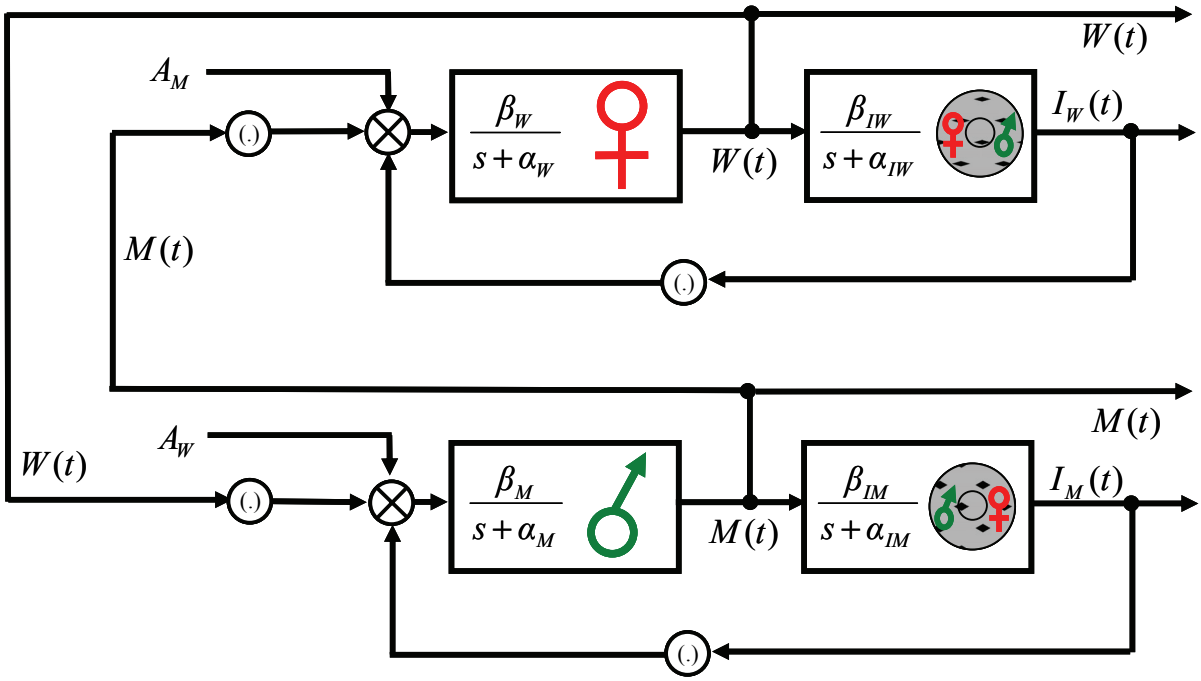

Figure 26. Simplified transfer function model for *Woman-Man Model* with linear influences.

As with the simplified Laura-Petrarch Model, the ODEs corresponding to the SFD in Figure 25 and to the transfer functions in Figure 26 are linear, and they show the full symmetry between woman's and man's emotions and inspirations:

$$\frac{dW(t)}{dt} = -\alpha_L W(t) + \beta_W M(t) + \gamma_W A_M$$
$$\frac{dI_W(t)}{dt} = -\alpha_{IW} I_W(t) + \beta_{IW} W(t)$$
$$\frac{dM(t)}{dt} = -\alpha_M M(t) + \beta_M M(t) + \gamma_M A_W$$
$$\frac{dI_M(t)}{dt} = -\alpha_{IM} I_M(t) + \beta_{IM} M(t)$$

Finally, the nonlinearities must be modelled in the FSD by auxiliaries, replacing the weighting parameters in the feedback, and in the transfer function model by nonlinear gains. Because of the symmetry in emotions and inspirations, the model makes use of two nonlinear cubic-like reaction functions for womans's and man'reaction to each other, $R_W(M)$ and $R_M(W)$, and of two nonlinear relations between inspiration, appeal, and emotion.

