MODELING OF COUNTERCURRENT HEAT EXCHANGERS

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Abstract

A heat exchanger is a device in which energy is transferred from one fluid to another across a solid surface. The base for elaboration of control systems is knowledge of static and dynamic characteristics of heat exchangers. This paper includes equations described heat exchange in countercurrent flow heat exchangers. The first Law of Thermodynamics, in rate form, applied to a control volume (CV) between crosses 1 – 1 and 2 – 2 is used. Taking the energy balance formula as a basis and dividing the heat exchanger into sections, the thermal balances of the cooled fluid, plate and heated fluid are prepared and pertinent system of three differential equations is derived. The ε -NTU method is used in the analysis.
Exemplary temperature profiles in steady state conditions are presented in a graphic form. Transfer functions and dynamic characteristic are determined. Values of temperatures in individual sections countercurrent flow exchangers could be computed using presented method of calculations. Response on step disturbance of inlet temperature is found. Responses on step or on sin disturbance of inlet temperature in steam condensation heat exchangers are found too.

Keywords: countercurrent heat exchanger, thermal balance, temperature profile.

Presenting Author's biography

Mariusz Adamski is a Doctor of Technical Sciences. He has finished Post Graduate Studies at Polish Academy of Sciences, Institute of Fundamental Technological Research in Warszawa. He works as senior lecturer at Faculty of Civil Engineering and Environmental Engineering, at Białystok Technical University, Poland.
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1 General

A heat exchanger is a device in which energy is transferred from one fluid to another across a solid surface. Exchanger analysis and design therefore involve both convection and conduction. Radiative transfer between the exchanger and the environment can usually be neglected unless the exchanger is uninsulated and its external surfaces are very hot. Exemplary countercurrent flow heat exchanger is presented in Figure 1.

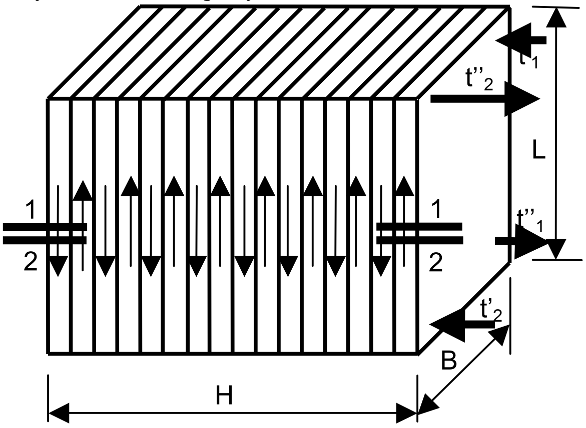

Fig. 1 Schema of plate type heat exchanger

In many industrial processes and operations, such as in nuclear reactors, power plants and chemical processes, precise simulations of the transient responses of heat exchangers are required. Optimal operation, treatment of accidents and real-time control and regulation also demand more accurate description of the time-domain behaviour of heat exchangers. With the computing power available today, modern control theories are also developed to control thermal systems using real-time dynamic models of heat exchangers [1]. Since the first dynamic model presented and solved [2] for the heat transfer between a porous medium and a fluid passing through it, many studies have been made. Historical reviews of earlier investigations on modelling and dynamics of heat exchangers were given in [3, 4]. In [5] was formulated the transient response problems of one-dimensional flow heat exchangers and summarized the available specific solutions for counterflow and cross-flow heat exchangers and thermal regenerators subjected to a step change in inlet temperature and/or flow rate of one or both fluids. New developments in dynamic analysis of heat exchangers were reviewed in [6]. Systematic description of the dynamic behaviour of heat exchangers was provided in [7], where the linear problems of heat exchangers are solved.

The base for elaboration of control systems is knowledge of static and dynamic characteristics of heat exchangers. Two important problems in heat exchanger analysis are: rating existing heat exchangers and sizing heat exchangers for a particular application.

2 Energy Considerations

The first Law of Thermodynamics, in rate form, applied to a control volume (CV) between crosses 1 – 1 and 2 – 2 (Fig. 1.), can be expressed as:

energy inflow = energy outflow + exchanged heat + + accumulated energy

or

$$Q_{\text{inf}} = Q_{out} + Q_{exch} + Q_{acc} \tag{1}$$

This simplified form of the First Law assumes no work- producing processes, no energy generation inside the CV, and negligible kinetic and potential energy in the fluid streams entering and leaving the CV. In steady state operation the energy residing in the CV is constant, meaning that Q_{acc} = 0. If, furthermore, the boundary of the CV is adiabatic (i.e., perfectly insulated), then Q_{exch} = 0.

