NUMERICAL SIMULATION OF DELAMINATION PHENOMENA IN FIBER REINFORCED COMPOSITE LAMINATES

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Abstract

Delamination mechanisms strongly affect the material integrity of composite structures, especially when the crack growth is generated by impact or dynamic loading conditions. In this framework, experimental observations have been shown that high speeds crack propagation characterizes the composite structures leading to catastrophic failure mechanisms. In this paper, the dynamic behavior of composite structures in the framework of steady-state crack growth is investigated. The proposed methodology considers the laminated structures as composed by first-order shear deformable plate elements interconnected by interface elements, whose constitutive relationships are based on fracture and contact mechanics. Analytical solutions of the relevant governing equations are proposed and closed form expressions for energy release rates (ERRs) are provided, emphasizing the influence of the beam formulation and the kinematic modeling on the crack tip solution. In particular, analytical expressions for total energy release rate and its mode components at the delamination front are provided in terms of the interface variables or the beam stress resultant discontinuities, emphasizing the influence of the inertial contributions on the ERRs. By means of these expressions the influence of transverse shear on interface fracture analysis is discussed and comparisons with other beam-based delamination models adopted in the literature are established. A parametric study, developed for mixed mode loading condition, is proposed to evaluate the effects produced on the ERRs by the inertial description of the structures and the speed of the crack tip front.

Keywords: delamination, dynamic energy release rate, mode partition, steady crack growth.

Presenting Author's biography

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1 General

Composite materials in the form of laminate structures are widely utilized for strategic applications in both mechanical and aerospace engineering fields. However, substantial experimental evidences have shown that delamination phenomena dramatically affect composite structures by means of catastrophic failure modes. During the last decades many efforts have been spent, mainly, to analyze static fracture behavior, giving rise to several studies devoted to predict the energy release rate and to simulate the crack growth phenomena. However, dynamic fracture in multilayered composite structures has not extensively investigated, because several complexities arise to identify properly the local crack tip behavior at high strain rate during the crack advance.

In the literature many works have been focused mainly on quasi static crack propagation for both monolithic or composite structures [1]. However, during the last decades, new advances in experimental mechanics provided enhanced techniques to analyze crack growth behavior at high speed ranges. In this context, experimental observations pointed out that for mode I loading condition and monolithic materials, the speed of the crack growth has been observed to attain almost half of the Rayleigh wave speed, because the crack branching frequently affects the specimen and as a result a reduction of the crack speed is observed experimentally [2]. However, this behavior has not verified in the context of laminate structures, where the interfaces represent weak planes, in which the crack growth is made possible along straight paths. As a result, elevated speeds of advance can be reached also close to the Rayleigh one [3].

For mixed-mode or shear (mode II) loading conditions, experimental results showed that the crack growth is able to exceed the shear wave speed, approaching to a stable velocity in the range of intersonic crack propagation(i.e. crack tip velocity between the shear and longitudinal wave speeds of the material). As far as dynamic crack growth is concerned, several experimental methods based on CGS interferometry and dynamic photoleasticity have been proposed [4]. The experimental records referred to above have been investigated by means of analytical approach developed in the context of 2D plane stress assumptions and linear elastic fracture mechanics. In particular, asymptotic fields around the crack tip have been analyzed, in which it has been observed that for mode I loading condition the crack growth is physically admissible since the speed propagation is theoretically less than Rayleigh wave speed [5]. At contrary, crack propagation in mode II loading condition is possible also in intersonic speed range, in which the shear cracks at first accelerate towards a specific speed and subsequently evolve under a steady-state crack propagation. The analyses referred to above have confirmed the presence of finite contact zones as well as stress discontinuous rays behind the interfacial crack, known as Mach waves, which strongly influence the crack growth phenomena [5].

In the literature only a limited number of analytical studies on the subject of dynamic crack propagation in fiber composite materials have been reported. Among these, Shahani & Forqani [1] have provided analytical expressions of dynamic ERRs in the context of mode I loading condition and under quasi-static crack growth. Wosu et al. [6] proposed specialized expressions for mixed mode open notch flexure scheme in the framework of quasi-static evolution of the crack. Moreover, asymptotic fields around an intersonic propagating crack tip are investigated for 2D strain model for both mode I, mode II and in mixed mode condition under steady state crack advance and remote loading conditions [2]. In particular, these analyses explained in detail, experimental investigations based on CGS and photoelastic fringe patterns developed by Rosakis and co-workers [4].

