# DYNAMICAL CHARACTERIZATION OF A VIBRATING MEMBER GYROSCOPE USING FINITE ELEMENT METHOD

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## Abstract

The subject of this paper is to show how a dynamic analysis of a micro machined vibrating-member gyroscope based on the finite element method (FEM) can be performed. Furthermore it is shown how to use results of the FEM analysis as input for a system simulation.
To get an overview of the dynamical behavior of the micro electro-mechanical system a modal analysis must be performed. To understand the dynamics of the structure under driving conditions the results of the harmonic response of the structure must be analyzed. In this paper we perform all these analysis steps with the help of FEM. We use ANSYS for preprocessing and solution procedures. A given device is simulated and the results are compared to measurements. In addition we mention good practice in performing different simulations and we report conditions where simplifications in our modeling and simulation procedure are allowed. We also want to stress the importance of using appropriate material models and meshing procedures.
Finally we propose a model of the gyro-structure which can be used for a system simulation. In this lumped parameter model approach we are able to simulate zero rate offset of the device and also effects due to different DC bias voltages.

**Keywords: Gyroscope, FEM, ANSYS, System model.**

### Presenting Author's Biography

Armin Satz. Graduated in 'Technical Physics' at the Vienna University of Technology. He finished his master thesis 'Lifetime Effects in Electron energy Loss Spectrometry' in April 2006. Since then he has been working towards his PhD at KAI Kompetenzzentrum Automobil und Industrie-Elektronik in Villach, Austria.

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## 1 Introduction

Finite Element Method (FEM) is widely used in the design of micro structures. For micro mechanical vibrating member gyroscopes the prediction of the dynamical behavior of the structure is crucial for the sensor application. On the next pages we show how FEM simulations for a dynamical analysis can be carried out for a given device, the so called ‘butterfly’-gyro structure [1]. In the first step the eigenfrequencies of the structure and the corresponding eigenvectors are calculated. Simulation results show that the appropriate choice of solid elements and proper material parameters is very important because they can change the results by more than 10%. Details of the meshing procedure are discussed to show how to get precise results by using the smallest number of elements as possible. Furthermore a harmonic analysis is carried out and the results are compared with measurements in order to prove the correctness of our modelling approach.

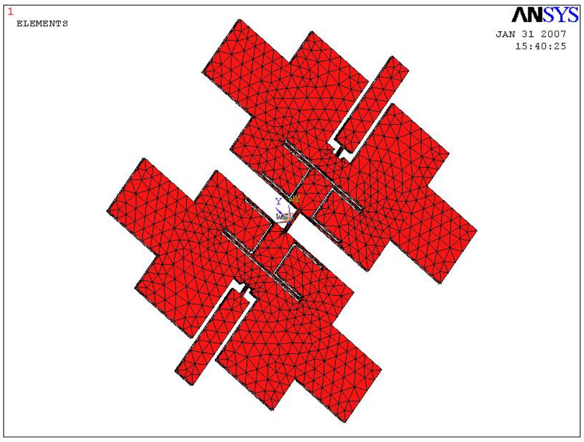


Fig. 1 The ‘butterfly’-gyro structure.

## 2 Sensor Principle

The butterfly gyro is an angular rate sensor. The sensor principle is based on the Coriolis acceleration. The effect of Coriolis acceleration can be explained by examining the expression for acceleration in a rotating reference frame. (see Eq. (1))

$$\overbrace{m\ddot{\vec{x}}_I}^{\text{Inertial system}} = \overbrace{m\ddot{\vec{x}}_R + \underbrace{2m(\dot{\vec{x}}_R \times \vec{\omega})}_{\text{Coriolis force}} + \underbrace{m\omega \times (\omega \times \vec{x}_R)}_{\text{Centrifugal force}}}^{\text{Rotating system}} \quad (1)$$

In Fig. 2 one can see how the Coriolis force is acting on a moving mass in a rotating system.

