ANALYSIS OF VARIOUS FORECASTING APPROACHES FOR LINEAR SUPPLY CHAINS BASED ON DIFFERENT DEMAND DATA TRANSFORMATIONS

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Abstract

Supply Chain Management (SCM) has become a popular topic in recent years. One of the main discussion points is the effective management of inventories along the supply chain. Several inventory policies have been analyzed and compared to overcome the traditional tradeoff between high service levels and high inventory costs and vice versa.
This paper describes how different demand data aggregation affects reorder point calculations in continuous inventory review systems and analyzes the impact on inventories, safety stocks, and service levels of the supply chain members. Based on a linear three-stage supply chain, four different forecasting scenarios based on simple means and variance calculations as well as moving average and moving variance estimations have been tested. To analyze potential effects for different supply chain settings, four demand patterns were implemented (stationary, season, trend, trend and season).
The simulations reveal different effects depending on demand data aggregation and customer demand structure. Since the assumption of normally distributed demand data is violated for upstream suppliers in linear supply chains, difficulties arise particularly in calculating safety stocks. Aggregating order data can mitigate some of the biases in several cases. It is shown that forecasting monthly aggregated orders outperforms the other strategies in terms of lower mean inventories and safety stocks, but may lead to slightly lower service levels.

Keywords: simulation, supply chain management, forecasting, inventory policies, safety stocks.

Presenting Author's biography

Roman Schmidt is working as research and teaching assistant at the University of Bern. His PhD research focuses on the application of simulation models in Supply Chain Management, with particular emphasis on the analysis of inventory policies.

1 Introduction

In the recent Supply Chain Management (SCM) discussion, control and management of inventories play important roles. Several analytical stochastic inventory models have been investigated and compared to determine optimal reorder points, safety stocks, and order quantities under uncertain conditions [1,2,3,4,5]. Research on the Bullwhip-Effect revealed substantial inefficiencies in the supply chain, resulting, amongst others, from information distortion and forecast updating [6,7]. Supply chain simulation studies often lead to the result that more accurate demand data (e.g., sharing point of sales data) allow upstream echelons to reduce their inventories [8,9,10,11,12,13]. However, the published results differ in the direction of the effects as well as in their intensity. One may assume that the effects are influenced by the implementation of basic inventory policies in the simulation models. As a large number of studies assume linear supply chains, the basic assumption of normally distributed demand data may be violated for upstream suppliers due to the fact that incoming orders arrive at discrete points in time. This paper investigates, how the aggregation of incoming order data affects reorder point calculation and thus, inventory levels, safety stocks and service levels of the supply chain partners.
The remainder of the paper is organized as follows. Section 2 discusses the basics of stochastic inventory policies, focusing on reorder point and order quantity calculations in continuous review systems. Section 3 describes the simulation model. In the final section the results are discussed and directions for future research are suggested.

2 Stochastic inventory policies

We focus on stochastic inventory policies with continuous inventory review [14,15,16]. The basic condition of such policies is the continuous measurement and monitoring of inventory position defined as

Inventory position (IP) = On-order inventory + On-hand inventory – Backorder.

An order is placed whenever IP drops below a critical reorder point to raise it up to a predefined level. If a fixed order quantity is assumed, this target level may be calculated as

$$S = s + Q, \tag{1}$$

where s is the reorder point and Q the order quantity. Fig. 1 illustrates the logic of stochastic inventory policies. The calculation of reorder points and order quantities is described in the following subsections.

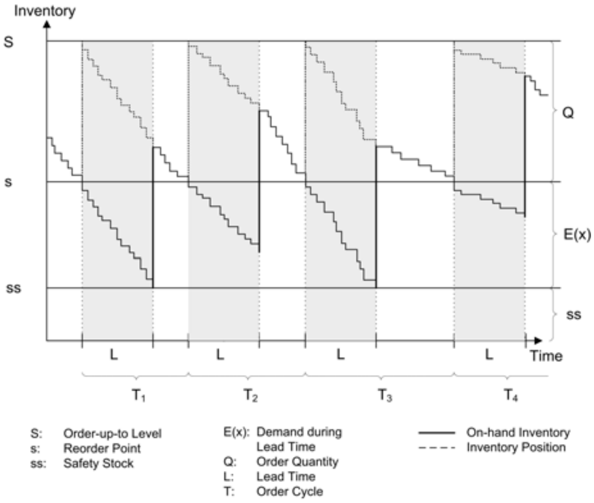

Fig. 1 Inventory levels resulting from application of stochastic inventory policies