In the present paper, dynamic fracture mechanics problem has been investigated in the context of the interface methodology and the plate/beam formulation, which provide an easy modeling to derive simple expressions related to the ERRs and to the crack advancing conditions. In particular, the crack growth is assumed to occur along a straight interface between layers, modeled as shear deformable beam elements. The use of the interface methodology allows to recover ERRs for interlaminar crack advance by taking the limit of the strain energy per unit interface at the crack tip when the interfacial stiffness approach to infinity. However, ERRs are obtained directly in terms of both interfacial or alternatively in terms of jumps in stress resultants. The easily way to analyze the crack tip behavior allows us to derive closed form solution of the ERR, by which it is possible to investigate the influence on the ERR of the speed of the crack front and the inertial effects of the laminate structure. The results have been illustrated by mean of the J-integral concept, in order to emphasize different contributions arising from the beam modeling and the cinematic assumptions, by which it is possible to verify that the use of the classical plate theory based on Eulero-Bernoulli formulation strongly underestimates ERRs and crack growth behavior. Finally, a parametric study developed for mixed mode loading condition is proposed to evaluate the effects produced on the ERRs by the inertial description of the structures and the speed of the moving crack tip front.

2 Delamination model

The structural model refers to a laminated structure, which is composed by unidirectional fiber reinforced plies connected by linear elastic interfaces (Fig.1). The delamination modeling is consistent with previous

Authors' works [7-8], mainly developed in the framework of quasi-static crack growth and here proposed in a generalized context, in which dynamic effects under a steady state crack growth are introduced in the governing equations. Each layer of the laminate is assumed to be homogeneous, orthotropic and linearly elastic, with orthotropy axes aligned with the global co-ordinate system. The upper and lower sublaminates are assumed to be perfectly bonded in the undelaminated region, by the use of interface methodology. In particular, denoting the interlaminar normal and shear stresses as σ_{xy} and σ_{yy}, the constitutive relationships for opening and transverse relative displacements, assume the following form :

$$\sigma_{yy} = k_y \Delta v, \quad \sigma_{xz} = k_{xy} \Delta u, \tag{1}$$

where $\left[k_z, k_{xy} \right]$ are the shear and the tensile stiffness of the interface. Consistently with a multilayer approach, the lower and the upper sublaminates are modeled by using with n_l and n_u elements, respectively. The kinematic of the *i-th* plate element is described in terms of the mid-surface in-plane displacements, (u_j, v_j) and the rotation about z axis (ψ_j), as:

$$\begin{aligned} U_i(x) &= u_i(x) + (y - y_i) \cdot \psi_i(x) \\ V_i(x) &= v_i(x) \end{aligned} \tag{2}$$

where $1 \leq i \leq n_u + n_l$. y_i is the y-coordinate of the *i-th* plate midplane element. The deformation state is described to the first order theory by the membrane, curvature and transverse shear strains, defined as:

$$\varepsilon_i = u', \quad \chi_i = \psi_i', \quad \gamma_i = \psi_i + v_i' \tag{3}$$

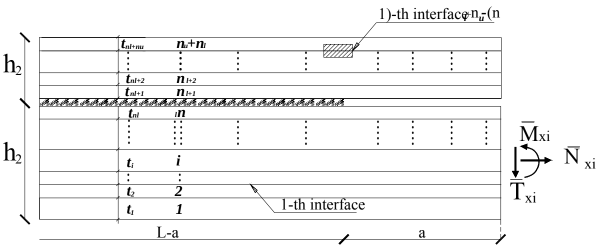

Fig. 1 Scheme of the composite structures

Moreover, by considering each plate as composed by one or several physical fiber reinforce plies with their material axes arbitrarily oriented, the constitutive relationships between stress resultants and corresponding strains are:

$$\begin{bmatrix} N_i \\ M_i \end{bmatrix} = \begin{bmatrix} A_i & B_i \\ B_i & D_i \end{bmatrix} \begin{bmatrix} \varepsilon_i \\ \chi_i \end{bmatrix} \tag{4}$$

where N_i is the membrane force resultant, M_i the moment resultant and T_i the transverse shear force resultant. Finally, the perfect adhesion conditions along the undelaminated planes are imposed by the use the Lagrange's method. In particular, the displacement continuity requirements between any two adjacent plate elements, i.e. i and $i+1$ (with $i \neq n_l$), are defined by the following relationships:

$$\begin{aligned} \Delta u_j &= u_j - \frac{t_i}{2} \psi_j - u_{j+1} - \frac{t_{i+1}}{2} \psi_{j+1} = 0, \\ \Delta v_j &= v_j - v_{j+1} = 0. \end{aligned} \tag{5}$$