Foucaults pendulum is a simple example for a vibrating member gyroscope. In this case the effect of the Coriolis acceleration is to alter the pendulums plane of oscillation.

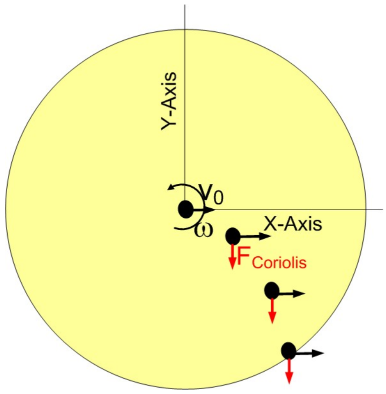


Fig. 2 The movement of a point mass like it is detected by an observer in the rotating reference system.

Most sensors use comb drives to excite a vibration in one direction and parallel plate electrodes are used to detect a perpendicular movement. The butterfly gyro does not need a comb drive, but uses beams with asymmetric cross sections so that the structure bends most easily in an inclined direction. For tuning the stiffness of the structure narrow tranches are used. Overall the butterfly gyro leads to a very complex structure and analytical solutions for the dynamic behavior are not feasible and methods like FEM must be applied.

## 3 Finite Element Method

A micro electro-mechanical system like the butterfly gyro is a system where a coupling between the structural and the electrical domain takes place. Modern FEM software tools support simulation of coupling effects like nonlinear electrical forces. We show that in a first order approach these coupling effects can be neglected.

To calculate the dynamics of the device the following equation of motion needs to be solved.

$$\{M\}\,\ddot{\vec{x}} + \{C\}\,\dot{\vec{x}} + \{K\}\,\vec{x} = \{F\} \quad (2)$$

| | | |
|---|---|---|
| $\vec{x}$ | ... | Nodal displacement vector |
| $\{M\}$ | ... | Mass matrix |
| $\{C\}$ | ... | Damping matrix |
| $\{K\}$ | ... | Stiffness matrix |
| $\{F\}$ | ... | Load vector |

We can neglect the damping matrix $\{C\}$ because the structure consists of crystalline silicon which means structural damping can be neglected. Additionally the device is encapsulated in Argon at a pressure of 0.55 mbar and therefore we can neglect squeeze film damping effects too.

### 3.1 Meshing Procedure

For discretization of the problem we use the so called SOLID186 element type. SOLID186 is a 3D 20 node solid element that exhibits quadratic displacement behavior. As one can see in Fig. 3 different element shapes are available. A simple approach to discretization is to use the tetrahedral element shape option. With

 

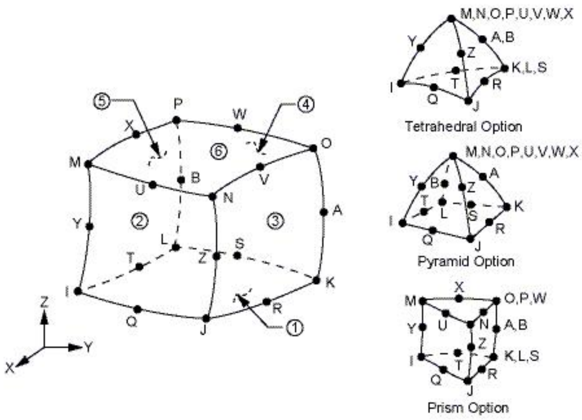


Fig. 3 SOLID186 element type. [2]

this option a more or less automatic meshing procedure can be performed by ANSYS. The result of this tetrahedral mesh can be seen in Fig. 1. The disadvantage of the tet-mesh is that the elements are distorted and have very acute angles in the region of the asymmetric springs and tranches.

A much better option for meshing is to use hexahedral shaped elements. This shape allows to mesh the complete geometry with well behaving element geometries and to mesh the crucial regions with a very high element density.

