MATHEMATICAL MODELING OF CLOSED-CIRCUIT HYDROSTATIC TRANSMISSION

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Abstract

Besides the efficiency, the control ability and the fast response of a hydrostatic transmission are very important. In this regard, a DC servomotor can be applied as an actuator of the variable displacement mechanism of hydrostatic units. In this way, fast responses and favorable control ability can be accomplished. This paper gives a mathematical model of the closed-circuit hydrostatic transmission, with the DC servomotor actuated variable displacement swash-plate pump and a fixed displacement motor. The dynamics response of the swash-plate angle, and consequently of the pump flow, mainly depends on torques acting on the swash-plate. Relatively large torque is required to move the swash-plate quickly to a new position. Here the second order model of the swash-plate dynamics is given. The total model of the closed-circuit hydrostatic transmission is given in a form suitable for the experimental parameterization, since most of the required parameters are unavailable at producers' data, and they should be verified experimentally. The presented model will be experimentally validated on the experimental setup of the hydrostatic transmission, which is designed at our Laboratory. The model will provide a necessary basis for the development and application of advanced control methods.

Keywords: Hydraulic actuator, Nonlinear model, Servomotor

Presenting Author's biography

Joško Petrić received the B.S., M.S. and Ph.D. degrees, all in mechanical engineering, from the University of Zagreb, Zagreb, Croatia, in 1987, 1991, and 1994, respectively. He is currently an Associate Professor in the Faculty of Mechanical Engineering and Naval Architecture at the University of Zagreb. His areas of teaching include automatic control, mechatronics, robotics, and fluid power. His current research interests include modeling and control of automotive systems. Prof. Petrić is a member of the ASME.

1 Introduction

There are two basic types of hydraulic systems regarding their control. These are resistance-controlled system and displacement-controlled system. In the first case, motion of the cylinder, or possibly hydraulic motor, is controlled by resisting the flow of fluid with a valve. In the second case, varying the supply of fluid controls motion. The hydrostatic transmissions are the displacement-controlled system. Due to the pressure "clamping" of the actuator, and a smaller volume of fluid, the resistance-controlled system has a faster response and a better accuracy then the comparable displacement-controlled system, where only one side of the actuator is pressurized. Furthermore, the damping coefficient of the resistance-controlled system is higher owing to throttle losses and higher internal leakage. Therefore, the displacement-controlled systems are more disposed towards oscillating behavior. However, for the same reason their efficiency is generally far better. The efficiency of the displacement-controlled system can approach, or even exceed 0.85, in relatively large operational area, while the efficiency of a resistance-controlled hydraulic system is good only at full-load operations.

Due to the actual requirements of energy savings, which is particularly emphasized in the mobile applications, the hydrostatic transmissions become more and more interesting. The recent examples of their applications are technologically advanced tractors (Fendt, Claas, Steyr, John Deere), hybrid hydraulic vehicles that operate in stop-and-go regime (urban delivery vehicles, garbage vehicles), all-axle drive at heavy trucks (MAN-HydroDrive), all-terrain motorbikes (Honda Rubicon 500ccm), and torque-vectoring differentials for passenger cars (Folsom Technologies).

Besides the efficiency, the control ability and the fast response of the hydrostatic transmission are very important. Therefore, some modern means of actuation of the variable displacement pumps are proposed.

In this paper, the DC servomotor is considered as the actuator of the variable displacement mechanism of the hydrostatic pump. The mathematical model of the closed-circuit hydrostatic transmission, with the DC servomotor actuated variable displacement swash-plate pump, is given in the paper. The nonlinear model is presented in the block-diagram, and it is suitable for the Simulink analysis. The model is in a form suitable for the experimental parameterization, since most of the required parameters are not provided at the available producers' data. They should be verified experimentally.

2 Mathematical modeling

2.1 Functional description

The simplified scheme of a pressure compensated variable-displacement swash-plate pump is given in Fig. 1 (note: the drive shaft usually goes through the swash-plate, but here is different for the sake of simplicity). Possible control devices (hydraulic or DC) for the swash-plate are added therein. The dynamics response of the swash-plate angle, and consequently of the pump flow, mainly depends on torques acting on the swash-plate. Relatively large torque is required to move the swash-plate quickly to a new position. In hydraulic adjusting mechanisms, that torque is controlled by the system pressure and area of the control piston. Due to the limitations of the pump construction and other possible issues (such as the pump stability), the applied torque on the swash-plate is limited, thus limiting the response time. Another important issue regarding the overall transmission dynamics is that the effect of load inertia usually causes an overshoot in the pressure response (hydrostatic systems usually have small damping coefficient), hence the stability can deteriorate.