The equations of motion and the associated growth law for the delamination shall be derived using a variational approach introducing proper functionals related to the strain and the kinetic energies, the interface displacements continuity for both undelaminated and delaminated planes. As a consequence the Hamilton's principle can be expressed as:

$$\int_{t_1}^{t_2} \delta \left(U - T \right) dt = 0 \tag{6}$$

where T and U are the kinetic and the potential energy of the whole dynamic system, respectively, and t_1 and t_2 define the observation period. Moreover, the equilibrium equations at a generic can be easily obtained taking the variation respect to the generalized cinematic variables:

$$\int_{t_1}^{t_2} \delta \left(\Phi + I + L - W - T \right) dt = 0 \tag{7}$$

where Φ is the strain energy of the plate elements in both delaminate and undelaminate zones, I is the strain energy of the interfaces representing a penalty functional, L is the Lagrangian functional related to the interface displacement continuity constraints between adjoining plate elements and T is the kinetic energy of the laminate. In view of the sublaminate modeling, the quantities referred to above can be expressed by the following relationships:

$$\Phi = \sum_{i=1}^{n_u + n_l} \int_0^L \varphi_i \left(u_i, v_i, \psi_i \right) dx \tag{8}$$

$$I = \lim_{k_y, k_{xy} \to \infty} \int_0^{L-a} \frac{1}{2} \left(k_y \Delta v^2 + k_{xy} \Delta u^2 \right) dx \tag{9}$$

$$L = \sum_{i=1, i \neq n_l}^{n_u + n_l - 1} \int_0^L \left(\lambda_{w_i} \Delta v_i + \lambda_{u_i} \Delta u_i \right) dx \tag{10}$$

$$T = \sum_{i=1}^{n_u + n_l} \int_0^L \tau_i \left(u_i, v_i, \psi_i \right) dx \tag{11}$$

where $\varphi_i = \frac{1}{2} \left[N_i \varepsilon_i + M_i \chi_i + T \gamma_i \right]$ is the strain energy density, $\tau_i = \frac{1}{2} \left[\mu \left(\dot{u}_i^2 + \dot{v}_i^2 \right) + \mu_0 \left(\dot{\psi}_i^2 \right) \right]$ are the kinetic energy densities of the i-th lamina, μ and μ_0 represent the mass and polar mass moments per unit length.

The evaluation of energy release rate in dynamic framework can be computed in terms of interface variables or alternatively by using plate strain and stresses. The equivalence between the approach referred to above have been proved in [7,8] and both formulations are able to point out a better understanding of the mechanics of delamination In order to derive analytical expressions of the ERR only stress resultants approach is here utilized, because the governing equations introduce less complexities and consequently are quite suitable to be solved.

The energy release rate is defined as the rate of mechanical energy flow out of the body and into the crack tip per unit crack advance [10]. The model used in this work extends the energy balance basis of ordinary linear elastic fracture mechanics to the dynamic situation by considering the energy dissipated at the crack tip at a rate which is primarily a material property. Energy not removed at the crack tip remains in the system as either strain energy or kinetic energy. In particular, according to the Fracture Mechanics framework, the crack advance is made possible only when the energy released by the body during the crack extension is at least equal to that absorbed by the extending crack. During the amount of crack advance, *da* at instantaneous speed *c*, the energy available for supporting crack extension is given by:

$$G=-\left[\frac{dU}{da}+\frac{dT}{da}\right]=-\frac{1}{c}\left[\frac{d}{dt}\left(U+T\right)\right] \tag{12}$$

Assuming a steady state crack evolution and by introducing a moving reference system at the crack tip front, here identified with the normalized coordinate X, with $X=x_{iud}-ct$ or $X=x_{iud}-ct-a$ for i=1,2, the dependence from the time variable is practically annihilated and all expressions can be written in terms of the normalized coordinated X only (Fig.2).

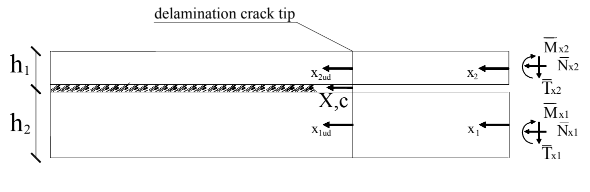

Fig.2 Two layer scheme – Moving and fixed coordinate systems.

Substituting, Eqs. (8)-(11) into Eq.(12), integrating by parts and taking into account equilibrium conditions arising from Eq.(7), the following expression for ERR has been recovered:

$$G=\sum_{i=1}^{n_u+n_l}\left[\!\left[\Phi+\mathrm{T}-\left[N_i u_i'+T_i\left(v_i'\right)+M_i\left(\psi_i'\right)\right]\right]\!\right] \tag{13}$$

where $\left(\bullet\right)'=\frac{\partial\left(\bullet\right)}{\partial X}$ is the derivative respect moving coordinate X, whereas $\left[\!\left[\bullet\right]\!\right]=\left(\bullet\right)^{+}-\left(\bullet\right)^{-}$ is the jump occurring at the crack tip a with the superscripts + and – denoting that the function is evaluated a $x=a^{+}$ and $x=a^{-}$.

