MULTIFORMALISM MBS MODELS FOR AN HIL APPLICATION IN VEHICLE DYNAMICS

Dr.-Ing. Martin Hahn[1], Dr.-Ing. Stephanie Toepper[2]

[1]iXtronics GmbH,
Technologiepark 11, 33100 Paderborn, Germany
[2]Bulachweg 14/5, 71229 Leonberg, Germany

martin.hahn@ixtronics.com (Martin Hahn)

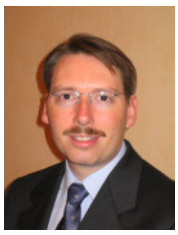
Abstract

In automotive engineering detailed multibody system (MBS) models are increasingly being employed. For the modelling process, various tools are available that typically generate the equations in the shape of differential-algebraic ones.
This sort of equations is not suitable for HiL applications, e.g., on test beds of vehicle components, because an iterative solution of the equations cannot comply with hard real-time conditions. For an alternative, there are multibody system formalisms that generate the equations on the basis of the nonlinear state-space representation. These equations can be solved at a fixed stepwidth; thus we can make sure that the time it takes to evaluate the differential equations is less than the sampling rate of the HiL system.
In this paper we will present and explain three different MBS formalisms that have different advantages and drawbacks – especially with regard to the complexity of the generated equations – and discuss the typical applications that the individual formalisms are suited for in particular.
Detailed vehicle models comprise submodels having different characteristics that one particular MBS formalism is especially suited for.
One development system allowing to use all three MBS formalisms is CAMeL-View. It is an object-oriented tool for the model-based design of mechatronic systems that provides components for the multibody system dynamics and the information-flow-based representation of elements from control engineering. One of the strong points of CAMeL-View is the physical-topological modelling that is suitable especially for MBS systems. Internal representation on the basis of modular-hierarchical graphs has made the implementation of different MBS formalisms possible. However, for an entire system just one MBS formalism could be employed so far.
In this paper we will demonstrate the way all three MBS formalisms are employed in a complex MBS vehicle model and then discuss the advantages this procedure has to offer.

Keywords: MBS Models, MBS Formalisms, HiL Application, Vehicle Dynamics

Presenting Author's biography

Dr.-Ing. Martin Hahn. Born in 1965. Studies in Mechanical Engineering at the University of Paderborn. PhD in 1999. Founder and CEO (since 1998) of iXtronics GmbH, manufacturer of the CAMeL-View simulation environment.



1 Introduction

In automotive engineering detailed multibody system models are increasingly being employed. For the modelling, various tools are available on the market that typically generate the equations in the shape of differential-algebraic ones (Eq. (1), (2)).

$$\dot{x} = f_1(x,u) \quad (1)$$

$$0 = f_2(x,u) \quad (2)$$

This sort of equations is not suitable for HiL applications, e.g., on test beds of vehicle components, because hard real-time conditions cannot be met due to the iterative solution of the equations.

As alternatives, there are a number of multibody system formalisms that generate the equations on the basis of the nonlinear state-space representation (Eq. (3), (4)):

$$\dot{x} = f(x,u) \quad (3)$$

$$y = g(x,u) \quad (4)$$

These equations can be solved at a fixed stepwidth; thus we can make sure that the time it takes to evaluate the differential equations is less than the sampling rate of the HiL system.

2 MBS Formalisms

In this paragraph the following MBS formalisms will be presented and explained:

- dynamical couplings
- minimal coordinates
- recursive formulation

As the different formalisms have several advantages as well as drawbacks – especially with regard to the complexity of the generated equations – we will demonstrate the typical applications that the individual formalisms are suited for in particular.

Particularly vehicle models comprise partial models with different characteristics. For every one of these partial models one of the three MBS formalisms is especially suited. A development system allowing the use of all three MBS formalisms is CAMeL-View.

2.1 Dynamical Couplings

The formalism of the dynamical couplings represents all joints by spring and damper elements. Every body with 6 degrees of freedom (DOFs) is subject to the forces and torques resulting from the relative motions between the bodies. In order to suppress the motion, high spring rates and damping constants have to be chosen for the disabled DOFs ([1] - [4]).