2.1 Calculation of reorder point

To ensure that enough inventories are available to meet customer demand during lead time, the reorder point must be higher than the average demand during lead time. As demand is typically stochastic, a safety stock is often added to protect against uncertainty. The reorder point s may be calculated as [17]

$$s = E(x) + z \cdot \sigma_x, \tag{2}$$

where $E(x)$ is the average demand during lead time with its standard deviation σ_x, and z represents a constant service factor often associated with a predefined service level. Under the assumption of normally distributed demand, frequently used service levels and their corresponding z-values are shown in Tab. 1.

Tab. 1 Service levels and their corresponding safety factor for normally distributed demand

Service level F(z)	*Safety factor z*
0.90	1.280
0.95	1.645
0.97	1.88

2.2 Calculation of order quantity

Different models have been analyzed and compared to determine optimal order quantities. The classical model is the calculation of an economic order quantity (EOQ) [18]. By minimizing a cost function consisting of fixed order costs and inventory holding cost, the optimal order quantity may be calculated as

$$Q = \sqrt{\frac{2Kd}{h}}, \tag{3}$$

where K are the fixed order costs per order placed, d is the demand per period, and h represents the inventory holding costs per item and period. For extensions of the basic EOQ model, e.g. with respect to backorder costs or capacity restrictions, see [19].

2.3 Forecasting demand during lead time

One of the major tasks in determining adequate reorder points is the estimation of demand mean and variance during lead time. Due to continuous demand, forecasting for retailers is straightforward. For upstream suppliers forecasting can be more complicated as demand becomes discrete. Even if the order quantity placed by retailers is constant, the inter-arrival times of these orders may vary, depending on the real end consumer demand. To illustrate the problems connected with forecasting demand for upstream suppliers in a linear supply chain consider the following example shown in Tab. 2. The incoming order quantity is always 500 units but the inter-arrival time is 4 or 5 days. By computing moving average and moving variance for 7 days, either one or two orders are incorporated in the forecast. Hence, the daily demand forecast becomes either 2*500/7 = 142.86 or 500/7 = 71.43.

Tab. 2 Sensitivity of moving average data depending on time frames

Day	Demand per day (incoming)	Moving Average (7 days)	Moving Variance (7 days)
1	500		
2	0		
3	0		
4	0		
5	0		
6	500		
7	0	142.86	243.98
8	0	71.43	188.98
9	0	71.43	188.98
10	500	142.86	243.98
11	0	142.86	243.98
12	0	142.86	243.98
13	0	71.43	188.98
14	0	71.43	188.98
15	500	142.86	243.98
16	0	142.86	243.98
17	0	71.43	188.98
18	0	71.43	188.98
19	500	142.86	243.98

The example shows, that even with zero variability in the order quantities, suppliers cannot accurately estimate demand. The problem is even more complicated if order quantities vary. One can assume that the importance of this phenomenon could eventually be reduced if order data is aggregated. In the following we analyze different aggregation scenarios.

3 Model specification

A simulation model was built by using the EXTEND simulation software (www.imaginethatinc.com). The experiments assume a three-stage supply chain with one manufacturer, one distributor, and one retailer. A linear supply chain was considered to investigate the impact of discrete demand on reorder point calculation and to examine, how the calculations are affected by different aggregation levels. The static model parameters as well as the experimental factors are described in the following sections.