Alternatively to Eq.(13), the ERR can be evaluated in terms of interface variables, starting from the regularized version of the Hamilton's functional and introducing the perfect adhesion conditions by means of the penalty technique. In particular, the limit procedure is performed for the interfacial stiffness parameters. As a result, the ERR is expressed in a decomposed form directly in terms of the interface strain energy, as:

$$\begin{aligned}G&=-\lim_{k_y,k_{xy}\to\infty}\frac{1}{2}\left[\!\left[\sigma_u\Delta v_{n_l}^2+\sigma_{yx}\Delta u_{n_l}^2\right]\!\right]_{X=0}=\\&=\lim_{k_y,k_{xy}\to\infty}\frac{1}{2}\left[k_y\Delta v_{n_l}^2\left(0\right)+k_{xy}\Delta u_{n_l}^2\left(0\right)\right]\end{aligned} \tag{14}$$

where $\Delta v_{n_l}\left(0\right)$ and $\Delta u_{n_l}\left(0\right)$ represent the opening and transverse displacements at the crack tip front.

Energy release rate mode partition is evaluated by using a specialized form of the virtual crack closure techniques (VCCT). Analytical expressions will be obtained concisely recalling only the main equations involved in the proof, which are useful in order to point out the mechanical parameter involved in the computation. According with VCCT the energy release rate during the crack advance is defined as the energy required to close the crack between two layers adjoining the delamination plane. In particular, based on an infinitesimal and virtual increment of crack length, the crack closure integral was proposed by Irvin [9] to calculate the strain energy release rate. By using results concerning the interface methodology in the context of strong interface given by Eq.(14), the ERR is decomposed by means of the following additive form:

$$\begin{aligned}G&=G_I+G_{II}=\\&=\lim_{k_y,k_{xy}\to\infty}\frac{1}{2}\left[k_y\Delta v_{n_l}^2\left(0\right)+k_{xy}\Delta u_{n_l}^2\left(0\right)\right]\end{aligned} \tag{15}$$

The previous equations can be easily proved starting from the J integral concept, which is reported in the canonical form by the following expression:

$$J=c\int_{\Gamma}\left[\left(U+T\right)n_1-\sigma_{ij}n_j\frac{\partial u}{\partial x}\right]d\Gamma \tag{16}$$

In particular, for a path with vanishing radius centered with respect to the crack tip, the expression of the J integral specializes to a corresponding one in which the terms arising from the kinetic energy, i.e. T, are practically zero along the path contour Γ. As a result, the J integral coincides with the strain energy of the interface elements at the crack tip.

Alternatively the Eqs.(15), the ERRs mode components can be evaluated by the virtue of the VCCT concepts expressed in the framework of the stress resultants approach. The mode I energy release rate component can be calculated as

$$G_I=\frac{1}{2\delta a}\lambda_y d\Delta v^{+} \tag{17}$$

where λ_y is the Lagrangian multiplier reflecting the singularities in normal interface stresses in the limit as the interface stiffness parameters approach to infinity (Fig.3a). In particular, taking into account the equilibrium requirements of an infinitesimal portion above or below the crack tip characterized by a moving crack tip front, the Lagrangian multiplier is defined by the following expression:

$$\lambda_y = -\sum_{i=1}^{n_l}\left[[\![T_i]\!] + [\![\mu_i c^2 v_i']\!]\right] = \sum_{i=n_l+1}^{n_l+n_u}\left[[\![T_i]\!] + [\![\mu_i c^2 v_i']\!]\right] \tag{18}$$

whereas the corresponding interlaminar separation can be expressed as

$$d\Delta v^+ = [\![\gamma_{n_l} - \gamma_{n+1}]\!]\, da \tag{19}$$

Substituting Eqs.(18),(19) into (17), after some algebraic manipulations the following expression is recovered:

$$G_I = \sum_{i=1}^{n_l+n_u}\left[\frac{1}{2}[\![T_i\gamma_i]\!] - T_i[\![\psi_i]\!] + [\![\Gamma_i^I]\!]\right] \tag{20}$$

with $\Gamma_i^I = \frac{1}{2}\left(\mu c^2 v'^2\right)$ representing the term in the kinetic energy related to vertical direction. It is worth noting that Eq.(20) corresponds to a generalization in the contest of steady stated crack growth of the closed form expressions previously developed by Author's work in the context of quasi-static delamination [7-8].

An analogous calculation gives the mode II energy release rate component as:

$$G_{II} = \frac{1}{2\delta a}\lambda_{xy}\, d\Delta u \tag{21}$$

where λ_{xy}, similarly to λ_y corresponds to Lagrangian multiplier related to relative transverse displacements. Taking into account of the equilibrium equations for an infinitesimal portion surrounding the crack tip the following expression can be derived:

$$\begin{aligned}\lambda_{xy} &= -\sum_{i=1}^{n_l}\left[[\![N_i]\!] + [\![\mu_i c^2 u_i' + \mu_{0i} c^2 \psi_i]\!]\right] = \\ &= \sum_{i=n_l+1}^{n_l+n_u}\left[[\![N_i]\!] + [\![\mu_i c^2 u_i' + \mu_i c^2 \psi_i]\!]\right]\end{aligned} \tag{22}$$

whereas the relative displacement at the crack tip is

$$d\Delta u = -\left(\varepsilon_{n_l+1} - \varepsilon_{n_l} + \chi_{n_l}\frac{h_{n_l}}{2} + \chi_{n_l+1}\frac{h_{n_l+1}}{2}\right)da \tag{23}$$