### 3.2 Modal Analysis

Knowing the frequencies and the shapes of the eigenmodes is very important and a modal analysis should be the first step in a dynamical characterization. In a

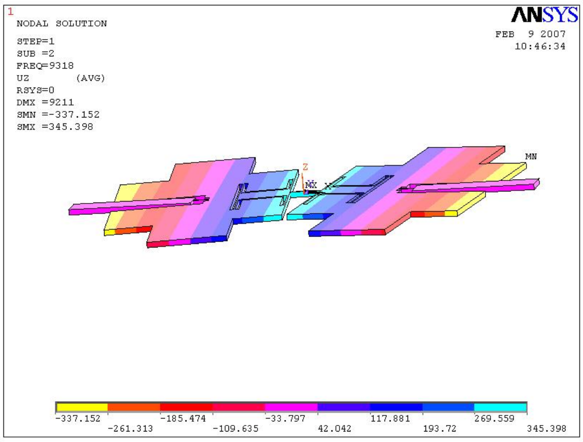


Fig. 4 Excitation mode.

modal analysis simulation the load vector in Eq. (2) is set to zero. The eigenfrequencies and the corresponding eigenvectors are governed by the following equation.

$$\left(\{K\} - \omega_i^2 \{M\}\right) [\phi_i] = \{0\} \tag{3}$$

The first four modes are torsion or bending modes of the connecting beams. The first mode at 7.4 kHz is a torsion mode of the two outer beams where the two butterfly masses are moving in phase. The second mode (excitation mode) shown in Fig. 4 together with the third mode (detection mode) shown in Fig. 5 is responsible for the sensor application. The fourth mode is a vertical bending mode of the beams.

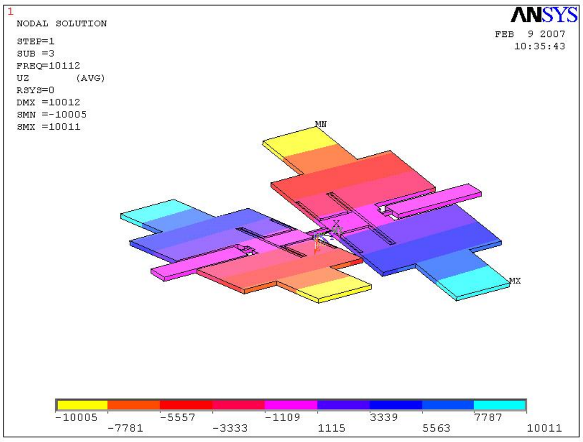


Fig. 5 Detection mode.

Mode number five and six are mass bending modes, which depend mostly on the thickness of the butterfly masses.

For an ideal angular rate sensor only the excitation and detection mode should contribute to the dynamics of the structure. The design of the shapes of the butterfly masses and the interconnecting beams should lead to an as much as possible idealized gyroscope, and therefore one can clearly see that a modal analysis is essential for achieving this aim.

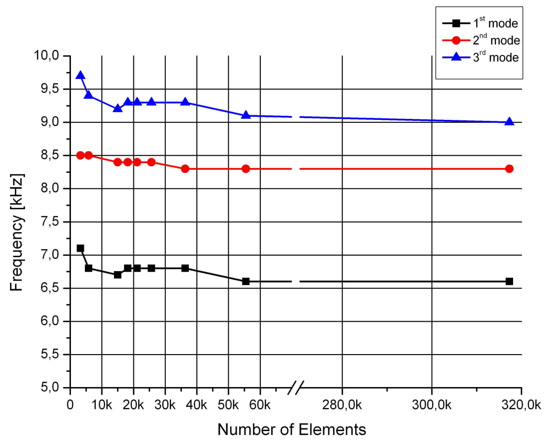


Fig. 6 Simulation results for the first three eigenmodes (tet-mesh).