The nonlinearities for the reactions become almost linear for big values for the parameters γ_W and γ_M, resp; the nonlinearities for the inspiration-appeal relation become linear, if the parameters δ_W and δ_M tend towards zero. Figure 27 presents this 'complete nonlinear *Woman-Man Model* in SD notation (SFD), Figure 28 shows the equivalent notation in transfer functions.

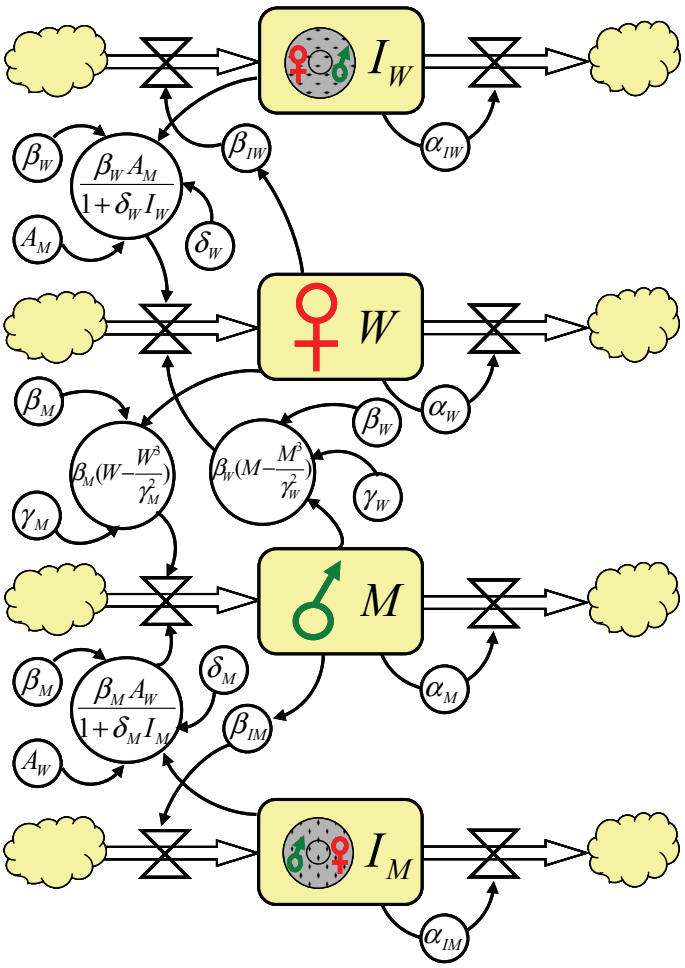

Figure 27: System Dynamics model: stock and flow diagram for *Woman-Man Model* with nonlinear reactions.

Compared with the Laura-Petrarch Model, the Woman-Man Model must make use of an increased number of parameters: four fading parameters (instead of three), four (linear) weighting factors for the cross-feedbacks (instead of three), two appeal parameters (instead of one), and four parameters in the nonlinear functions (instead of two) – in sum 14 parameters. An analysis of *Man-Woman Model* is almost impossible, but numerical experiments may give interesting insight into love dynamics.