Substituting Eq.s (22),(23) and into Eq.(21) the mode II energy release rate component assumes the following expression:

$$G_{II} = \sum_{i=1}^{n_l+n_u}\left[\frac{1}{2}[\![N_i\varepsilon_i + M_i\chi_i]\!] + [\![\Gamma^{II}{}_i]\!]\right] \tag{24}$$

with $\Gamma_i^{II} = \frac{1}{2}\left(\mu_i c^2 u_i'^2 + \mu_{0i} c^2 \psi_i'^2\right)$ corresponding to the kinematic energy expression related the bending and axial contribution of the *i-th* layer.

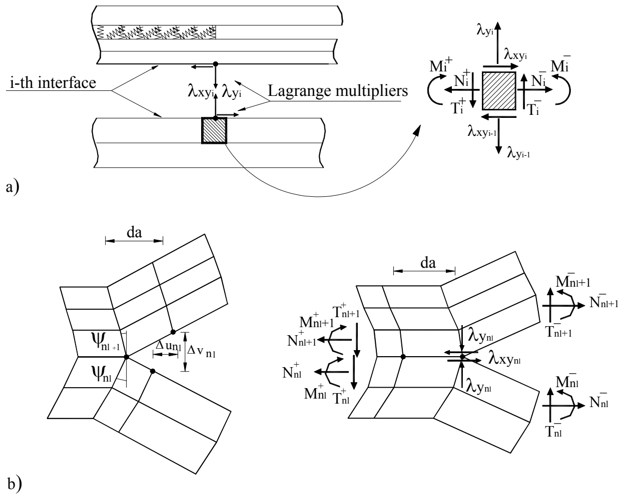

Fig.3 a)Lagrangian forces at the crack tip front. b) Virtual crack closure methods.

3 Results and discussion.

The results will be presented starting from pure mode I and Mode II loading conditions, whereas a general example involving mixed mode will analyzed subsequently. A finite crack length is assumed to propagate along the fiber direction at constant crack tip speed. The formulation refers to a two layers scheme, where the main governing equations can be handled from analytical point of view. However, more accurate results could be derived by the use of the multilayer approach, which requires a numerical based techniques to solve the main equations [7,8].

The governing equations for mixed mode loading condition are here summarized, with respect to a two layer DCB scheme. In particular, starting from Eq.(7) the following relationships describe the equilibrium equations in the context of a steady state crack growth condition:

Undelaminated zone: X≥0 and X≤L-a

$$\begin{aligned}&\left(A_1 - \mu c^2\right)\overline{u}_1'' + \left(A_2 - \mu_2 c^2\right)\overline{u}_2'' = 0\\ &\left(H_1 - \mu_1 c^2\right)\overline{v}_1'' + \left(H_2 - \mu_2 c^2\right)\overline{v''}_2 + H_1\overline{\psi'}_2 + H_2\overline{\psi'}_2 = 0\\ &\left(D_1 - \mu_{01} c^2\right)\overline{\psi''}_1 - H_1\left(\overline{\psi}_1 + \overline{v}_1\right) = 0\\ &\left(D_2 - \mu_{02} c^2\right)\overline{\psi''}_2 - H_2\left(\overline{\psi}_2 + \overline{v}_2\right) = 0\end{aligned} \tag{25}$$

Delaminated zone X≤0 and X≥ -a

$$\left(A_i - \mu_i c^2\right)u_i'' = 0$$
$$\left(H_i - \mu_i c^2\right)v_i'' + H_i\psi_i' = 0 \qquad \text{with i=1,2} \qquad (26)$$
$$\left(D_i - \mu_i c^2\right)\psi_i'' - H_i\left(\psi_i + v_i'\right) = 0$$

Introducing, boundary conditions at left and right ends of the laminate as well as at the delamination crack tip front, analytical expressions of the displacement fields could be recovered solving for the constants introduced from the general solution of the differential equations system given by Eqs. (25) ,(26). Subsequently, substituting in Eqs. (20) and (24), ERR mode components, not here reported for the sake of brevity, can be derived, analytically.

At first, the equations referred to above are specialized for a double cantilever beam scheme (DCB) in a pure mode I loading condition with two opening end forces, namely *F*. In particular, a simple symmetrical composite structure with the same mechanical and geometrical characteristics (i.e. $h_1=h_2=h$, $D_1=D_2=D$, $H_1=H_2=H$) is considered. The material and geometrical properties of the laminated structures are reported in Tab.1.

