PIEZOELECTRIC FUNCTIONALLY GRADED LAYERS IN VIBRATION CONTROL OF LAMINATED PLATES

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Abstract

The aim of this study is to develop the modelling of active laminated plates with conventional piezopolymer sensor layers and newly proposed actuator layers made of a piezoelectric functionally graded (PFG) material. The electromechanical properties of the PFG layers can be tailored by a smooth variation of piezoelectric material fraction across the thickness direction to achieve the satisfactory operational efficiency of the actuator while minimizing the interlaminar shear stresses and also a risk of delamination. The exponential and parabolic functions are used to describe volume fractions of constituents. The dynamic analysis is based on the classical laminated plate theory and concerns a steady-state out-of-plane vibration, which is actively reduced due to the constant gain velocity feedback control. In order to model inner energy dissipation the elastic-viscoelastic correspondence principle is applied to predict complex moduli for composite components. The effects of the applied transversal distribution of the PFG actuator properties on the structural response of the plate including the changes in natural frequencies and resonant amplitudes are numerically examined and discussed. It is also shown that a sufficient control effectiveness of the considered active system with the PFG actuator layers can be obtained.

Keywords: Laminated plate, Piezoelectric control, Functionally graded actuator.

Presenting Author's biography

Marek Pietrzakowski is an assistant professor at the Faculty of Automobiles and Machinery Engineering of Warsaw University of Technology. He received his PhD and ScD degrees in mechanical engineering. His main fields of interest are: dynamics of discrete and continuous systems, smart materials applications and intelligent structures. His recent papers are devoted to vibration control of laminated plates with piezoelectric layers of a monolithic or composite form i.e. piezoelectric fiber composites and functionally graded materials.

1 Introduction

Piezoelectric sensor and actuator layers have found a relevant role in vibration control of thin-walled laminated structures. Satisfactory control effectiveness requires control forces, which are produced by a relatively large deformation of piezoelectric actuator layers. Thus, the interaction between the piezoelectric layer and the main structure creates severe interlayer shear stresses, which may cause a damage of the control system by cracks and the local delamination. The effect of the shear stress concentration is enhanced by a sharp change in elastic properties of bonded piezoceramic layers, which are widely used for actuation, and classic layers of the main structure. Using piezocomposite layers whose electromechanical properties vary in the thickness direction may reduce a risk of the control system failure. The concept of functionally graded materials (FGMs) has been applied as a thermal barrier, which is designed to reduce high thermal stresses and prevent decreasing of strength and stiffness of the structure. Well-known FGMs are a mixture of ceramics and metal with continuously varying volume fractions. This kind of materials is used in air- and spacecrafts, reactor vessels and other technical applications in high-temperature environments. The mechanical behaviour of FGMs has been the topic of interest of a large number of researches during the last decade. They investigated static and dynamic responses of functionally gradient ceramic-metal plates cf [1,2,3] and cylindrical shells cf [4,5]. Active control of the FGM plate with integrated monolithic piezoelectric sensor/actuator layers is discussed in [6] and a finite element model of the FGM plate with piezoelectric fiber composite (PFC) for static analysis can be find in [7].

In order to increase the operational reliability and durability of piezoelectric devices a new kind of materials, known as piezoelectric functionally graded (PFG) materials, has been developed. The static transverse displacements and stress field in PFG laminates composed of layers whose electro-mechanical properties vary from layer to layer is analysed in [8]. The vibration control of laminated plates using multi-layered functionally graded PFC actuators is studied in [9]. The dynamic stability analysis of the PFG plate under the time-dependent thermal load is presented in [10]. The finite element modelling for the static and dynamic analysis of the PFG bimorph can be found in [11].