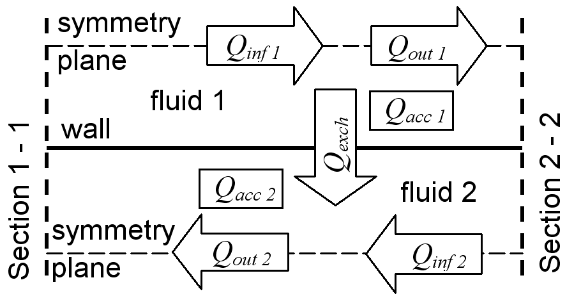

Fig. 2 Control volumes of cooled fluid (marked fluid 1) and of heated fluid (fluid 2) between sections 1-1 and 2-2 in the countercurrent flow heat exchangers

3 The thermal balances of single stream of cooled fluid, of plate and of heated fluid

The following simplifying assumptions are introduced in the analysis:

- heat conductivity along the plate is disregarded,
- densities and heat capacities are constant in considered temperature range,
- overall heat transfer coefficient is constant in all points of heating surface,
- heat losses to environment are disregarded.

3.1 The thermal balance of a single stream of the cooled fluid

The ingredients of the energy balance for a single stream of the cooled fluid are as follows:

$$Q_{inf} = G'_1 c_1 \vartheta_1 dt \tag{2}$$

$$Q_{out} = G'_1 c_1 (\vartheta_1 + \frac{\partial \vartheta_1}{\partial l} \partial l)\, dt \tag{3}$$

$$Q_{exch} = \alpha_1 S_1 (\vartheta_1 - \vartheta_w)\, dl\, dt \tag{4}$$

$$Q_{acc} = A_1 \rho_1 c_1 \frac{\partial \vartheta_1}{\partial t}\, dl\, dt \tag{5}$$

By substitution equations (2 ÷ 5) into (1) we get

$$A_1\rho_1 c_1\frac{\partial\vartheta_1}{\partial t}dl+G_1 c_1\frac{\partial\vartheta_1}{\partial l}=\alpha_1 S_1(\vartheta_w-\vartheta_1)dl \tag{6}$$

thus

$$T_1\frac{\partial\vartheta_1}{\partial t}+v_1 T_1\frac{\partial\vartheta_1}{\partial l}=\vartheta_w-\vartheta_1 \tag{7}$$

where:

$T_1=\dfrac{A_1\cdot\rho_1\cdot c_1}{\alpha_1\cdot S_1}=\dfrac{D_{h1}\cdot\rho_1\cdot c_1}{4\cdot\alpha_1}$, $\quad D_{h1}=\dfrac{4\cdot A_1}{S_1}$,

$G'_1=\dfrac{G_1}{i}=\dfrac{A_1\cdot i\cdot\rho_1\cdot v_1}{i}=A_1\cdot\rho\cdot v_1=\dfrac{W'_1}{c_1}$.

and

α – heat transfer coefficient [W m^{-2} K^{-1}]

A – area of one duct cross – sectional [m^2]

B, L, H – width of heat exchanger, length of duct, length of exchanger, respectively [m]

c – specific heat [J kg^{-1} K^{-1}]

dl – element of length [m]

D_h – hydraulic diameter [m]

ρ – density [kg m^{-3}]

S – wetted periphery of one duct [m]

ϑ – temperature [°C]

t – time [s]

T – time constant [s]

$G\ c = W$– thermal capacity of flow rate [W K^{-1}]

v – velocity [m.s^{-1}]

1, 2, w – cooled, heated fluid, plate, respectively,

G', W', F'– connected with one channel or plate

3.2 The thermal balance of plate

The thermal balance of plate takes the form:

$$Sg\rho_w c_w\frac{\partial\vartheta_w}{\partial t}dl=\alpha_1 S(\vartheta_1-\vartheta_w)dl-\alpha_2 S(\vartheta_w-\vartheta_2)dl \tag{8}$$

and

$$\frac{\partial\vartheta_w}{\partial t}=\frac{1}{T_{1s}}(\vartheta_1-\vartheta_w)-\frac{1}{T_{2s}}(\vartheta_w-\vartheta_2) \tag{9}$$

where:

g – wall thickness [m],

and

$T_{1s}=\dfrac{(S\cdot g\cdot\rho_w)\cdot c_w}{\alpha_1\cdot S}$, $\quad T_{2s}=\dfrac{(S\cdot g\cdot\rho_w)\cdot c_w}{\alpha_2\cdot S}$