To check whether the discretization is not too fine grained but still produces accurate results one needs to perform simulations with a varying number of elements to find an optimum. For getting the results reported in Fig. 6 the structure was meshed with tetrahedral shaped elements of constant volume. This regular tet-mesh leads to a satisfying convergence behavior for a number of elements larger than 60k. A comparison

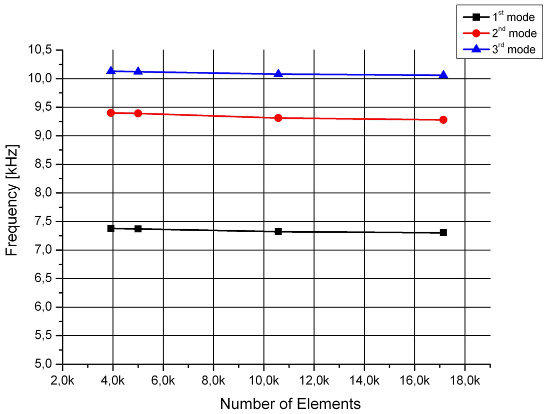


Fig. 7 Simulation results for the first three eigenmodes (hex-mesh).

of Fig. 6 and Fig. 7 shows that the hexahedral shape option leads to a much better convergence.

As already mentioned the gyro structure is manufactured of mono crystalline silicon (Si). The elastic constants like Young's modulus (E), Shear modulus (G) and Poisson's ratio ($\nu$) are anisotropic material properties because of the fcc crystal structure of Si. E for example is varying between 120 and 180 GPa [3] depending on the direction in the crystal. We simulated the first four eigenfrequencies as a function of E in order to see the influence of the anisotropy.

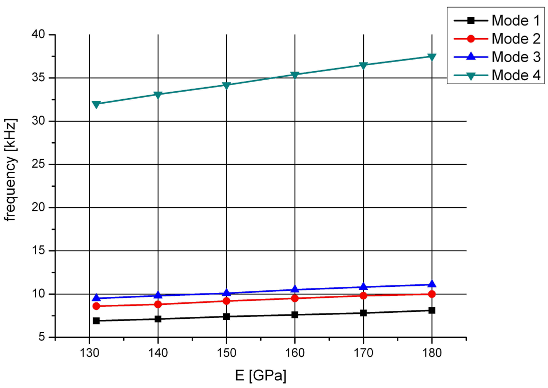


Fig. 8 The first eigenfrequencies as a function of E.

In Fig. 8 one can see that without taking into account anisotropic effects the simulation error can be expected to be in the range of approximately 10 % compared to measurements.

### 3.3 Harmonic Analysis

Performing a frequency response analysis of the device is a very important step for the device characterization. For driving and sensing an ensemble of twelve electrodes is used. The structure is driven electrostatically and the differential capacitance between the corresponding electrodes is measured with the help of a signal analyzer. The detection electrodes give the frequency response shown in Fig. 9. The detection

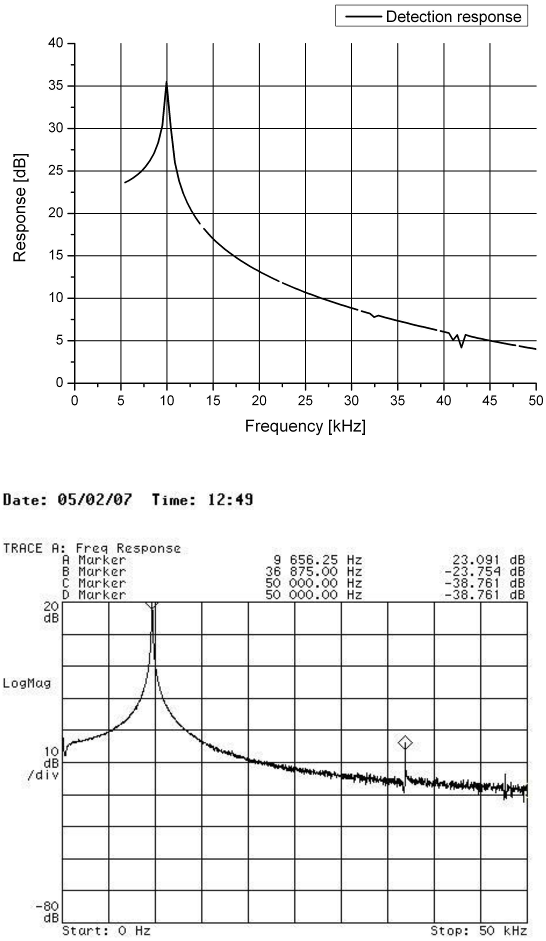


Fig. 9 The measured and simulated detection response of the structure.