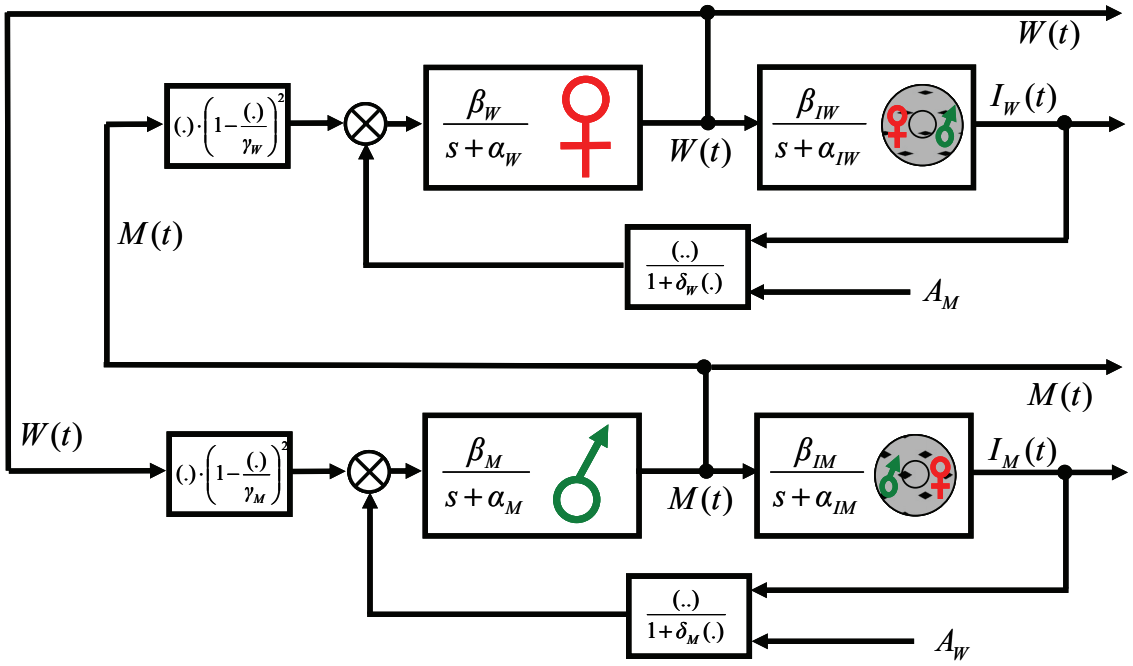

Figure 28: Transfer function – based *Woman–Man model* describing the love dynamics $W(t)$ for a woman, $M(t)$ for a man, and the inspiration variables $I_W(t)$ and $I_M(t)$ communicating the love to public.

The ODE model equivalent to the SFD in Figure 27 and to the transfer function model in Figure 28 shows now typical symmetric structure:

$$\frac{dW(t)}{dt} = -\alpha_L W(t) + \beta_W M\left(1-\left(\frac{M}{\gamma_W}\right)^2\right) + \beta_W \frac{A_M}{1+\delta_W I_W(t)}$$

$$\frac{dI_W(t)}{dt} = -\alpha_{IW} I_W(t) + \beta_{IW} W(t)$$

$$\frac{dM(t)}{dt} = -\alpha_M M(t) + \beta_M W\left(1-\left(\frac{W}{\gamma_M}\right)^2\right) + \beta_M \frac{A_W}{1+\delta_M I_M(t)}$$

$$\frac{dI_M(t)}{dt} = -\alpha_{IM} I_M(t) + \beta_{IM} M(t)$$

As the Laura–Petrarch model, the Woman–Man model has been implemented in Maple (ODE model) and in MATLAB/Simulink (transfer functions model). Experiments can be controlled by a simplified MATLAB GUI / Maple Maplet, which suggest case studies. Figure 29 shows results of two case studies in a GUI, with predefined parameters characterising typical parameter configurations ('Everyday Boring' and 'Pretty and Ugly').