The objective of the present study is to develop models of active laminated plates with PFG actuator layers, which are made of two-phase material being a mixture of PZT (lead-zirconate-titanate) ceramics and matrix material (e.g. epoxy resin). The electromechanical properties of each PFG layer are defied by a variation in the volume fraction of the material constituents across the thickness direction. Two types of distribution patterns are considered: exponential and parabolic. The dynamic analysis is based on the classical laminated plate theory (CLPT) and concerns steady-state problem. The active reduction of transverse vibration of the rectangular laminated plate is achieved applying the velocity feedback control strategy.

2 Formulation of the problem

A simply supported rectangular symmetrically laminated plate of dimensions a and b is considered. The plate is composed of classic orthotropic layers (e.g. graphite-epoxy) and piezoelectric sensor and actuator layers polarized in the transverse direction. The sensor layers are assumed as the PVDF (polyvinylidene fluoride) film, while the actuators are of the PFG material. The electroelastic properties of each PFG lamina vary continuously in the thickness direction due to the exponential or parabolic gradation of the PZT fraction. To produce the bending mode action the midplane symmetric PFG actuators (with the midplane symmetric gradient of material properties) are with either opposite polarization or opposite applied electric field. The actuators are supplied with the voltage generated by the sensor layers and transformed according to the constant gain velocity feedback algorithm. The steady-state vibration of the considered active plate is analysed taking into account Kirchhoff's simplifications of the deformation field cf [12].

2.1 PFG actuator relations

The actuator behaviour is described by the constitutive equation of the reverse piezoelectric effect. Assuming a plane stress state in the actuator layer the constitutive relation in the principal material axes 1, 2, 3 can be reduced to the following matrix form

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} - \mathbf{e}\mathbf{E} \qquad (1)$$

where $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \tau_{12}]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, \gamma_{12}]^T$ is the in-plane stress and strain representation, respectively, $\mathbf{E} = [E_1, E_2, E_3]^T$ is the external electric field, $\mathbf{c}$ is the stiffness matrix, $\mathbf{e}$ is the matrix of piezoelectric coefficients, T denotes matrix transposition.

In the considered piezoelectric layers the poling direction is along the 3-axis, which is perpendicular to the layer and coincides with the z-axis of the laminate. Thus, the electric field components E_1 and E_2 are of zero value. It is assumed that the electric potential function is linear through the thickness and the electric field E_3 is constant for each actuator layer. This electric potential representation has been identified as acceptable for relatively thin layers.

The PFG material of the actuator is a two-phase isotropic composite and can be defined by a variation of the volume fraction of piezoelectric and matrix components throughout the thickness. Two patterns are under consideration: the exponential function given by the formula

$$v(z_l) = v_0 \exp\left(\lambda_e\left(z_l + \frac{h_a}{2}\right)\right) \text{ with } \lambda_e = \frac{1}{h_a}\ln\frac{v_1}{v_0} \quad (2)$$

and the parabolic distribution written as

$$v(z_l) = v_1 - \lambda_p z_l^2 \text{ with } \lambda_p = \left(\frac{2}{h_a}\right)^2\left(\frac{v_1}{v_0} - 1\right)v_0 \quad (3)$$

where z_l is the local co-ordinate measured from the middle surface of the layer, h_a is the layer thickness, v_0 and v_1 are the minimal and maximal PZT volume fractions, respectively.

The parameters λ_e and λ_p describe the inhomogeneity of the PFG material across the thickness for the exponential and parabolic distribution, respectively.

The effective properties of the PFG material i.e. the Young's modulus and piezoelectric coefficient are determined according to the rule of mixtures. Therefore, for the considered two-phase material the following general formula can be used

$$P_{eff}(z_l) = (P_p - P_m)\, v(z_l) + P_m \quad (4)$$

where P_p indicates properties of a pure piezoelectric material (PZT), P_m denotes properties of a matrix material (epoxy resin).

The effect of the Poisson's ratio gradation in the actuator layer on the laminate mechanics is much less than that of Young's modulus and can be ignored.