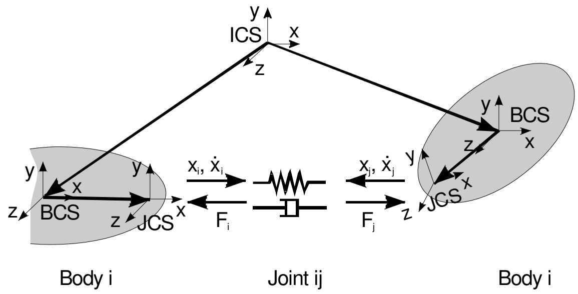


Fig. 1: Dynamical couplings represent all joints by spring and damper elements

Table 1. Advantages and drawbacks of the formalism with dynamical couplings

Advantages: • structure of the system is preserved • decoupling of the masses • representation of real components (rigid bodies) • physical implication of the couplings • suitability for parallelisation
Drawbacks: • high system order (*13n* differential equations and *z* force equations with n: number of rigid bodies z: number of joint constraints) • spring stiffnesses and damping constants to be adjusted in dependence of the system • high eigenfrequencies due to "stiff couplings" can lead to problems in the numerical integration

2.2 Minimal Coordinates

The formalism with minimal coordinates is a Lagrange formalism based on ideal kinematic linkings. On the basis of the Lagrange equation gradual insertion and the use of Jacobian matrices generate a system of differential equations comprising coordinates that are independent of one another (minimal form). The number of DOFs is the sum of the DOFs determined for the joints by the user ([5], [6]).

This formalism yields the system equations in a very concise representation:

$$M(q)\ddot{q} + D(q)\dot{q} + C(q)q + h(q,\dot{q}) = F(q) \quad (5)$$

with q: generalised coordinates,
$M(q)$: mass matrix,
$D(q)$: damping matrix,
$C(q)$: stiffness matrix,
$h(q,\dot{q})$: centrifugal and Coriolis forces
$F(q)$: generalized forces

Transformed into state-space representation Equation 5 becomes:

$$\dot{\begin{bmatrix} q \\ \dot{q} \end{bmatrix}} = \begin{bmatrix} \dot{q} \\ M^{-1} \cdot (F - D \cdot \dot{q} - C \cdot q - h) \end{bmatrix} \tag{6}$$

Table 2. Advantages and drawbacks of the minimal-coordinate formalism

Advantages:

- minimal order of the system:
  f = 6 n – z
  with n: number of rigid bodies
  z: number of joint constraints
- fast numerical evaluation
- system equations can be used for the control

Drawbacks:

- suited only for systems with a tree structure
- structure of the system is no longer discernible
- large systems yield extended and complicated equations
- computational costs increase cubically with the number of DOFs

2.3 Recursive Formulation

In the case of a recursive formulation of the equation of motion, the first recursion consists of computing the kinematic variables of the body from those of the preceding body and the relative motion between the two bodies. In the subsequent recursion the masses and constraint forces are determined before the relative accelerations are computed in the third recursion ([7] - [14]).

Table 3. Advantages and drawbacks of the formalism of the recursive formulation

Advantages:

- computational costs increase linearly with the number of DOFs
- structure of the system is preserved
- efficient numerical processing
- suitable for parallelisation only to a limited extent (parallel branches of a tree structure)
- open and closed systems

Drawbacks:

- equations of motion are available only as extended, highly intricate partial equations
- numerical processing with many intermediate computations

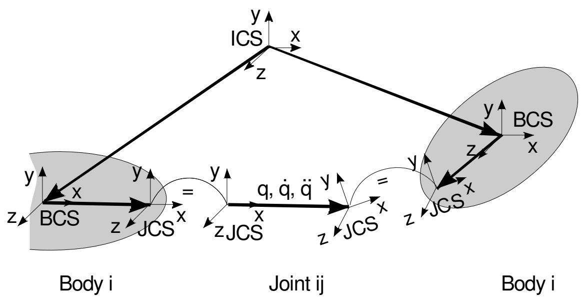


Fig. 2: Kinematics of the relative motion between the two bodies

2.4 Computational Benefits of the Formalisms

The preceding tables show all of the 3 derivation formalisms having their particular advantages and drawbacks. This leads us to the pros and cons of a certain generation method that depend on the respective physical model.

If there are elasto-kinematic linking elements in a physical model, it will be possible to represent their physical properties by means of dynamical couplings. Here small deformations in the joints are tolerated and the resulting forces act on the components. These elasto-kinematic connecting elements can e.g. be found in rubber-metal bearings in wheel suspensions.

If it is possible to study the system behaviour without taking into account elasto-kinematics, rigid-body models with idealised kinematic couplings will be made up. The equations describing the system behaviour are generated in minimal coordinates or in recursive formulation.

Generation of the system equations in minimal coordinates yields a very concise representation (see Equation 5). Here we have the mass-, stiffness-, and damping matrices that can be analysed. Transformation of this standard representation form of mechanics (Equation 5) into nonlinear state-space representation (Equations 3 and 4) requires inversion of the mass matrix (Equation 6). As the expense for a symbolic computation of the inverse mass matrix increases cubically with the latter's dimension the „minimal formalism“ can only be employed with a limited number of DOFs that depends on the system structure. In the case of a simple chain (cf. example 1) where 2 masses each are connected via a rotational DOF, the limit is about 20.