In Fig.5, the relationship between dimensionless ERR and speed of crack tip front, namely $\overline{G} = G\text{EBh}^2/F^2$ and c/c_{sh} (with c_{sh} denoting the shear wave speed of the laminate) is investigated for different ratios of the normalized delaminated length. It is observed that the ERR decreases for increasing valued of the crack tip front speeds. As far as the speed approaches to the Rayleigh one, the ERR denotes a singular behavior. This is in agreement with several works in the literature, who established that from the molecular dynamic simulation of dynamic fracture, the Rayleigh speed consists to limiting crack tip velocity in mode I crack propagation [11]. Moreover, before to reach the Rayleigh speed, the ERR function tends to values close to zero, denoting contact phenomena at the crack faces. This is confirmed by the results reported in Fig.6, where the normalized vertical displacement around the crack tip became negative exactly when the ERR approach to zero.

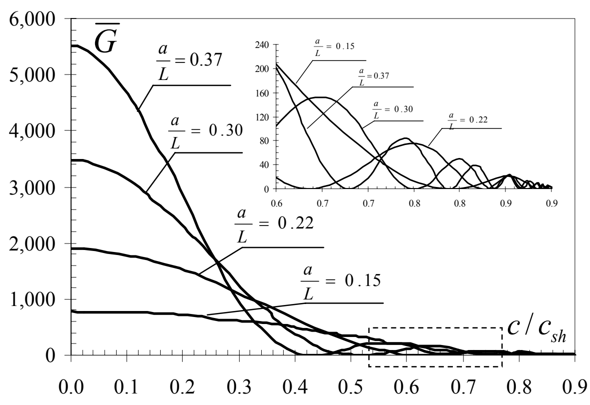

Fig.5 Dimensionless ERR for different speed of the crack front and initial delaminated length..

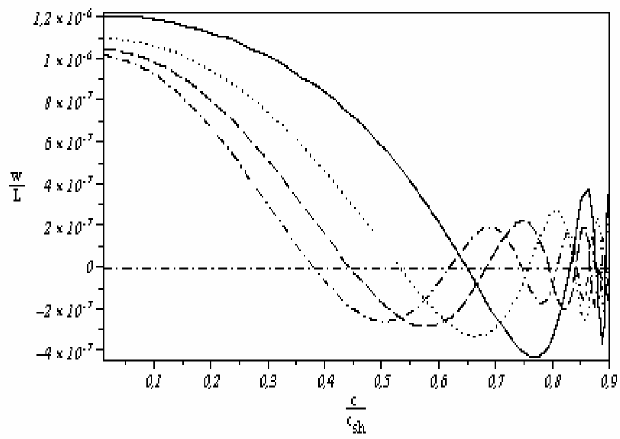

Fig.6 Mode I loading conditions: Dimensionless relative displacements close to the crack tip front.

Tab. 1 Material properties

E_1 [N/mmq]	G_{xy} [N/mmq]	ρ [Kg/mc]
149.25E3	5.45E3	1500
a/L	h/a	
0.148	0.125	

The effect of the shear deformability on the ERR evaluation is discussed in the context of a steady state crack growth. In particular, with the aid of the J integral concept, assuming a path surrounding the crack tip close to the crack itself, the expression of the ERR assumes the following form:

$$J = J_M + J_T \qquad (27)$$

with

$$J_M = \frac{M_{tip}{}^2}{DB}\left(1 - \frac{\rho c^2}{E}\right) \qquad (28)$$

$$J_T = \frac{T_{tip}{}^2}{HB}\left(1 - \frac{\rho c^2 h}{H}\right) + \\ -\frac{2T_{tip}}{B}\psi_1(0)\left(1 - \frac{\rho c^2}{E}\right) - \frac{\rho c^2}{B}\psi_1(0)^2 h \qquad (29)$$

in which J_M and J_T denote the bending, shear terms, respectively. An analogous expression could be obtained in the context of the classical delamination models (CDM), where the shear effects are completely neglected by the use of an Euler-Bernoulli (EB) formulation. As a result, the expression obtained of the ERR assumes the following relationship:

$$J_{CDM} = J_M = \frac{M_{tip}{}^2}{DB}\left(1 - \frac{\rho c^2}{E}\right) \qquad (30)$$

In order to investigate, the influence of the shear effects, comparisons between the proposed formulation and that concerning EB assumption is proposed in Fig.7, in which the single terms in dimensionless form (i.e. $\overline{J}_i = (\cdot)\,\mathrm{EBh}^2/F^2$, with $(\cdot) = J_{tot}, J_M, J_T$) arising form Eq.(27) are reported.