3.1 Static model parameters

3.1.1 Order policy

Based on a continuous inventory review system, each echelon determines a reorder point

$$s = E(x) + z \cdot \sigma_x . \tag{4}$$

For each echelon a safety factor of z = 1.88 was chosen, which correspond to a service level of 97% (for normally distributed demand). Lead time parameters are set as follows: for the manufacturer 14 days, the distributor 7 days, and the retailer 3 days.
When inventory position falls below the reorder point, an order is placed to raise the inventory position to the target level, which is the sum of the reorder point and the order quantity chosen. To guarantee, that orders are not placed too frequently, the order quantities are set higher than the expected reorder points and are chosen to be 10,000 for the manufacturer, 2,000 for the distributor, and 400 for the retailer. However, as demand occurs in batches, it is possible that the inventory position is lower than the reorder point. Thus, the effective order quantities placed may vary. To ensure that sufficient inventory is available at the beginning of the simulation, initial inventories are set for the manufacturer to 5,000, for the distributor to 1,000, and for the retailer to 200 units.

3.1.2 Delivery policy

As long as on-hand inventory is sufficient, the quantity ordered by the customer is completely delivered. For the deliveries, no restrictions in transportation capacity are considered. In out of stock situations suppliers deliver the available quantity and note backorders for the unfilled demand. Backorders are delivered as soon as inventory becomes available.

3.2 Experimental factors

3.2.1 Demand patterns

Four different demand patterns of customer demand were generated based on the following formula [13]:

$$\begin{aligned} Demand_t &= base + trend \cdot t \\ &\quad + season \cdot \sin\left(\frac{2\pi}{360} \cdot t\right) \\ &\quad + noise \cdot snormal(), \end{aligned} \tag{5}$$

where $Demand_t$ represents the demand for day t= (1,2,3,…N). The daily demand consists of an initial mean demand factor (base) with a standard deviation factor (noise) as well as trend and seasonal factors to generate non-stationary demand patterns. For the stationary pattern daily demand is normally distributed with a mean demand of 100 and a standard deviation of 20. To ensure that mean demand is approximately 100 also for demand patterns with trends, initial mean demand is set to 40. Parameter values for generating the four different demand patterns are shown in Tab. 3.

Tab. 3 Four parameter settings of demand patterns

	Parameter settings			
	base	*trend*	*season*	*noise*
stationary	100	0	0	20
season	100	0	20	20
trend	40	0.1	0	20
trend & season	40	0.1	20	20

3.2.2 Demand data aggregation

The focus of this paper lies in the analysis of different demand data aggregation levels and their impact on supply chain performance. Simple mean and variance computations are examined as well as moving average (MA) and moving variance (MV) estimations. Four different scenarios are investigated:

1. Mean and variances based on weekly orders
2. Mean and variances based on monthly orders
3. MA and MV of weekly orders (12 weeks)
4. MA and MV of monthly orders (3 months)

The impact of the four aggregation scenarios are tested for the distributor as well as for the manufacturer. As the retailer has daily demand data available, he can estimate the demand using MA and MV based on actual data. The forecast is based on the past 30 days.

3.3 Performance measures

To evaluate the four aggregation scenarios under different demand settings, the following performance measures were analyzed:

- mean inventory,
- mean service level, and
- mean safety stock.

4 Simulation results

For each of the four demand patterns, 10 simulation runs were performed under varying aggregation scenarios. As aggregation scenarios may be implemented either for the manufacturer or for the distributor, a total of 4x4x10x2=320 runs were executed and compared, where one run consists of 1000 days.

For the statistical analysis the first 200 days of each simulation run were deleted to account for warm-up effects. Statistical calculations (ANOVA and Tukey-HSD tests) were conducted using the SPSS statistical software.

4.1 Results for stationary demand

For stationary demand the mean inventory as well as the mean safety stock is significantly lower with monthly aggregated than with weekly demand for the manufacturer as well as for the distributor. The maximum reduction for both partners is approximately 50% in safety stocks resulting in a 15% reduction of mean inventory (Tab. 4).