Moreover, for a mathematical model in minimal coordinates it is the centrifugal and Coriolis terms that determine its size and evaluation time. Disregard of these terms can considerably enlarge the field of application of the „minimal coordinates“ and minimize the time necessary for evaluating the system equations. In order to check if this simplification is admissible one has to perform a comparative analysis. For this purpose – especially with long subchains – a mathematical model in recursive formulation is of use. If, in a physical model, closed structures occur that have no elasto-kinematic connecting elements where the kinematics can be split with the help of a dynamical coupling, one will generate the mathematical model in recursive formulation. The latter determines the kinematical coupling equations (constraints) and derives the constraint forces in effect. As computation of the masses and constraint forces requires also an inversion of a matrix it is equally restricted and only possible with closed structures that have to keep 20 constraints at most within a closed substructure with several nested loops.

The two advantages the recursive formulation has to offer are the computing effort that increases linearly with the numer of DOFs and the possibility to compute closed-loop structures. This is why it is especially suited for chains and chain drives.

Of particular importance for stable simulations in HiL applications are evaluation time and the maximum possible stepwidth that vary with the respective generation formalism used resp. the mathematical description form. Thus, with the same modelling depth, i.e., without loss of the physical properties mapped, one or the other formalism may be better suited for the respective simulation time.

In the following, 2 simple examples which were simulated on a dSPACE DS1006 processor board in the CAMeL-View TestRig environment, are used to compare the time it takes to evaluate the system equations of chains of a certain length.

2.4.1 Example 1: Simple Chain

The chain links are connected by joints with a rotatory DOF. Their coupling points are not twisted and are in line with the center of gravity.

Fig. 3 shows that up to a chain length made up of 12 rigid bodies (equivalent to 12 DOFs) using minimal coordinates and taking into account the Coriolis and centrifugal terms yield smaller evaluation times and smaller code size. Only from a chain length of 13 on does the model in recursive formulation display better behaviour.

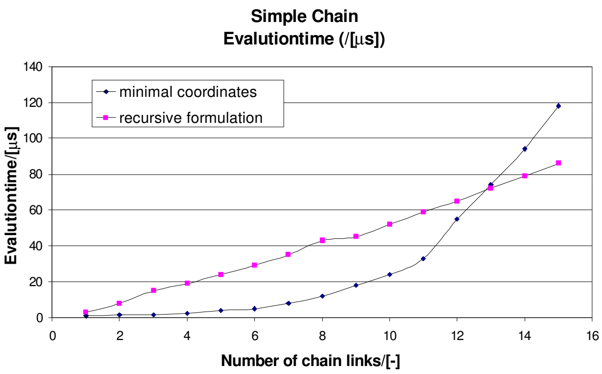


Fig. 3: Evaluation time of the simple-chain model generated by “minimal coordinates” and “recursive formulation”

2.4.2 Example 2: Chain with Cardan Joints

The chain links are connected by joints with 2 rotatory DOFs. The coupling points of the chain links are not twisted and lie in line with the center of gravity. The 2 DOFs formulated for each joint make the describing equations considerably longer. When formulated in minimal coordinates they increase in size disproportionately compared to the recursive formulation. Fig. 5 shows that with this chain even in the case of 5 chain links (10 DOFs) the recursive formulation is advantageous as compared to that in minimal coordinates.

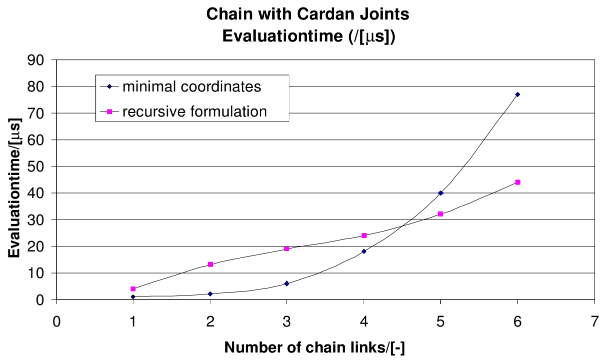


Fig. 4: Evaluation time of the chain with cardan joints generated by “minimal coordinates” and “recursive formulation”

3 CAMeL-View

CAMeL-View TestRig ([15] - [17]) is a consistent object-oriented toolchain for the design of mechatronic systems that supports the model-based design of mechatronic products in the model-, testbed-, and prototype phases.

CAMeL-View allows study of different solutions variants on the basis of the simplified structure of the assembly.