3.3 The thermal balance of a single stream of the heated fluid

The thermal balance of the heated fluid takes the form:

$$A_2\rho_2 c_2\frac{\partial\vartheta_2}{\partial t}dl-G'_2 c_2\frac{\partial\vartheta_2}{\partial l}dl=\alpha_2 S_2(\vartheta_w-\vartheta_2)dl \tag{10}$$

thus

$$T_2\frac{\partial\vartheta_2}{\partial t}-v_2 T_2\frac{\partial\vartheta_2}{\partial l}=\vartheta_w-\vartheta_2 \tag{11}$$

where:

$T_2=\dfrac{A_2\cdot\rho_2\cdot c_2}{\alpha_2 S_2}=\dfrac{D_{h2}\cdot\rho_2\cdot c_2}{4\cdot\alpha_2}$, $D_{h2}=\dfrac{4\cdot A_2}{S_2}$,

$G'_2=\dfrac{G_2}{i+1}=\dfrac{A_2\cdot i\cdot\rho_2\cdot v_2}{i+1}=A_2\cdot\rho_2\cdot v_2=\dfrac{W'_2}{c_2}$.

4 Static characteristics of plate heat exchangers

If we disregard time derivatives in the balance equations (6, 8, 10), we obtain differential equations for steady state conditions:

$$W'_1\frac{\partial\vartheta_1}{\partial l}=\alpha_1 S_1(\vartheta_w-\vartheta_1) \tag{12}$$

$$0=\alpha_1(\vartheta_1-\vartheta_w)-\alpha_2(\vartheta_w-\vartheta_2) \tag{13}$$

$$-W'_2\frac{\partial\vartheta_2}{\partial l}=a_2 S_2(\vartheta_w-\vartheta_2) \tag{14}$$

Boundary conditions are as follows:

$$\text{for } l=0\quad \vartheta_1=t'_1, \tag{15}$$

$$\text{for } l=L\quad \vartheta_2=t'_2. \tag{16}$$

where t' – inlet temperature [°C].

If we solve equations (12, 13, 14) taking into account boundary conditions (15 ÷ 16), we obtain of formulae describing temperatures for the steady state conditions:

$$\vartheta_1(l)=t'_1-(t'_1-t'_2)Z(l) \tag{17}$$

$$\vartheta_2(l)=t'_2+(t'_1-t'_2)\frac{W'_1}{W'_2}(Z(L)-Z(l)) \tag{18}$$

$$\vartheta_{sc}(l)=\frac{\alpha_1\cdot\vartheta_1(l)+\alpha_2\cdot\vartheta_2(l)}{\alpha_1+\alpha_2} \tag{19}$$

where (20)

$$Z(l)=\frac{1-\exp((W'_1/W'_2-1)\cdot k\cdot F'(l)/W'_1)}{1-(W'_1/W'_2)\exp((W'_1/W'_2-1)\cdot k\cdot F'(L)/W'_1)},$$

$$\frac{1}{k}=\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{g}{\lambda}+R_Z,\quad F'(l)=2\cdot B\cdot l,$$

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$$\frac{F'(l)}{W'_1} = \frac{2 \cdot B \cdot l}{A_1 \cdot v_1 \cdot \rho_1 \cdot c_1} = \frac{4}{D_{h1} \cdot v_1 \cdot \rho_1 \cdot c_1} \cdot l$$

and F' – heat transfer area of one plate [m^2]

k – overall heat transfer coefficient [W m^{-2} K^{-1}]

λ – thermal conductivity coefficient [W m^{-1} K^{-1}]

R_Z – additional thermal resistance of calcium precipitations [W^{-1} m^2 K]