Tab. 4 Output for the stationary demand pattern

	Manufacturer			*Distributor*		
	inv.	*SS*	*SL*	*inv.*	*SS*	*SL*
Scenario 1	8505	2756	1.00	1549	430	1.00
Scenario 2	7238	1432	1.00	1365	250	1.00
Scenario 3	8575	2878	1.00	1547	440	1.00
Scenario 4	7319	1578	0.99	1327	214	0.99
inv: mean inventory level, SS: safety stock, SL: service level						

Despite the significant inventory reductions the service levels are almost identical in all aggregation scenarios and are higher than the expected 97% by setting a safety factor of $z = 1.88$ in determining the safety stock. Thus, aggregating demand data may lead to better estimations of means and variances to calculate reorder points more adequately.

4.2 Results for seasonal demand

The seasonal demand pattern shows quite similar results as the stationary demand pattern. Scenario 4 has lower inventories and safety stocks than the other scenarios (Tab. 5). The service level is equal or higher than 97%, while the inventory reductions are approximately 15% for both the manufacturer and the distributor again. Using monthly aggregated order data in combination with a forecast technique (scenario 4) the safety stocks can be reduced for the manufacturer by up to 51%, whereas the reductions for the distributor are about 43%.

Tab. 5 Output for the seasonal demand pattern

	Manufacturer			*Distributor*		
	inv.	*SS*	*SL*	*inv.*	*SS*	*SL*
Scenario 1	8600	2768	1.00	1581	442	1.00
Scenario 2	7352	1497	1.00	1623	480	1.00
Scenario 3	8569	2847	1.00	1537	414	1.00
Scenario 4	7321	1390	0.97	1394	276	0.99
inv: mean inventory level, SS: safety stock, SL: service level						

4.3 Results for trend demand

As can be seen in Tab. 6, the results obtained for trend demand show again striking benefits from order data aggregation. The safety stocks can be massively reduced by aggregating order data by approximately 58% for both the manufacturer and the distributor. The inventory reductions are about 21% for the manufacturer and 10% for the distributor. The slightly lower service levels may be explained by the use of moving average, which reacts slowly to trends in demand. By applying a more appropriate forecasting method (e.g., exponential smoothing) the service levels are expected to be higher.

Tab. 6 Output for the trend demand pattern

	Manufacturer			*Distributor*		
	inv.	*SS*	*SL*	*inv.*	*SS*	*SL*
Scenario 1	7799	2453	1.00	1351	416	0.98
Scenario 2	6655	1324	0.99	1437	511	0.99
Scenario 3	8463	2805	1.00	1477	359	0.99
Scenario 4	7035	1181	0.97	1330	214	0.97
inv: mean inventory level, SS: safety stock, SL: service level						

4.4 Results for trend and seasonal demand

The demand pattern with seasonality and trend shows almost identical results as the trend demand pattern without seasonality. Tab. 7 shows the absolute output measures. Aggregating information leads to a maximum safety stock reduction of over 50%. The manufacturer can reduce the mean inventory by 24%, whereas the reductions for the distributor are only 8%.

Tab. 7 Output for trend and seasonal demand pattern

	Manufacturer			*Distributor*		
	inv.	*SS*	*SL*	*inv.*	*SS*	*SL*
Scenario 1	7815	2473	1.00	1395	448	0.98
Scenario 2	6610	1356	0.98	1516	558	0.99
Scenario 3	8679	2773	1.00	1475	366	0.99
Scenario 4	7100	1337	0.98	1402	275	0.98
inv: mean inventory level, SS: safety stock, SL: service level						

4.5 Sensitivity analysis

To ensure that the results are not biased by the parameter settings chosen, different order quantities have been tested. In a first execution retailer's order quantity was increased from 400 to 600 units by unchanged order quantities for the distributor and the manufacturer. In a second execution the order quantity placed by the distributor was decreased from 2000 to 1000 units by unchanged order quantities for the retailer and the manufacturer.

One important finding was that the results are strongly influenced by the order quantities placed by downstream customers. An increase in retailer's order quantity does not lead to any positive effect of aggregating order data for stationary demand and results in a massive service level reduction for all demand patterns for the distributor. Interestingly, the highest service levels can be achieved by reorder point calculations based on weekly forecasts (scenario 3).

In contrast, the decrease in distributor's order quantity (from 2000 to 1000) has no significant impact on manufacturer's results. A detailed discussion of these rather surprising results is presented in the following section.