mode (3$^{rd}$-mode) gives a peak at approximately 10 kHz. The electrodes are arranged in such a way that they are not sensitive to contributions of undesired eigenfrequencies in the observed frequency range. The model used for the simulation is completely symmetric therefore the simulation results show only one peak. The real device will always have a geometry which is not perfectly symmetric and so measurement show always small peaks coming for modes which are not completely suppressed.

For the excitation response the same statements as for the detection response are valid. Fig. 10 shows that for the excitation response measurement and FEM simulation match.

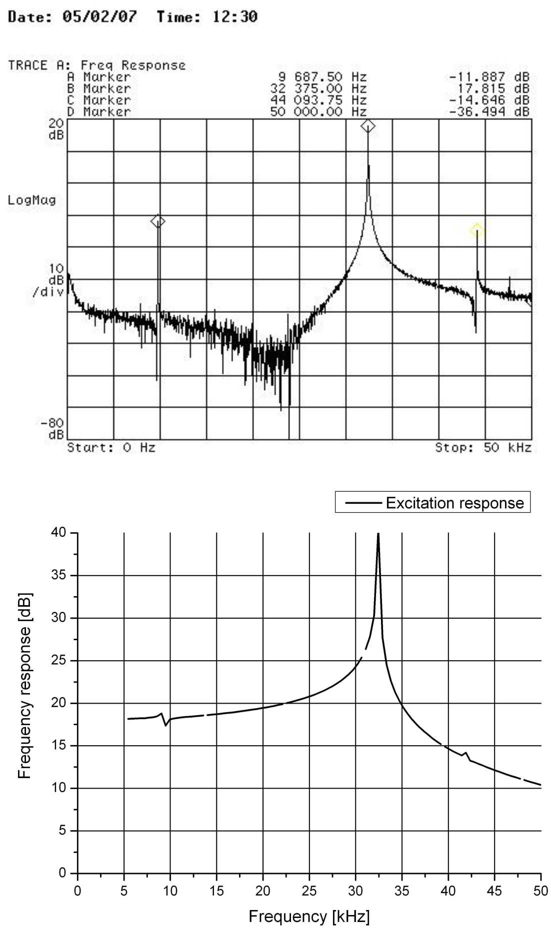


Fig. 10 Measured and simulated excitation response of the structure.

## 4 System Simulation

We present a lumped parameter model which is used for system simulation. In this model two eigenmodes are used and modeled as second order systems. These two modes are coupled via the Coriolis force [4].

$$\begin{aligned} M_1\ddot{x} + \xi_1\dot{x} + K_1x - C_2\omega\dot{y} &= F(t) \\ M_2\ddot{y} + \xi_2\dot{y} + K_2y + C_1\omega\dot{x} &= 0 \end{aligned} \tag{4}$$

At the system simulation level the zero-rate offset of the device can be modeled. Zero-rate offset in a vibrating member gyroscope comes from an undesired coupling between the excitation and detection mode. Ideally, the only coupling should be by the Coriolis force but asymmetries in the sensor cause several types of cross-coupling and therefore one has to define different cross-