A procedure of this kind is not sufficiently supported by other design tools as these are usually limited to their respective domain, e.g., MBS simulation. Yet, mechatronic systems comprise also elements from hydraulics, control engineering, and software engineering whose dynamic behaviour has to be taken into account from the outset.

Additionally, a holistic design – beyond mere simulation – requires further analysis and synthesis methods, such as frequency response or optimization of parameter vectors.

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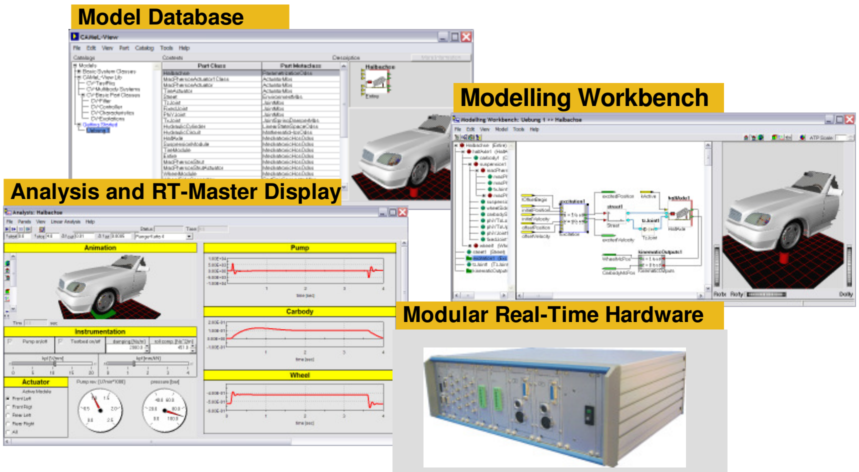

Fig. 5: CAMeL-View TestRig design environment

For supporting the individual development tasks in the different design phases, CAMeL-View TestRig offers the following components:

- CAMeL-View basic development environment, consisting of model-database browser, model editor (composition editor), simulator (analysis- and RT-master display), and code generator
- toolboxes for MBS- and hydraulic elements, instrumentation components, and the animation of mechatronic systems
- TestRig hardware with the CAMeL-View TestRig toolboxes for HiL- and prototype applications

The works of the CAMeL-View users focus on a continuous adaptation, improvement, and extension of system models via downstream analysis and synthesis steps. Only optimal support by suitable models and modelling methods allows consistent application in all three phases of the development process.

The core element of CAMeL-View is an object-oriented modelling [18] of physical-topological systems which will be detailed in the following. To learn this methods tutorials and prepared models are available online ([19], [20]).

3.1 Object-oriented Modelling of Mechatronic Systems with CAMeL-View

The CAMeL-View modelling environment allows an engineering-related description of mechatronic systems and for this purpose offers a comfortable interactive procedure. CAMeL-View allows the build-up of complex mechatronic systems and supports an export of the models for subsequent use with Matlab/Simulink or with HiL environments (e.g., iXtronics, dSPACE, etc.) [21].

CAMeL-View modelling boasts the following properties which are indispensable for successful use in a model-based development of mechatronic systems:

- For the modelling of mechatronic systems CAMeL-View provides discipline-specific description elements, i.e., components from mechanics, hydraulics, control engineering, etc., can be used for the modelling. The centre of the modelling is the object-oriented model-description language Objective-DSS [1]:

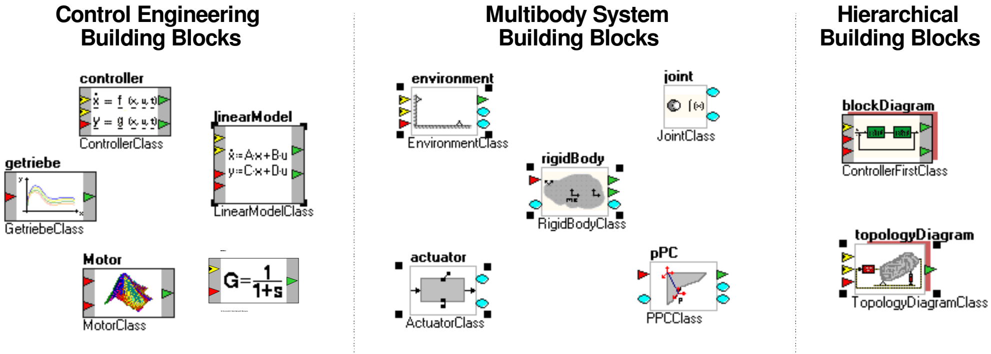

Fig. 6. CAMeL-View description elements for MBS dynamics, control engineering, and for hierarchical elements (components)

- In order to allow modelling of the structure and the properties of mechanical subsystems, CAMeL-View provides elements for the description of the multibody system dynamics (rigid bodies, joints, actuators, etc.).