For example, based on the formula (4) combining with the exponential distribution of the PZT volume fraction, Eq. (2), and taking into account electroelastic isotropy of the PFG material, the Young's modulus $Y(z_l)$ and the piezoelectric coefficient $e_{31}(z_l)$ varying in the thickness direction are given by the relations

$$Y(z_l) = (Y_p - Y_m)v_0 \exp\left(\lambda_e\left(z_l + \frac{h_a}{2}\right)\right) + Y_m \quad (5)$$

$$e_{31}(z_l) = \frac{1}{1-\nu}\begin{bmatrix} (Y_p - Y_m)\, d_{31} v_0{}^2 \exp\left(2\lambda_e\left(z_l + \frac{h_a}{2}\right)\right) + \\ d_{31} Y_m v_0 \exp\left(\lambda_e\left(z_l + \frac{h_a}{2}\right)\right) \end{bmatrix} \quad (6)$$

where Y_p, Y_m are Young's moduli of piezoelectric and matrix material, respectively, ν is Poisson's ratio, d_{31} is the piezoelectric strain constant of the PZT material.

The piezoelectric coefficient function, Eq. (6), is formulated assuming a non-piezoelectric matrix material and the stiffness matrix related to the mechanically isotropic material with the following elements

$$c_{11} = c_{22} = \frac{Y(z_l)}{1-\nu^2},\ c_{12} = \frac{\nu Y(z_l)}{1-\nu^2},\ c_{66} = \frac{Y(z_l)}{2(1+\nu)} \quad (7)$$

The analysis concerns the two-dimensional actuation effect being transferred from the PFG actuator layers to the main structure. Assuming that the actuator layers are perfectly integrated with the laminate the interaction may be described by the control moment distributed along the activated area. Taking into account the electric part of the constitutive equation (1) the moment resultant $\mathbf{M}^E$ for m PFG actuators symmetrically located about the laminate midplane is defined as

$$\mathbf{M}^E = \begin{bmatrix} M_x^E \\ M_y^E \\ M_{xy}^E \end{bmatrix} = \sum_{k=1}^{m} \int_{z_{k-1}}^{z_k} \mathbf{e}^k \mathbf{E}^k z\, dz \quad (8)$$

The moment resultant $\mathbf{M}^E$ is formulated with respect to the x, y, and z reference plate axes with the z co-ordinate measured from the midplane in the thickness direction. Assuming the applied external electric field through the thickness direction the interaction of the considered actuator system may be reduced to the components

$$M_x^E = M_y^E = \sum_{k=1}^{m} E_3^k \int_{z_{k-1}}^{z_k} e_{31}^k(z) z dz \quad (9)$$

In Eq. (9) the piezoelectric coefficient function $e_{31}^k(z)$ of the kth actuator depends on the applied distribution of the PZT volume fraction and is transformed to the plate reference axis z. Since the PFG material isotropy is considered, the twisting moment component M_{xy}^E vanishes and the two-dimensional bending actuation occurs.

2.2 Sensor relations

The sensor equation is formulated based on the constitutive law of direct piezoelectric effect, which for the transversally polarized layer with the material axes 1, 2, and 3, after eliminating the external electric field becomes

$$D_3 = \mathbf{e}^T \boldsymbol{\varepsilon} \quad (10)$$

where D_3 is the electric displacement in the 3-axis direction.

The voltage produced by the kth sensor layer deformed with the structure is obtained by integration the charge stored on the electrodes. In the study only the flexural strains (bending mode) of the perfectly bonded sensor are considered. Assuming that the material axes 1, 2, 3 coincide with the plate reference axes x, y, z, respectively, and the electrodes fully cover the sensor faces, after using the standard equation for capacitance, and the geometric relation between strain and transverse displacement the following formula is derived

$$V_s^k = -\frac{h_s\, z_0^k}{\in_{33}\, A_s}\mathbf{e}^T \int_0^a\int_0^b \left[\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial x\,\partial y}\right]^T dxdy \quad (11)$$

where A_s is the sensor effective electrode area, h_s is the layer thickness, $\in_{33}$ is the permittivity constant, z_0^k is the distance of the *k*th sensor layer from the laminate midplane.