In balanced heat exchangers $\frac{W'_1}{W'_2} = 1$. Hence from equations (17, 18) we obtain:

$$\vartheta_1(l) = t'_1 - (t'_1 - t'_2) \cdot \frac{k \cdot F'(L)/W'_1}{1 + k \cdot F'(L)/W'_1} \cdot \frac{l}{L} \quad (17a)$$

$$\vartheta_2(l) = t'_2 - (t'_1 - t'_2) \cdot \frac{k \cdot F'(L)/W'_1}{1 + k \cdot F'(L)/W'_1} \cdot \left(1 - \frac{l}{L}\right) \quad (18a)$$

In steam heat exchangers, if $W'_2 < W'_1 = \infty$ and from equations (17, 18) we obtain:

$$\vartheta_1(l) = t'_1 \quad (17b)$$

$$\vartheta_2(l) = t'_2 + (t'_1 - t'_2) \cdot \left(\exp\left(\frac{k\,(F'(L) - F'(l))}{W'_2} \right) - 1 \right) \quad (18b)$$

Taking into account that:

$$t''_1 = \vartheta_1(L), \quad (21)$$

$$t''_2 = \vartheta_2(0) \quad (22)$$

the outlet temperatures t''_1, t''_2 can be calculated from system of equation (17, 18), in balanced heat exchangers from equations (17a, 18a) and in steam heat exchangers from equations (17b, 18b).

The above equations describe stationary conditions in individual channels of the counter flow heat exchanger. If we disregard the influence of first and last plate it is possible to apply this equation to calculations of temperature profiles in every sections whole exchangers. Then thermal capacity of flow rate in one duct we substitute for thermal capacity of flow rate of proper fluid and heat transfer area of one plate we substitute for heat transfer area of whole exchanger.

Temperature profiles in individual sections for chosen particular cases are presented on drawings 3a –3d.

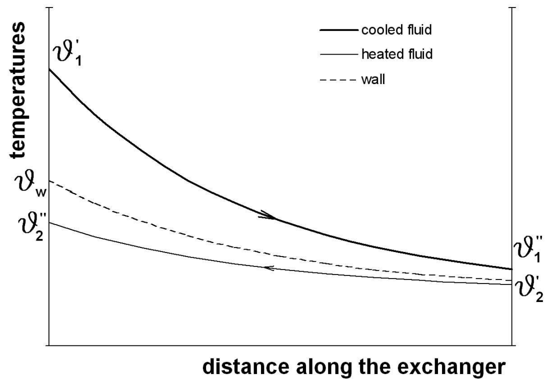
5 Dynamic characteristics of heat exchangers

Plate heat exchanger is considered as element with two input and two output quantities:

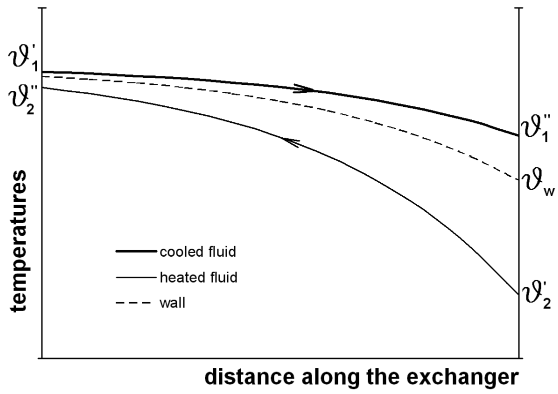
$$Y_2 = G_{2 \times 2} X_2 \quad (23)$$

X_2 = **X** [t'_1, t'_2] – input signal, two–dimensional vector

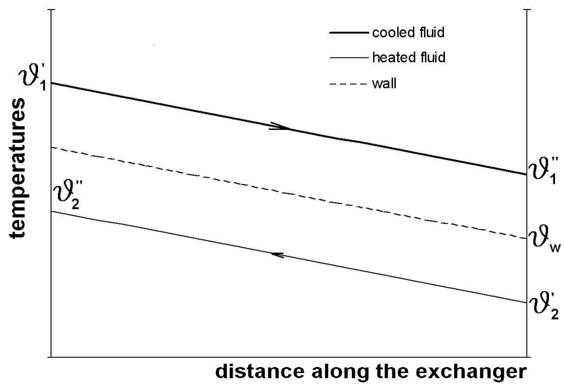
Y_2 = [t''_1, t''_2] –output signal, two–dimensional vector