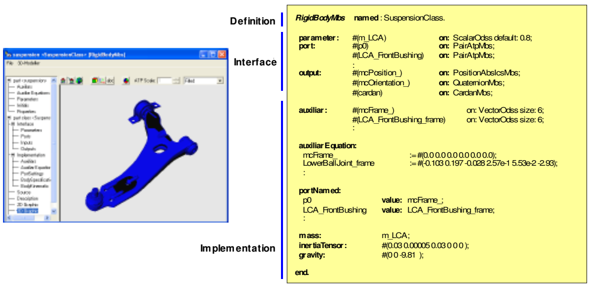

Fig. 7: MBS description element of the lateral control arm of a wheel suspension

- As regards control engineering, CAMeL-View offers description elements on the basis of nonlinear and linearized state-space representation. All variables can also be formulated as vectors and matrices by means of vector- and matrix functions.

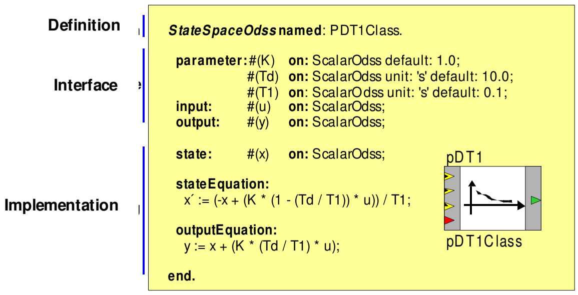

Fig. 8: State-space description element of a SISO controller

- Beyond the modelling phase, the testbed- and the prototype phases are also supported by CAMeL-View TestRig which provides components for the connection to real-time hardware:

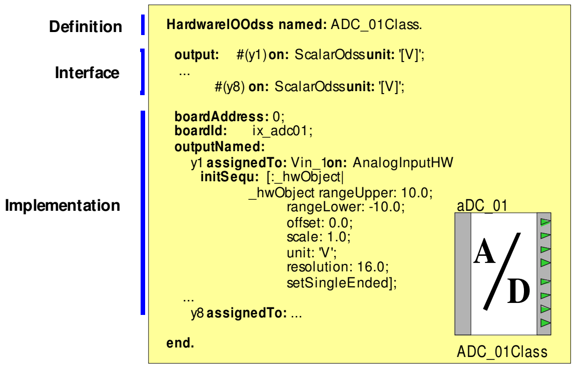

Fig. 9: Example of the description elements of an A/D converter

- The modelling of hierarchical systems (aggregates resp. components) in CAMeL-View is carried out in the shape of extended block diagrams (topology diagrams) which provide not only signal flows but also physical couplings. In addition to informational blocks and couplings, topology diagrams in CAMeL-View can also comprise components from for instance mechanics and hydraulics that can be connected by means of physical (undirected) couplings.

3.2 Deriving the Mathematical Model

Due to the entirely object-oriented approach the CAMeL-View code generator is able to derive the mathematical equations describing the dynamical behaviour of the system and to make them available in the shape of models that are suitable for simulation.

To do so, the code generator sets up equations for continuous systems on the basis of the nonlinear state-space representation. For an automatical generation of the equations describing the system on the basis of the topological interlinking of components, the MBS formalisms are supported, which are described in more detail in the previous Chapter. Starting from the system structure, these MBS formalisms generate the describing differential equations.

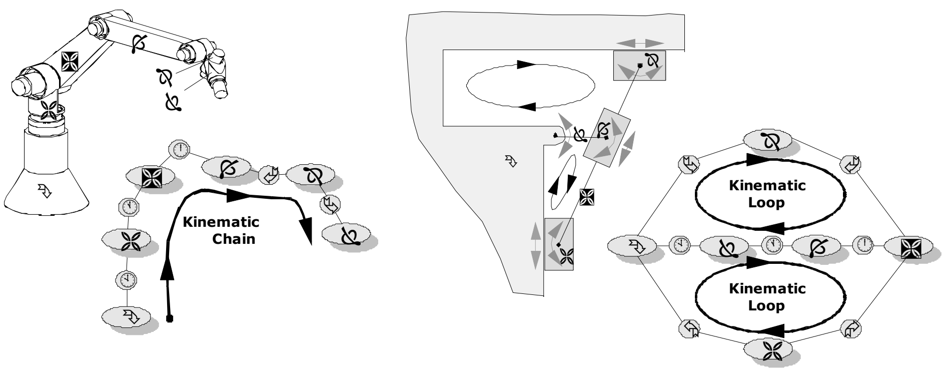

Fig. 10: Topological basic structures of MBS systems

One of the strong points of CAMeL-View is the physical-topological modelling that is presented especially with MBS systems. Internal representation on the basis of modular-hierarchical graphs has made the implementation of different MBS formalisms possible. However, for an entire system just one MBS formalism could be employed so far. An extension has made it possible to apply all three MBS formalisms to one model; thus for every partial model the most appropriate formalism can be employed.