The voltage produced by the PVDF sensors is transformed due to the applied control algorithm and then drives the actuators.

2.3 Equation of motion and the solution

Based on the classical laminated plate theory the transverse vibration $w(x,y,t)$ of the active symmetrically laminated orthotropic plate subjected to the external distributed load $q(x,y,t)$ can be described as follows

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + \tilde{\rho}\frac{\partial^2 w}{\partial t^2} = q(x,y,t) - p(x,y,t) \quad (12)$$

where D_{ij} (i, j = 1, 2, 6) are the elements of the bending stiffness matrix, $\tilde{\rho}$ denotes the equivalent mass density parameter, $p(x,y,t)$ is the loading produced by the piezoelectric control system.

The bending stiffness D_{ij} for the n-layered laminate is defined as the following sum of integrals

$$D_{ij} = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \bar{c}_{ij}^k z^2 dz \quad (13)$$

where $\bar{c}_{ij}^k$ are the stiffness coefficients related to the *k*th layer of thickness $t_k = z_k - z_{k-1}$ and determined with respect to the plate reference axes *x*, *y*.

Viscoelastic behaviour of the laminate is approximated according to the Voigt-Kelvin model. Based on the principle of elastic-viscoelastic correspondence the complex moduli for the composite constituents are predicted. In general case the complex modulus can be written in the form $Y^* = Y(1 + j\mu\omega)$, were Y and μ indicate elastic modulus and retardation time, respectively. In consequence the stiffness matrix elements D_{ij} become complex.

In the present analysis the control loading $p(x,y,t)$ produced by the PFG actuators located symmetrically about the middle of the laminated plate is given by

$$p(x,y,t) = \frac{\partial M_x^E}{\partial x^2} + \frac{\partial M_y^E}{\partial y^2} \quad (14)$$

The solution to the plate motion equation Eq. (12) is obtained for the steady-state case. Assuming harmonic in time single frequency external loading the response of the plate can be predicted to be harmonic with the same frequency ω as the excitation

$$w(x,y,t) = W(x,y)\exp(i\omega t) \quad (15)$$

where W is the spatial function related to the boundary conditions.

By solving the governing equation (12) with simply supported boundary conditions and the closed loop control with velocity feedback the transverse displacements of the active laminated plate are obtained and may be expressed in terms of frequency response functions.

3 Results and discussion

Calculations are performed to recognize the influence of the applied PZT material distribution on the gradient of elastic and piezoelectric properties through the PFG actuator thickness and also the global structural response of the considered active plate including changes in the natural frequencies and the control system effectiveness.

A rectangular laminated plate of the dimensions a = 600 mm, b = 400 mm and the total thickness h = 2.8 mm is considered. The plate is simply supported and composed of classic graphite-epoxy layers of thickness h_1 = 0.2 mm and PVDF sensor and PFG actuator layers of thickness h_s = 0.1 mm and h_a = 1 mm, respectively. The symmetric stacking order $[S/A/90°/\bar{0}°]_s$ is applied. In the notation a bar indicates a layer, which is split at the midplane, symbols “S” and “A” refer to the sensor and actuator, respectively. The stiffness parameters of the graphite-epoxy composite are following: $Y_{11} = 150$ GPa, $Y_{22} = 9$ GPa, G_{12} = 7.1 GPa, and the equivalent mass density is equal $\rho = 1600$ kg/m^3. The electromechanical properties of the piezoelectric materials and matrix are listed in Table 1.