4.6 Discussion of results

The analysis of the simulation output reveals huge safety stock and inventory reductions by aggregating order data. However, the striking benefits of reorder point and safety stock calculations based on aggregated order data is less impressive with different order quantities considered than in the original simulation model. Furthermore, sensitivity analysis revealed a massive reduction of service level using forecast based on aggregated order data.

The main influencing factor is the inter-arrival time of incoming orders. In the original simulation model the inter-arrival time for the distributor is about 4.5 days for all demand patterns with a slightly higher standard deviation for demand patterns of higher complexity. An inter-arrival time of 4.5 days results in strong fluctuation of weekly orders as the number of incoming orders varies between one or two orders in each week. Thus, demand estimation based on weekly data is inefficient. Fig. 2 shows the probability distribution of retailer's weekly and monthly incoming orders for stationary demand.

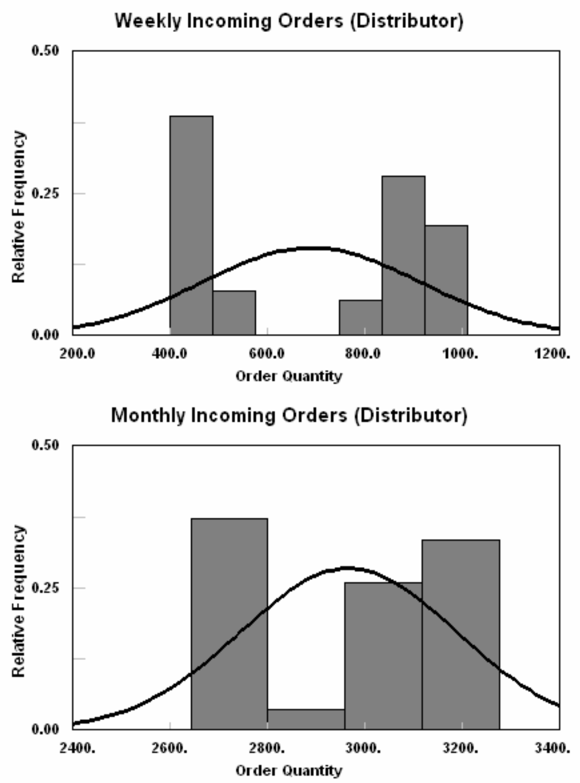

Fig. 2 Distribution of differently aggregated order data

As can be seen, the order quantity chosen in the original model leads to two peaks of weekly incoming order quantities. As the inter-arrival time of 4.5 leads to changing weekly orders, an aggregation to monthly data may strongly change the probability distribution.

A modified (increased) order quantity of 600 units for the retailer examined in the sensitivity analysis leads to an inter-arrival time for the distributor of 6.5 days for all demand patterns. Consequently, the distributor may estimate the mean demand more exactly by using weekly order data than aggregating the data into larger time units. Nevertheless, at several points in time two orders are placed in one week resulting in incorrect forecast.

The sensitivity analysis does not influence the results for the manufacturer significantly as the distributor's order frequency is too low. Using aggregated order data is therefore beneficial in the original simulation setting as well as with modified order quantities investigated in the sensitivity analysis.

5 Conclusions

The analysis of a linear three-stage supply chain has shown that inventories and safety stocks may be reduced by aggregating weekly incoming orders into larger time units. As order data is not normally distributed, the computation of reorder points and safety stocks is biased. An aggregation of order data can mitigate some of these biases in several cases. However, the results are strongly influenced by the order quantities considered. Depending on the order quantities, changing inter-arrival times of incoming orders have a strong impact on the effectiveness of demand forecasts.

For analyzing linear supply chains, the application of simple stochastic inventory policies is not appropriate for two reasons. First, due to a violation of the assumption of normally distributed incoming orders, and second, due to the strong impact of order quantities considered. Therefore, results of simulation studies assuming linear supply chains may be strongly biased by inappropriate computations of reorder points and safety stocks.

Future simulation research should be aware of biasing factors in computing reorder points. Furthermore, the effects of information sharing strategies should be analyzed in depth with regard to the results discussed in this paper.

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