4 Example: MBS Vehicle-Dynamics Model

This section will show how all three MBS formalisms can be employed with a complex MBS vehicle model consisting of the following components:

- vehicle
    - vehicle body
    - 4 independent wheel suspensions as double wishbone axle with wheel carrier, rims, and spring actuators, the tyres being described by an easy-to-use tyre model
    - drive train for rear-wheel drive with switchable front-wheel drive, consisting of clutch, gearbox, differentials, cardan- and drive shafts
    - braking and steering input
- excitation model for the specification of different driving manoeuvres, e.g., step-, sine- or torsion excitation in vertical direction as well as load change, sudden steering input or lane change
- evaluation model, yielding roll-, float-, and pitch angles as well as lateral and longitudinal accelerations, among other things

Fig. 11, 13, and 15 show the 3-D representation of the entire vehicle

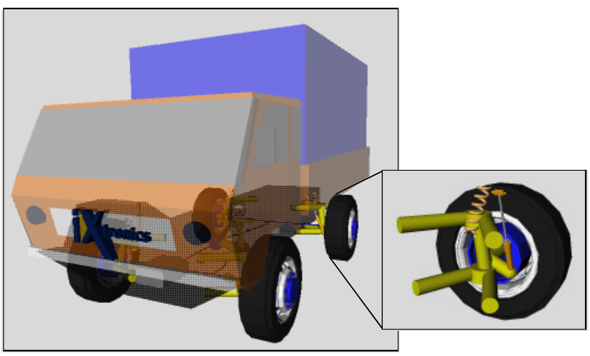

Fig. 11. 3-D representation of the overall model and one wheel suspension

All components are mapped in the computer and interlinked to make up a hierarchical entire system. Fig. 12 displays the structure and the complexity of the mechanical part of the entire system.

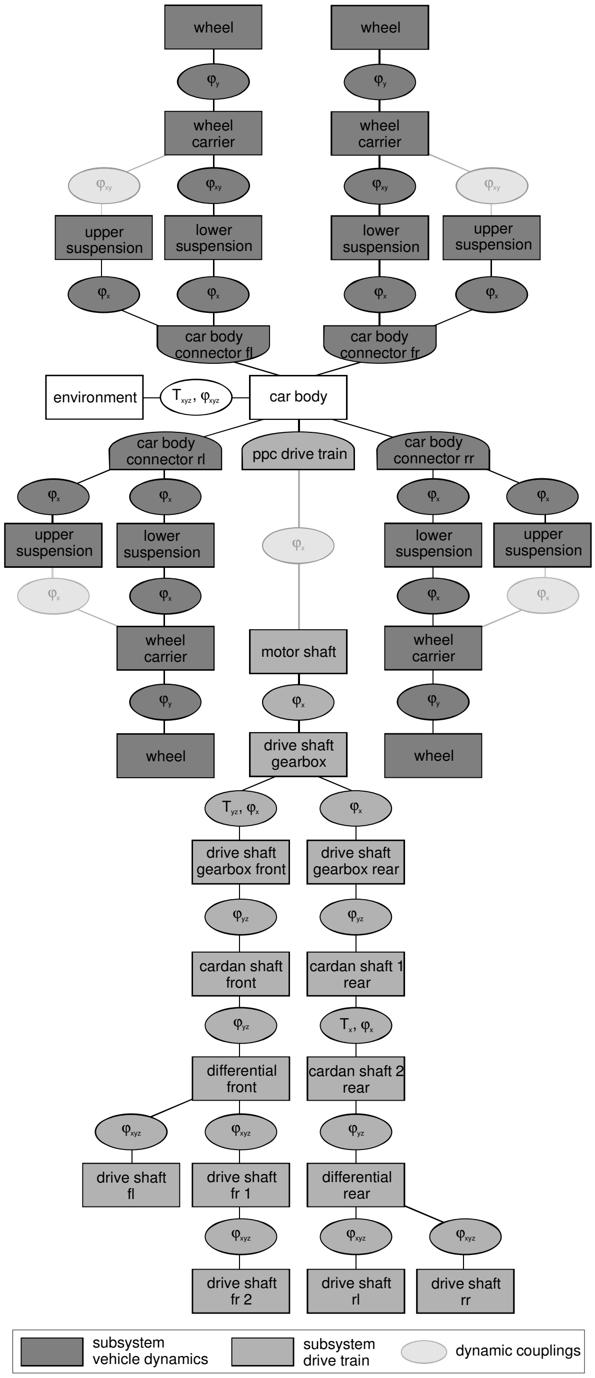

Fig. 12. Structure of the mechanical subsystem

4.1 Subsystem „Vehicle Dynamics“

The subsystem „Vehicle Dynamics“ consists of the car body that the 4 wheel suspensions are attached to.