Tab. 1 Material properties

Parameter	PVDF	PZT	Matrix
ρ [kgm^{-3}]	1780	7650	1780
Y [GPa]	2	63	6
ν	0.3	0.28	0.3
d_{31} [m/V]	$23\cdot10^{-12}$	$190\cdot10^{-12}$	-
d_{32} [m/V]	$3\cdot10^{-12}$	$190\cdot10^{-12}$	-
$\in_{33}/\in_0$	12	1200	-

Internal damping is involved to limit the resonant amplitudes. For simplification the equivalent damping of the plate composite material refers to the orthotropic graphite-epoxy layers and is described by the Voigt-Kelvin model with the following retardation time values: $\mu_1 = 10^{-6}$ s, $\mu_2 = \mu_{12} = 4{\cdot}10^{-6}$ s.

The harmonic excitation of the amplitude intensity $q_0 = 1$ Nm^{-2} is uniformly distributed over the plate surface. The velocity feedback of the constant gain $k_d = 0.02$ s is applied.

The gradation of the PZT material in the PFG actuator is described by the exponential function Eq. (2) or the parabolic function Eq. (3) with the maximal PZT volume fraction v_1 and the minimal fraction v_0 varying according to the inhomogeneity parameter R. The parameter R is defined as the ratio of the limiting volume fractions, $R = v_1 / v_0$. In the case of the exponential PFG actuator its electroelastic properties decrease toward the middle of the plate. It is obvious that the gradient of elastic and piezoelectric properties is strictly associated with the applied PZT fraction distribution.

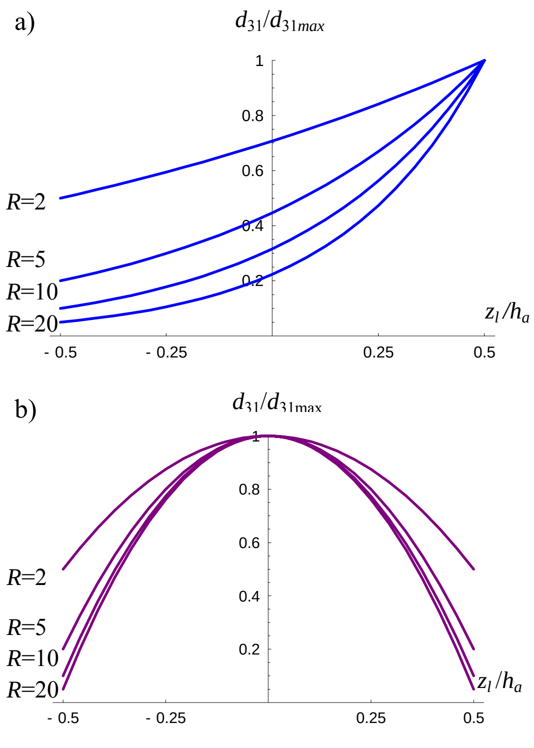

Fig. 1. Changes in the piezoelectric coefficient values of the PFG layer depending on the parameter R for a) exponential and b) parabolic distributions

For example, the variation of the piezoelectric strain coefficient d_{31} in the thickness direction calculated for exponential and parabolic distributions are shown in Fig. 1a and b, respectively. Here, the non-dimensional co-ordinates are used. The non-dimensional piezo-electric coefficient relates to its maximal value d_{31max}, which is obtained due to the rule of mixtures for the largest PZT fraction of the constant value $v_1 = 0.8$.

The similar plots may be obtained comparing the Young's modulus distributions. Thus, for the applied gradations of the PZT inclusions the inhomogeneity of the electroelastic properties increases significantly when the volume fraction ratio R (called the inhomogeneity parameter) becomes greater.

The stiffness and mass density, which vary through the PFG layer thickness, change the natural frequencies of the laminate depending on the PZT fraction distribution. The plots of the fundamental frequency ω_{11} versus the inhomogeneity parameter R calculated for the exponential and parabolic distributions are presented in Fig. 2a and b, respectively. The plots also show the effect of the maximal PZT volume fraction v_1 on the natural frequency.