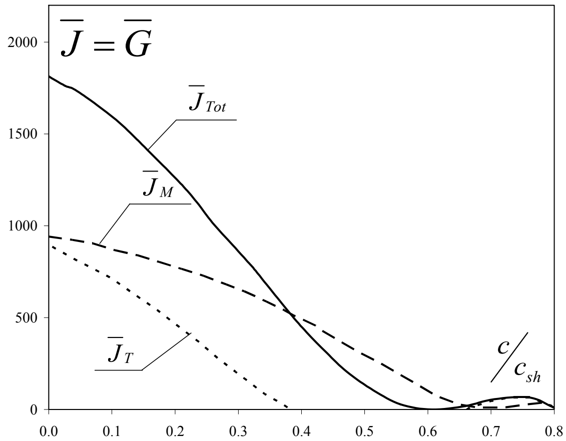

Fig.7: Mode I loading condition: dimensionless contributions in the J-integral expression.

The convergence of the ERR in the penalty procedure is investigated in Fig.8, where a logarithmic scale (base 10) is used for the horizontal axis. The results are proposed for different speeds of the crack front for a pure mode I loading condition. Note that the ERR denotes a stable and convergent behavior as far as the penalty parameter approaches to relatively high values.

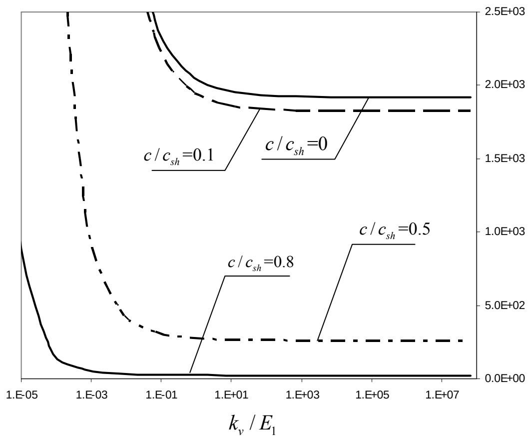

Fig.8 Convergence of the ERRs in the penalty procedure.

Sensitivity analyses have been developed with respect to the end loading split configuration (ELS), which is well known to produce delamination phenomena in pure mode II. The relationship between normalized ERR and speed of the crack front is reported in Fig.9, for different value of the initial delaminated length. Analogously to Mode I case, the ERR tends to decrease rapidly for increasing values of the speed of the crack front. The antisymmetrical loading condition avoids as a result interfacial compenetration effects along the crack faces.

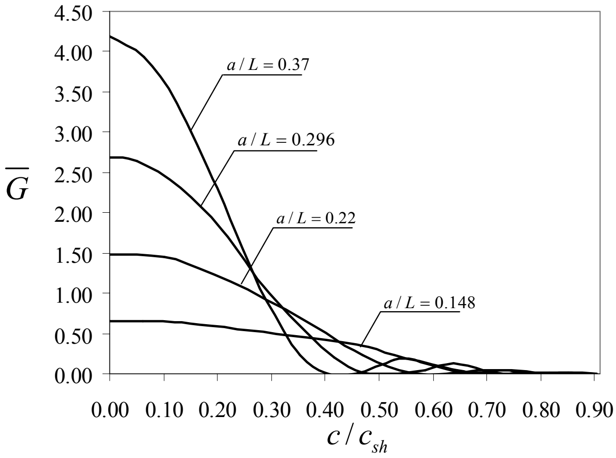

Fig.9 Mode II loading conditions: Dimensionless ERRs for different speed of the crack front.

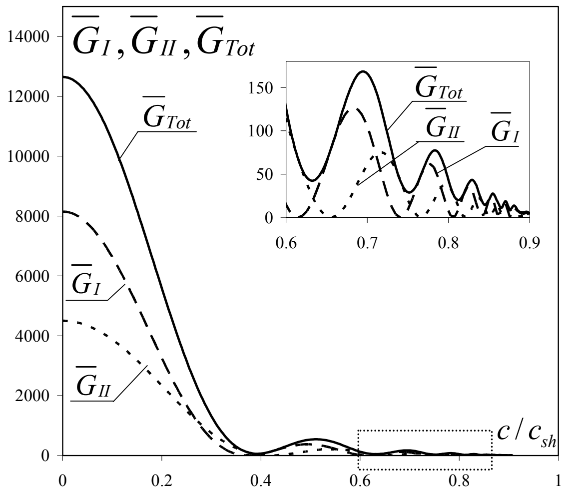

Fig.10 Mixed mode loading conditions: Dimensionless ERRs for different speed of the crack front.

The investigation has been proposed under mixed mode loading condition. In particular, In Fig.10, the relationships between ERR and speed of crack tip front is investigated. An oscillating behavior for the ERRs is noted and for increasing values of the speed both components approach to zero. It is worth noting that the speed of advance influences the mode mix ratio, which is strongly altered respect to the static case, i.e. c→0 (Fig.11).