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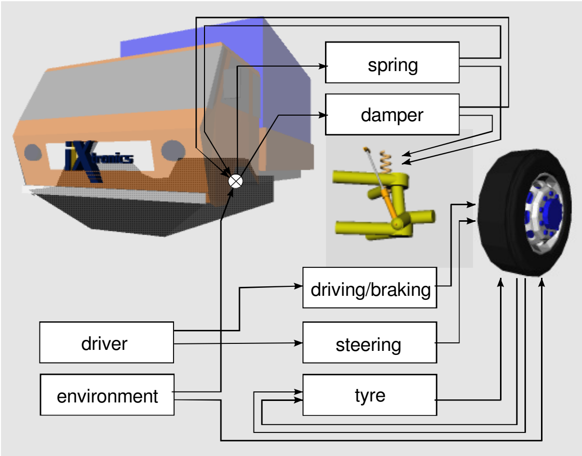

Fig. 13. Modelled effects, displayed by a front-wheel suspension

The rear wheel suspensions consist each of an upper and a lower lateral control arm that are attached to the car body by means of swivel joints, with the wheel carrier being attached to the upper and the lower lateral control arm also by means of swivel joints.

The front wheel suspensions are built up the same way, with the wheel carrier being attached to the upper and the lower lateral control arm by means of cardan joints with 2 rotational DOFs which make steering possible.

The rim is connected to the wheel carrier by a pivot bearing. The tyres are described by an easy-to-use tyre model.

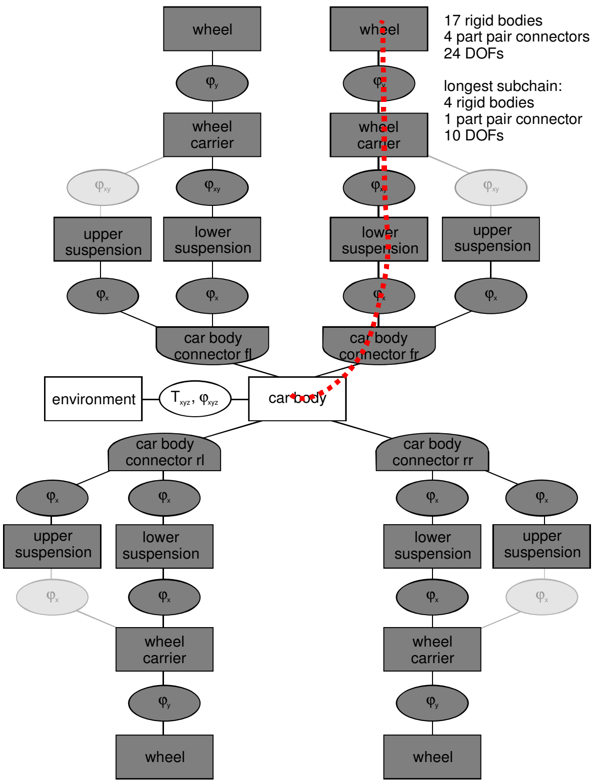

Fig. 14. Structure of the vehicle-dynamics model

The mechanical subsystem consists of 17 rigid bodies and 24 DOFs. In it the joints between the upper lateral control arm and the wheel carrier are represented as elasto-kinematic connecting elements by dynamical couplings so that there are no kinematic loops but a broad tree structure whose longest chains comprise 4 rigid bodies and 10 DOFs. For this the description of the mathematical model in minimal coordinates is suited best.

4.2 Subsystem „Drive Train“

The drive train consists of the engine shaft that can be disconnected from the gear shaft by a clutch. The revolutions of the drive gear shaft are transformed according to the chosen gear and transmitted to the rear driven gear shaft or – if the front-wheel drive is switched on – to the one in front.

The rear-wheel drive is effected by means of a cardan shaft that is split in two for the purpose of lengh compensation, a differential, and the two drive shafts.

The front-wheel drive consists of a simple cardan shaft, a differential, and the drive shafts, with the right-hand-side drive shaft being again split in two because the front cardan shaft is not located in the center of the vehicle.