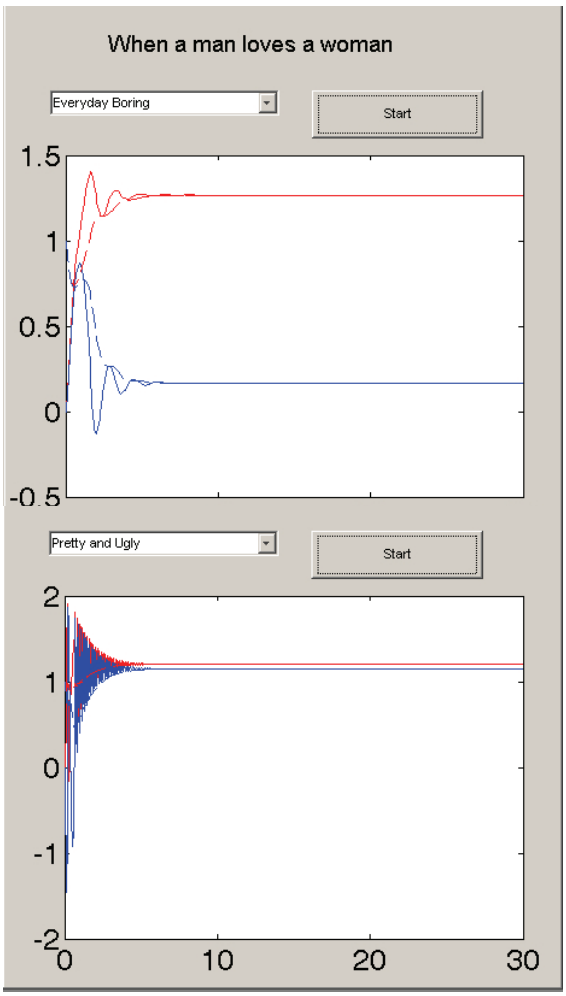

Figure 29. Experiment with *Woman–Man Model*s: case studies 'Everyday Boring' and 'Pretty and Ugly' (red/blue – woman's/men's love emotion; red/blue dashed – woman's/men's love inspiration).

11 Woman-Man Model with dynamic appeal parameters

Does the Woman – Man model reflect reality? The model is able to mimicry different situations, but with one assumption: the appeal parameters A_M and A_W are constant. This assumption may not meet reality; the appeal for each other may change and may be controlled.

A dynamic appeal can be easily modelled by time-dependent appeal variables $A_M(t)$ and $A_W(t)$, resulting in a small change in the ODE model:

$$\frac{dW(t)}{dt} = -\alpha_L W(t) + \beta_W M\left(1 - \left(\frac{M}{\gamma_W}\right)^2\right) + \beta_W \frac{A_M(t)}{1 + \delta_W I_W(t)}$$

$$\frac{dI_W(t)}{dt} = -\alpha_{IW} I_W(t) + \beta_{IW} W(t)$$

$$\frac{dM(t)}{dt} = -\alpha_M M(t) + \beta_M W\left(1 - \left(\frac{W}{\gamma_M}\right)^2\right) + \beta_M \frac{A_W(t)}{1 + \delta_M I_M(t)}$$

$$\frac{dI_M(t)}{dt} = -\alpha_{IM} I_M(t) + \beta_{IM} M(t)$$

Case studies may become now very complicated, because not only 14 parameters have to be chosen appropriately, but also the function $A_M(t)$ and $A_W(t)$ have to be provided meaningful. An extended version of the MATLAB GUI / Maple Maplet presented in Figure 20 allows additionally providing predefined appeal functions. Figure 30 and Figure 31 show results for perhaps interesting cases: the appeal decreases exponentially – should happen, and second, the appeal $A_M(t)$ is an increasing step function - could model a plastic surgery.