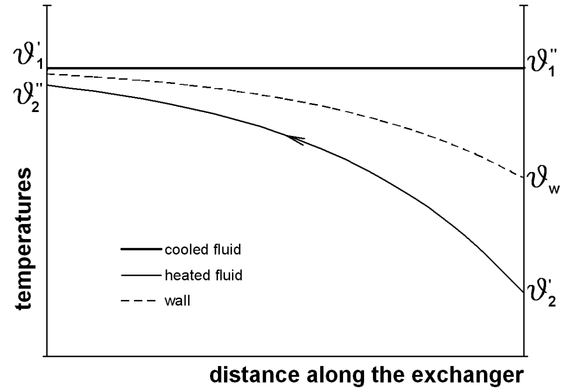
a) $W_1 < W_2$



b) $W_2 < W_1$



c) balanced heat exchanger $W_1 = W_2$ (eq. 17a, 18a)



d) steam heat exchanger $W_2 < W_1 = \infty$ (eq. 17b, 18b)

Fig. 3 Temperature profiles in counter flow heat exchanger

G – transmittance matrix:

$$\mathbf{G} = \mathbf{G}_{2x2} = \begin{Vmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{Vmatrix} = \begin{Vmatrix} \frac{t''_1(s)}{t'_1(s)} & \frac{t''_1(s)}{t'_2(s)} \\ \frac{t''_2(s)}{t'_1(s)} & \frac{t''_2(s)}{t'_2(s)} \end{Vmatrix}.$$

If we disregard heat accumulation in plates of exchanger (9) in the thermal balance equations (7, 9, 11), we should obtain system of two differential equations:

$$T_1 \frac{\partial \vartheta_1}{\partial t} + v_1 T_1 \frac{\partial \vartheta_1}{\partial l} = \vartheta_2 - \vartheta_1 \qquad (24)$$

$$T_2 \frac{\partial \vartheta_2}{\partial t} - v_2 T_2 \frac{\partial \vartheta_2}{\partial l} = \vartheta_1 - \vartheta_2 \qquad (25)$$

where T_1 and T_2 are calculated for overall heat transfer coefficient.

Let's consider small changes of temperatures from state of equilibrium:

$$\vartheta_{1,2} = \overline{\vartheta_{1,2}} + \Delta\vartheta_{1,2}$$

where $\overline{\vartheta}$– temperature in state of equilibrium, $\Delta\vartheta = \Theta$ – change of temperature ϑ.

Let's define:

$$L^*_1 = \frac{L}{v_1 T_1}, \quad L^*_2 = \frac{L}{v_2 T_2}, \quad v^* = \frac{v_1}{v_2}, \quad t^* = \frac{v_1 \cdot t}{L},$$

$l^* = \frac{l}{L}$, i. e.: $dt^* = \frac{v_1}{L} dt$, $dl^* = \frac{dl}{L}$.

Then equations (24, 25) can be written in dimensionless form [8]:

$$\frac{\partial \Theta_1}{\partial t^*} + \frac{\partial \Theta_1}{\partial l^*} = L^*_1 \left(\Theta_2 - \Theta_1\right) \qquad (26)$$

$$v^* \frac{\partial \Theta_2}{\partial t^*} - \frac{\partial \Theta_2}{\partial l^*} = L^*_2 \left(\Theta_1 - \Theta_2\right) \qquad (27)$$

We solve system of equations (26, 27) by the Laplace – transform method. Boundary conditions (15, 16) are transformed too and included into solution. Then, the transfer functions can be written as [9]:

$$G_{11} = \frac{2q \exp(b+q)}{q - p + (p+q)\exp(2q)} \qquad (28)$$

$$G_{12} = L^*_1 \frac{1 - \exp(-2q)}{p + q - (p-q)\exp(-2q)} \qquad (29)$$

$$G_{21} = L^*_2 \frac{1 - \exp(-2q)}{p + q - (p-q)\exp(-2q)} \qquad (30)$$

$$G_{22} = \frac{2q \exp(-b+q)}{-p + q - (p+q)\exp(2q)} \qquad (31)$$

where:

$$b = -\left(\frac{1-r}{2} s + \frac{b_1 - b_2}{2}\right), \qquad p = \left(\frac{1+r}{2} s + \frac{b_1 + b_2}{2}\right),$$

$$q = \sqrt{p^2 - b_1 b_2}.$$

6 Simulations example

Change of outlet temperature of cooled fluid $\Delta\vartheta''(t)$, which is response to step change of inlet temperature of heated fluid $\Delta\vartheta'(t)$ could be calculated from equation:

$$\Delta\vartheta''(t) = \frac{M(0)}{N(0)} + \sum_{k=1}^{n} \frac{M(s_k)}{s_k N'(s_k)} e^{s_k t} \qquad (32)$$

where $M(s_k)$ is numerator, $N(s_k)$ denominator of the taken into consideration transfer function, s_k are roots of an equation $N(s_k) = 0$ and $N'(s_k) = [dN(s)/ds]_{s=sk}$.