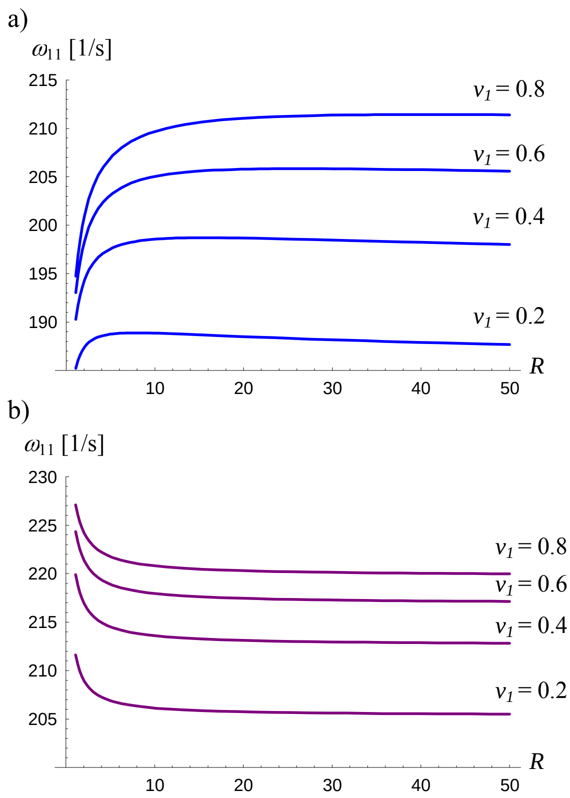

Fig. 2. Natural frequency ω_{11} versus the parameter R for a) exponential and b) parabolic distributions. Influence of the maximal PZT volume fraction.

Comparing the plots for both the considered distributions it can be seen that with increasing the maximal PZT volume fraction v_1 the frequency ω_{11} becomes greater with tendency to reduce its increment for larger values of v_1. The influence of the parameter R on the natural frequency differs depending on the applied distribution pattern. For the exponential distribution (Fig. 2a) the curves increase sharply when the values of R are relatively small and then show the extreme while for the parabolic distribution (Fig. 2b) they are decreasing functions starting with a significant slope. The natural frequency modification depends on the correlation between changes of the global stiffness and mass of the laminate, which are created by the applied volume fraction arrangement.

The dynamic responses presented in the Figs. 3 and 4 are calculated at the plate point $x = y = 100$ mm assuming the maximal PZT volume fraction $v_1 = 0.8$.

The frequency response functions shown in Fig. 3a and b, which are calculated for the active plate near its first mode region, confirm the influence of both the distribution function and the parameter R on the natural frequency changes. Comparing the plots it can be noticed that with increasing inhomogeneity of the PFG actuator the resonant amplitudes rise; thus, the operational effectiveness of the control system becomes lower.

a)

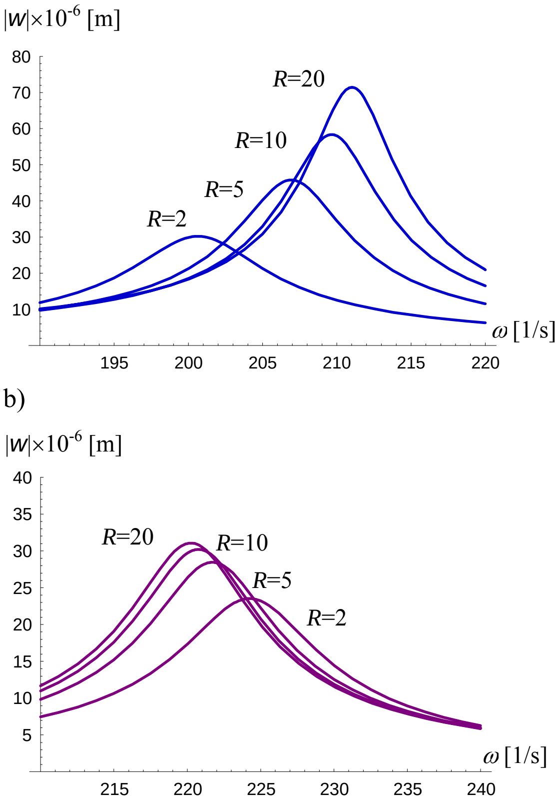

Fig. 3. Effects of variations in the parameter R near the first resonance region for a) exponential and b) parabolic distributions

A qualitatively similar effect may be observed for another resonance regions of the plate.