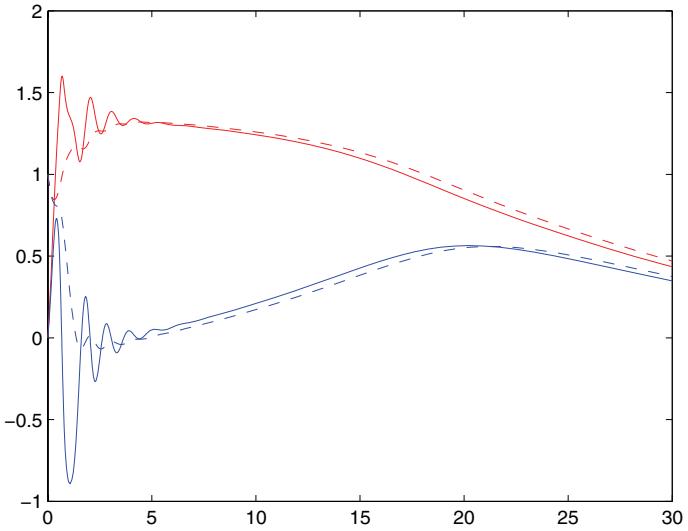

Figure 30. Experiment with Woman–Man Model: exponentially decreasing appeal functions (red/blue – woman's/men's love emotion; red/blue dashed – woman's/men's love inspiration)

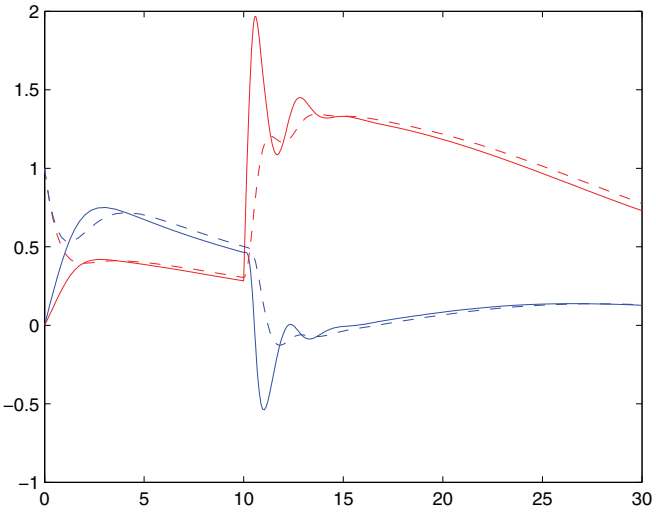

Figure 31. Experiment with Woman–Man model - positive step in appeal at ten years (time courses as in Figure 27).

12 Conclusion

In principle, the contribution could show,

- that it is possible to model in some detail the love dynamics between two person by System Dynamics or by transfer functions, resulting in nonlinear ODE system,
- that additionally a poetic inspiration caused by love emotions can be modelled in System Dynamics, or by transfer functions, extending the ODE system for the love dynamics,
- and that the model can be validated with data from history, reflecting the emotions between Petrarch and Laura.

Of course, this contribution presents serious investigations. But is it possible to investigate the dynamics of love, perhaps the most important phenomenon concerning our lives, seriously by methods of mathematics and engineering? One could also conclude, it might be better not to tackle the secrets of love, because described and controlled by formula, it is not love anymore longer. In this view, the contribution might be seen as reference to Petrarch and the most beautiful love poems the author ever read.

And finally, not ODEs, not System Dynamics, and not transfer functions are the appropriate tools to model and to document emotions. The most appropriate tool is poetry – proven by Francesco Petrarca in *Sonnet LXII* (Listing 13).

Benedette le voci tante ch'io *chiamando il nome de mia donna ò sparte,* *e i sospiri, et le lagrime, e 'l desio;* *et benedette sian tutte le carte* *ov'io fama l'acquisto, e 'l pensier mio,* *ch'è sol di lei, sí ch'altra non v'à parte.*	*And blessed be all of the poetry* *I scattered, calling out my lady's name,* *and all the sighs, and tears, and the desire;* *blessed be all the paper upon which* *I earn her fame, and every thought of mine,* *only of her, and shared with no one else.*

Listing 13. Sonnet LXII from *Canzoniere* or *Rerum Vulgarium Fragmenta* (*Fragments of Vernacular Matters*).

13 References

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[2] Breitenecker, F., Judex, F. Popper, N., Breitenecker, K. Mathe, A., and Mathe, A.: *Love Emotions between Laura and Petrarch - an Approach by Mathematics and System Dynamics.* Journal of Computing and Information Technology - CIT, **16** (2008), 4; S. 255 - 269.

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