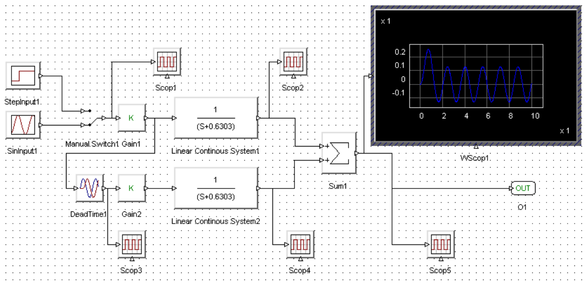
First summand in right part of equation (32) represents steady-state conditions, following – transient components.

The transfer functions G_{12}, G_{21} are interesting particularly. In case steam condensation heat exchangers b_1 b_2 $= 0$, hence $p = q$ and the transfer functions G_{21} takes the form:

$$G_{21} = L^*_2 \frac{1 - \exp(-b_2)\exp((1+r)s)}{(1+r)s + b_2} \qquad (33)$$

Supplementary schema of heat exchanger under equation (33) with sine-response function is showed in Figure 4.

Fig. 4 Supplementary schema of heat exchanger under



equation (33) with sine-response function

7 Response to temperature disturbance

Taking into consideration the system of three differential equations (7, 9, 11) and dividing the heat exchanger into N sections (Fig. 5), we obtain as follows:

Inside, j = 2, 3, 4 …N

$$T_1 \frac{d\vartheta_{1j}}{dt} + v_1 \cdot T_1 \frac{\left(\vartheta_{1j+1} - \vartheta_{1j-1}\right)}{2 \cdot \Delta x} = \vartheta_{wj} - \vartheta_{1j} \qquad (34)$$

$$\frac{d\vartheta_{wj}}{dt}=\frac{1}{T_{w1}}\left(\vartheta_{1j}-\vartheta_{wj}\right)-\frac{1}{T_{w2}}\left(\vartheta_{wj}-\vartheta_{2j}\right) \quad (35)$$

$$T_2\frac{d\vartheta_{2j}}{dt}-v_2\cdot T_2\frac{\left(\vartheta_{2j+1}-\vartheta_{2j-1}\right)}{2\cdot\Delta x}=\vartheta_{wj}-\vartheta_{2j} \quad (36)$$

Ends, j = 1 or N+1

$$T_1\frac{d\vartheta_{1N+1}}{dt}+v_1\cdot T_1\frac{\left(\vartheta_{1N+1}-\vartheta_{1N}\right)}{\Delta x}=\vartheta_{wN+1}-\vartheta_{1N+1} \quad (37)$$

$$\frac{d\vartheta_{w1}}{dt}=\frac{1}{T_{w1}}\left(\vartheta_{11}-\vartheta_{w1}\right)-\frac{1}{T_{w2}}\left(\vartheta_{w1}-\vartheta_{21}\right) \quad (38)$$

$$\frac{d\vartheta_{wN+1}}{dt}=\frac{1}{T_{w1}}\left(\vartheta_{1N+1}-\vartheta_{wN+1}\right)-\frac{1}{T_{w2}}\left(\vartheta_{wN+1}-\vartheta_{2N+1}\right) \quad (39)$$

$$T_2\frac{d\vartheta_{21}}{dt}-v_2\cdot T_2\frac{\left(\vartheta_{22}-\vartheta_{21}\right)}{\Delta x}=\vartheta_{w1}-\vartheta_{21}\,. \quad (40)$$