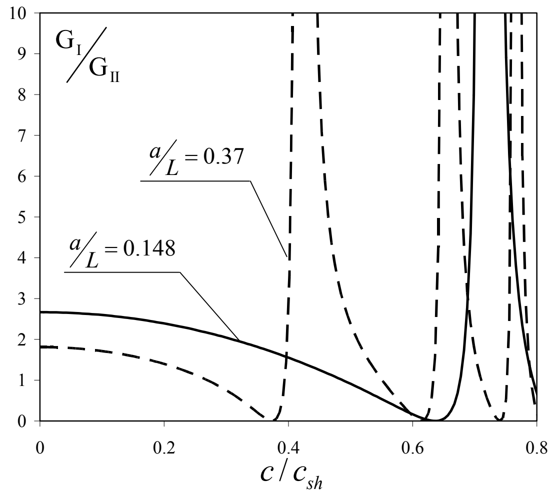

Fig.11: Mode mix ratio versus speed of the crack front.

Finally, in Figs.12-14 the dimensionless ERRs mode components over the maximum value are reported as a function of the crack extension. In particular, the comparisons denote that as far as the speed of the crack front is increased the ERR mode components are strongly reduced involving a more oscillating behavior during the delamination path.

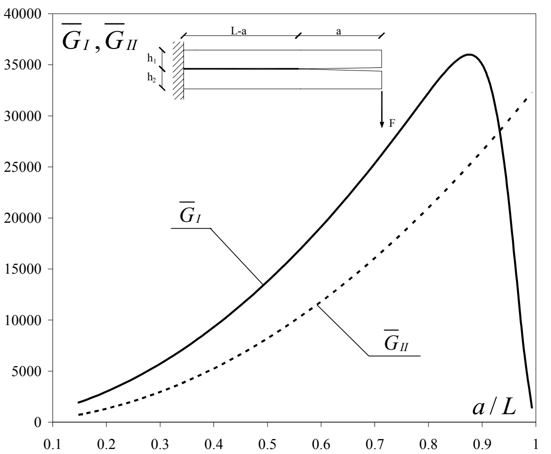

Fig.12 Dimensionless ERR mode components as a function of the crack extension: c/c_{sh}.=0, static case.

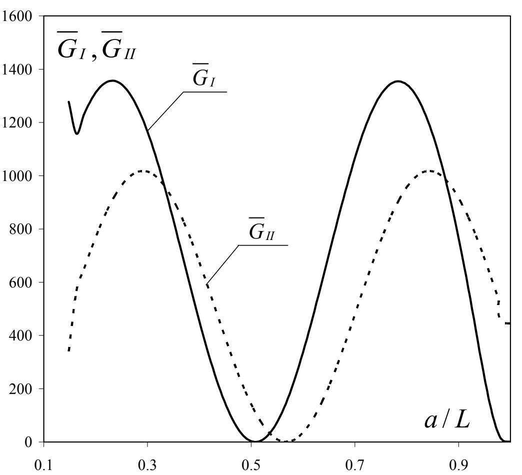

Fig.13 Dimensionless ERR mode components as a function of the crack extension: c/c_{sh}.=0.3

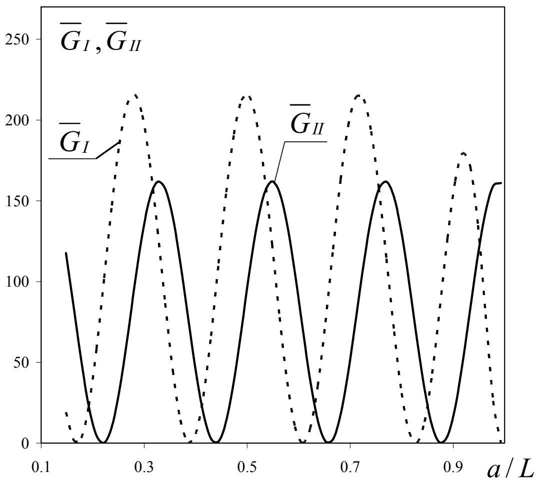

Fig.14 Dimensionless ERR mode components as a function of the crack extension: c/c_{sh}.=0.6

4 Conclusions

The dynamic behavior of composite structure is investigated in the context of the steady-state crack advance. The delamination phenomena in layered composite structures has been analyzed by using an interface model and appropriate kinematic formulation of the layers to achieve an accurate evaluation of the Energy Release Rate. The investigations have been proposed to point out the influence on the crack phenomena of the speed of the crack front and the inertial effects of the composite structures. The Energy release rate appears quite dependent from the speed of the crack. Comparisons between the interface model and classical delamination models are proposed to point out the influence of the shear effects on the Energy release rate evaluation. Moreover, a parametric study is proposed for pure mode I, mode II and mixed mode loading conditions to investigate the influence on the dynamic energy release rate and the corresponding mode partition of the crack growth speed and the inertial contributions of the laminate structure.

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