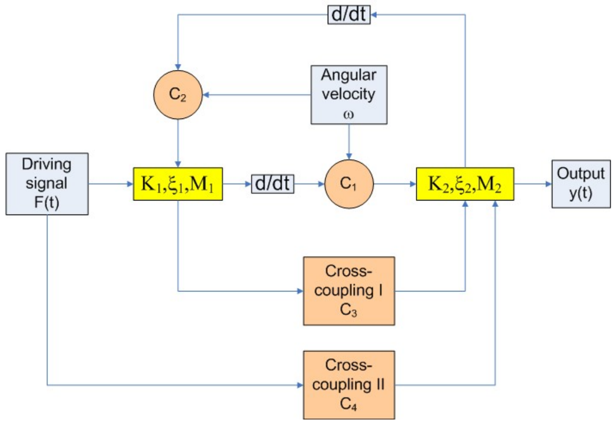


Fig. 11 Schematics of a system simulation.

coupling parameters in the system simulation. (see Fig. 11)

The butterfly gyro uses a DC-bias voltage to tune the eigenfrequency of the detection mode. In the system model approach the detection mode is characterized by three parameters, namely a mass constant M, damping constant $\xi$ and a stiffness constant K. The effect of changing the bias voltage can be translated to the system model by a change of the stiffness constant $K_2$. A comparison of Fig. 12 and Fig. 13 shows that as a first order approach the shift of the eigenfrequencies can be modeled quite accurately. The measured frequency shift shows also a non-constant damping behavior which is due to the fact that an increased bias voltage leads also to a higher damping constant for the detection mode. This effect shows that the first order approach of a variable stiffness constant can be insufficient.

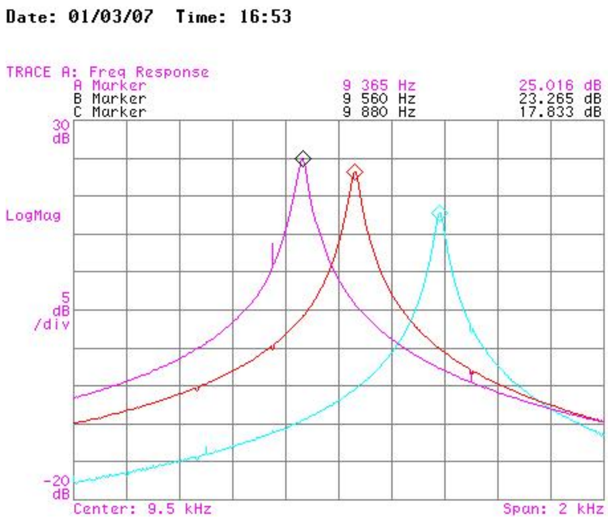


Fig. 12 Measured eigenfrequency shift of the detection mode due to bias voltages from 1.4-2.6 Volts.

 

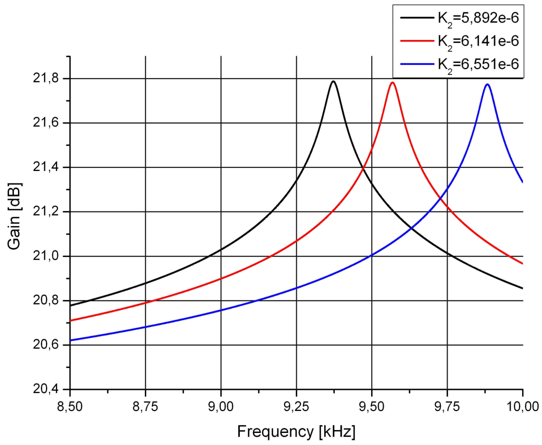


Fig. 13 Simulated eigenfrequency shift of the detection mode due to different stiffness constants $K_2$.

## 5 Conclusion & Outlook

We have shown that the FEM simulation of the gyro dynamics is able to produce characteristical frequency spectra. A further target is the extraction of coupling coefficients for the Coriolis coupling from FEM simulations. The effect of different DC-bias voltages is simulated in a lumped parameter model which is well suited for describing this effect.

For system level simulation the lumped parameter model can be expanded to implement cross-coupling effects for modeling zero-rate offset behavior. For further investigations it is necessary to find a proper way for extracting these cross-coupling coefficients out of FEM simulations.

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