From the above equations we obtain

$$\frac{d\vartheta_{1j}}{dt}=-v_1\frac{\vartheta_{1j+1}-\vartheta_{1j-1}}{2\cdot\Delta x}+\frac{1}{T_1}\left(\vartheta_{wj}-\vartheta_{1j}\right) \quad (41)$$

$$\frac{d\vartheta_{2j}}{dt}=v_2\frac{\vartheta_{2j+1}-\vartheta_{2j-1}}{2\cdot\Delta x}+\frac{1}{T_2}\left(\vartheta_{wj}-\vartheta_{2j}\right) \quad (42)$$

and

$$\frac{d\vartheta_{1N+1}}{dt}=-v_1\frac{\vartheta_{1N+1}-\vartheta_{1N}}{\Delta x}+\frac{1}{T_1}\left(\vartheta_{wN+1}-\vartheta_{1N+1}\right) \quad (43)$$

$$\frac{d\vartheta_{21}}{dt}=v_2\frac{\vartheta_{22}-\vartheta_{21}}{\Delta x}+\frac{1}{T_2}\left(\vartheta_{w1}-\vartheta_{21}\right) \quad (44)$$

and equations (35, 38, 39) remain without change.

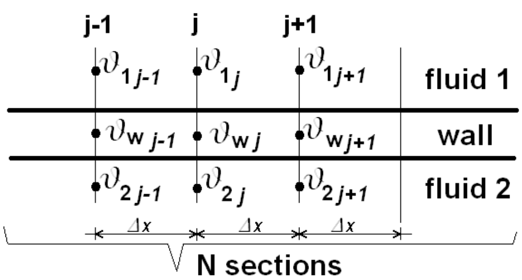


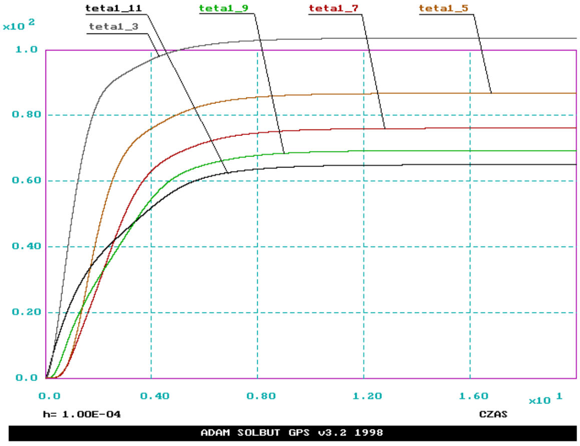
Fig. 5 Schema of the divided into N sections heat exchanger

It has been used generator of simulation programs GPS v.3.2 described in works [10, 11]. The data assumed for computation were as follows:

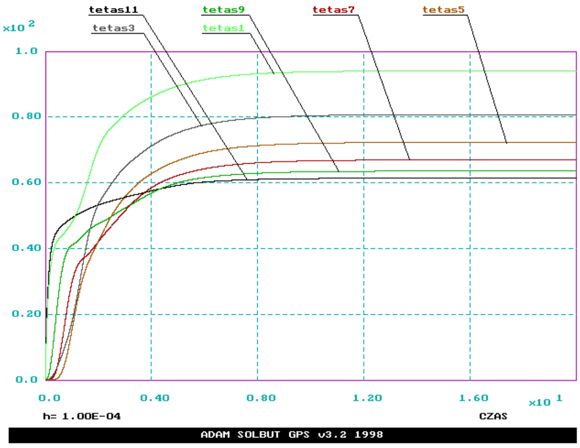
$t'_1=\vartheta_1(0)=\vartheta_{1\_j=0}=130$ °C,

$t'_2=\vartheta_2(L)=\vartheta_{2\_j=N+1}=60$ °C,

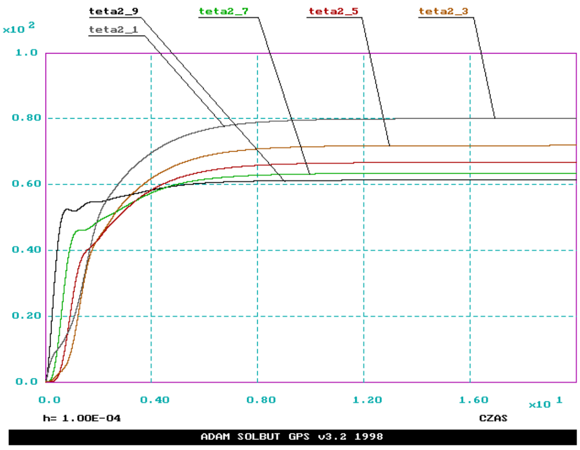
$v_1=0.462$ m/s, $v_2=1.067$ m/s, $L=1.30$ m,



a) temperatures of the cooled fluid



b) temperatures of the considered points of wall



c) temperatures of the heated fluid

Fig. 5 The counter current heat exchanger is warmed up

$k\cdot F'(L)/W'_1=3.326$, $k\cdot F'(L)/W'_2=1.023$,

$T_1=0.84539$ s, $T_2=0.33382$ s,

$T_{w1}=0.34828$ s, $T_{w2}=0.13565$ s.