The frequency response functions of the considered plate with the exponential and parabolic PFG actuators calculated within a wide frequency range are presented in Fig. 4a and b, respectively. Here, the parameters of the PZT gradation are $v_1 = 0.8$ and $R = 10$. Due to the plate geometry and the applied external load the resonance picks occur at the following natural mode frequencies: ω_{11}, ω_{31}, ω_{13}, ω_{33} and ω_{51}. For comparison the uncontrolled response of the plate is indicated by a dotted line. It is evident that the proposed control system may be used to achieve the active reduction of the plate transverse vibration (solid line). Comparing the plots it can be seen that the parabolic PFG actuator layers (Fig. 4b) offers better control effectiveness for the same limiting volume fractions of piezoceramic material. The reason is a greater amount of the PZT component and in consequence the stronger actuation forces generated.

a)

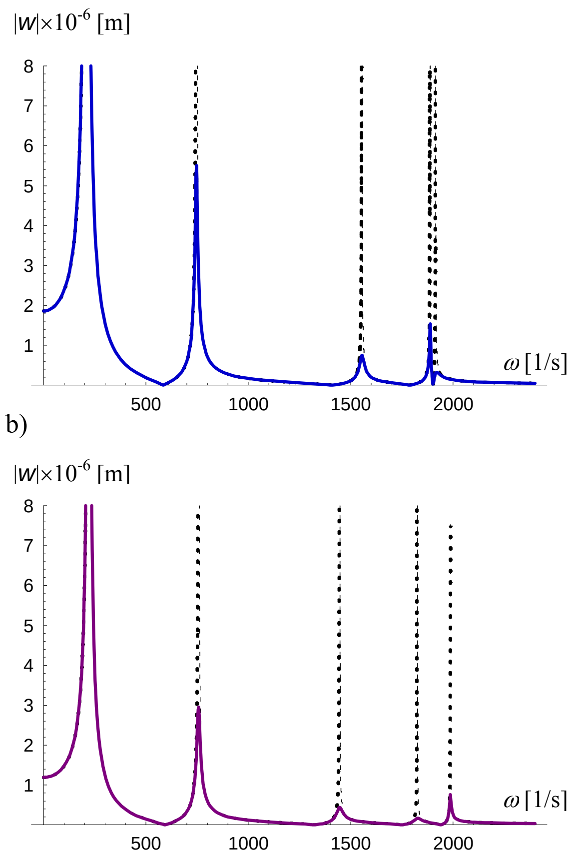

Fig. 4. Frequency responses for the laminated plate with PFG actuators with a) exponential and b) parabolic distributions. Dotted line – no control.

The amplitudes of the higher modes are significantly reduced due to both the active damping and the passive form of energy dissipation related to the applied material damping model.

4 Concluding remarks

The model of the actuator layer with functionally graded mechanical and piezoelectric properties has been formulated and applied for transverse vibration control of the laminated plates. The concept of the PFG actuator is based on the two-phase material with the exponential or parabolic volume gradation of the PZT fillers through the thickness. The results of simulation confirm the system modelling correctness. They show the influence of the applied PZT fraction distribution pattern and its parameters on the gradient of electromechanical properties and the plate structural response including the comparison of changes in both the natural frequencies and resonant amplitudes. It is also shown that the PFG actuator layers give a satisfactory operational effectiveness of the control system. Therefore, the use of PFG actuators with the suitable distribution of electromechanical properties offers sufficient actuation forces and also may reduce interlayer stresses minimizing the damage hazard comparing with the traditional monolithic and composite piezoceramic actuators.

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