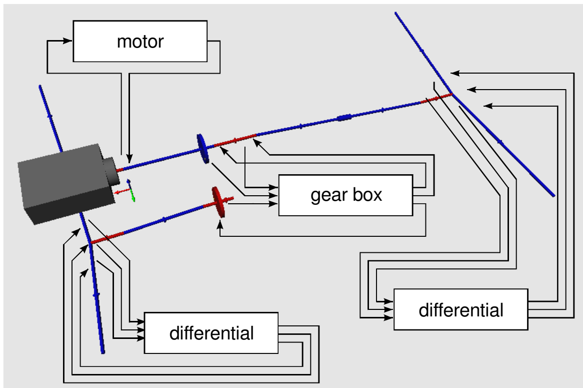

Fig. 15. 3-D representation of the drive train

This subsystem (fig.16) has 14 rigid bodies with a total of 30 DOFs. The lean structure of this system has few ramifications but long branches: the longest subchain consists of 7 rigid bodies with 12 DOFs. Due to the length of this subchain a mathematical description in recursive shape yields the smallest code as well as the shortest time necessary to evaluate the system equations.

4.3 Entire System

Using dynamic couplings, one can divide the mechanical entire system into two independent, kinematically coupled subsystems: the chassis and the drive train. For each of these subsystems the describing equations will be generated with the MBS formalism that is especially suited for it (see above).

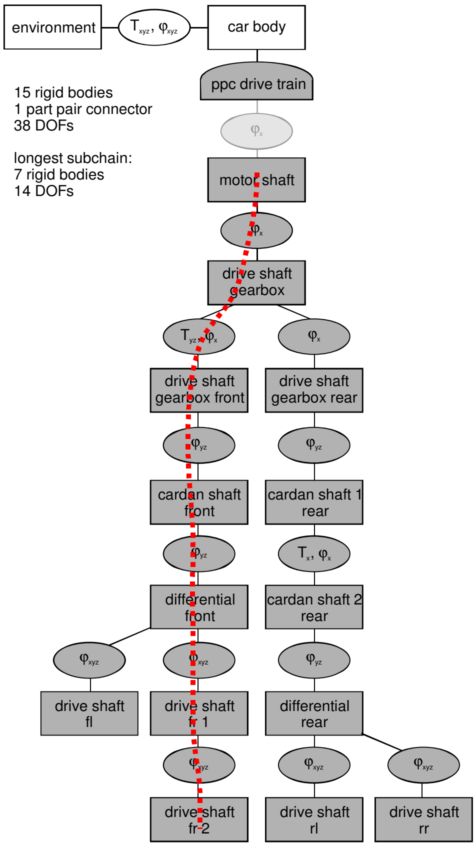

Fig. 16. Structure of the drive train

The result will be an equation system that preserves the structure of the entire system. For every subsystem there is an appropriate description that is independent of the other subsystems. The subsystems will be interlinked by means of a coupling of physical input and output variables. See the following figure for simulation results:

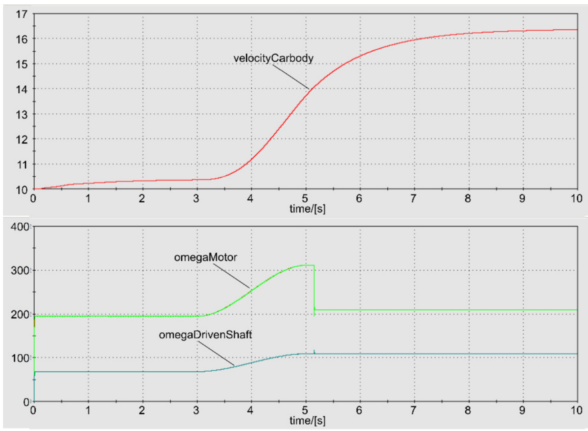

Fig. 17. Simulation results of multiformalism MBS "vehicle-dynamics model"

5 Conclusion

In order to use detailed multibody systems with HiL applications the equations of motion of the MBSs have to be available in nonlinear state-space representation and evaluable in real time. For generating these equations, there are different well-known derivation formalisms having various advantages and drawbacks that will yield different mathematical descriptions depending on the structure of the respective MBS. With the help of the CAMeL-View TestRig environment that supports the model-based design of mechatronic systems it is possible to apply a derivation formalism with every mechanical subsystem of a complex entire system, this formalism being particularly suited for the respective subsystem. The example of the MBS "Vehicle-dynamics Model" shows that for the mechanical subsystem "Vehicle Dynamics" a mathematical model in minimal coordinates yields the smallest code that can be evaluated in the shortest time while with the long chains of the drive train the recursive formulation is the most appropriate description. The equations of the entire system model are now generated by processing the two mechanical subsystems with the formalism that is best suited for each of them. The result is a mathematical model of the entire system that is optimized in view of HiL applications as regards code size and evaluation time.

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