The counter current heat exchanger has been divided into N = 10 sections and

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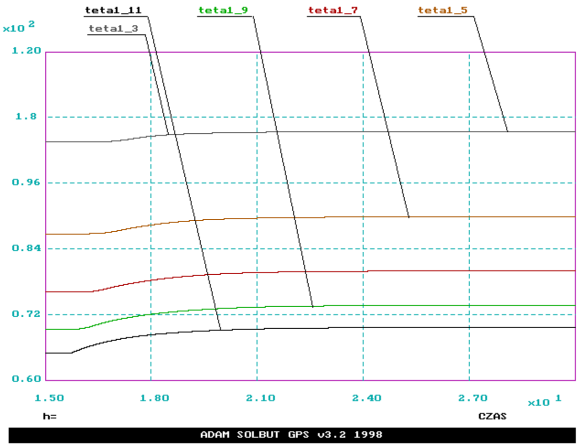
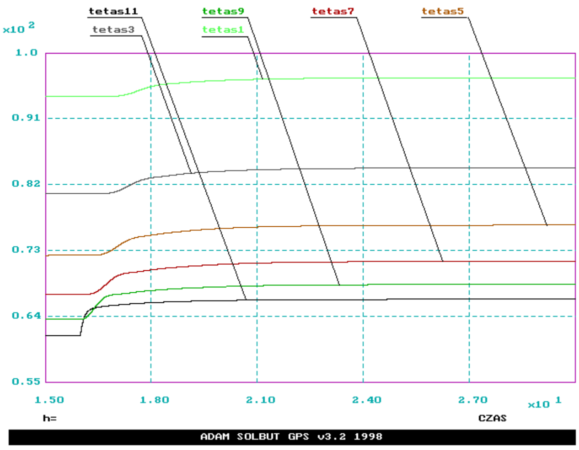
$$\Delta x = \frac{L}{N} = \frac{1.30}{10} = 0.13\text{m}.$$

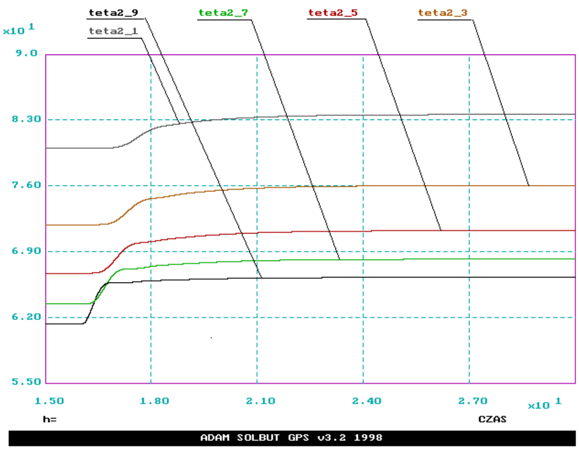
First are computed temperatures ϑ_{1_j}, ϑ_{w_j}, ϑ_{2_j} in steady state conditions (Fig. 6a-c), next is simulated response to temperature t'_2 step disturbance. The inlet temperature t'_2 is changed from 60 °C to 65 °C.



a) temperatures of the cooled fluid



b) temperatures of the considered points of wall



c) temperatures of the heated fluid

Fig. 6 Computed response to temperature t'_2 step disturbance

8 Conclusions

Values of temperatures in individual sections countercurrent flow exchangers and responses to temperature disturbance could be computed using presented methods of calculations.

Important element of this computations is determination of heat transfer coefficients α_1, α_2, overall heat transfer coefficient k and time constants T_1, T_2, T_{w1}, T_{w2}.

Better knowledge of specificities enables farther optimization heat exchangers and exchanger control systems.

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