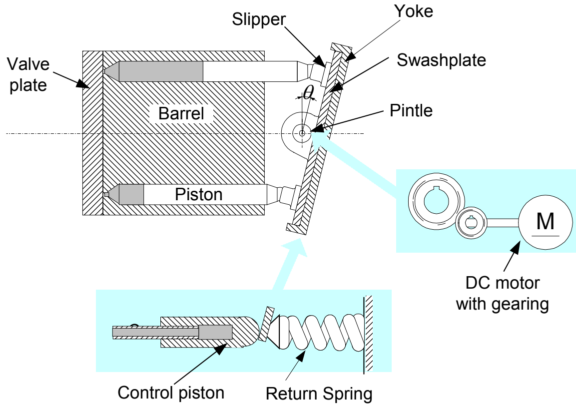

Fig. 1 Swash-plate axial piston pump

2.2 Model of hydrostatic transmission

A lumped model of the hydrostatic transmission with the variable displacement pump and the fixed displacement motor is given in [1] or [2]. Several assumptions had been made therein, but none of them are restrictive. That is a rather elementary mathematical model of a hydrostatic transmission, which is sufficient for majority of analysis. The stability range of linearized model for a hydrostatic transmission, in which the swash-plate dynamics is approximated as a first order system, is given in [3]. That model can be a good basis for system parameter estimation. Several important parameters (like effective bulk modulus, leakage coefficient, torque efficiency, viscosity factor, etc) usually have been approximated as constant, although they can be

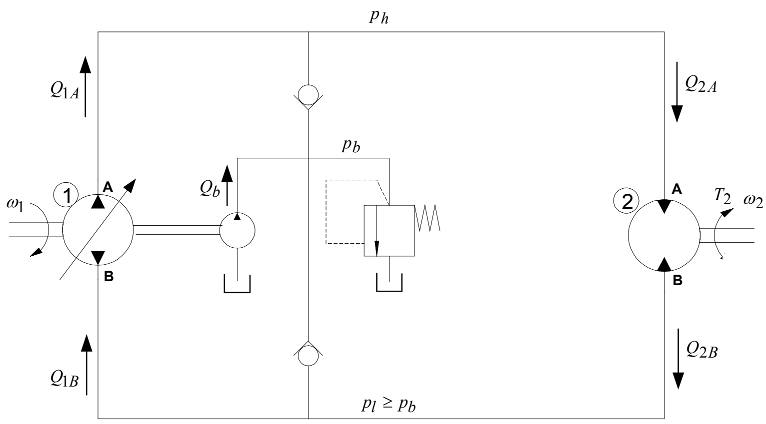

Fig. 2 Schema of hydrostatic transmission
(1 – pump; 2 – motor; A and B - ports;
h – high-pressure side; l – low-pressure side; b - boost pump variables)

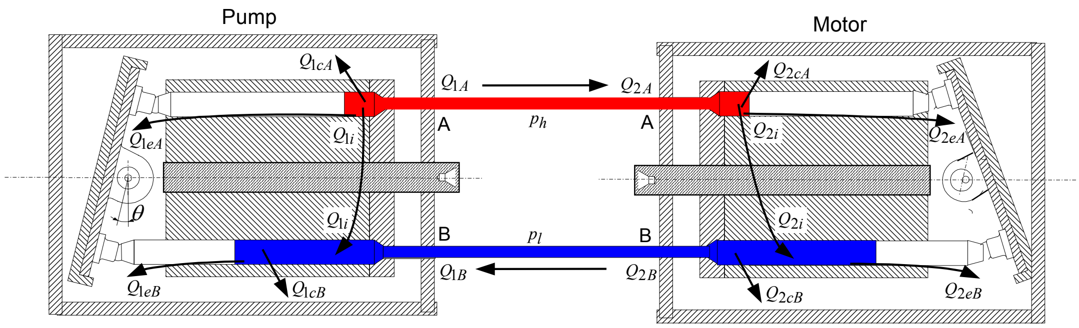

Fig. 3. Pump and motor flows

different at different operating points. Modeling and identification of hydrostatic transmission hardware-in-the-loop system is given in [4].

A mathematical model of a hydrostatic transmission with a variable displacement pump and a fixed displacement motor is presented below. The ongoing research at the University will verify the model experimentally. The simplified schema of the transmission with associated variables is shown in Fig. 2.

The continuity equations yield:

$$Q_{1A} - Q_{2A} = \frac{V_h}{\beta} \cdot \frac{dp_h}{dt} \qquad (1)$$

$$Q_{2B} - Q_{1B} + Q_b = \frac{V_l}{\beta} \cdot \frac{dp_l}{dt} \qquad (2)$$

Where are:
Q – flow [m^3/s]
β – bulk modulus of system (include fluid, entrapped air and piping compliance) [Pa]
V – volumes of chambers where oil is under pressure (h for high-pressure side and l for low-pressure side; V includes one side of pump and motor, connecting line and additional volumes associated with check valves and pressure relief valves) [m^3]
p – pressure [Pa] (p_h high pressure, p_l low pressure and p_b boost pump pressure).

Input and output flows of a pump or a motor are given by following terms in Table 1.

Tab. 1 Input and output flows of pump and motor

	Pump
Input	$Q_{1B} = Q_{1\_ideal} - Q_{1i} + Q_{1eB} + Q_{1cB}$
Output	$Q_{1A} = Q_{1\_ideal} - Q_{1i} - Q_{1eA} - Q_{1cA}$
	Motor
Input	$Q_{2A} = Q_{2\_ideal} + Q_{2i} + Q_{2eA} + Q_{2cA}$
Output	$Q_{2B} = Q_{2\_ideal} + Q_{2i} - Q_{2eB} - Q_{2cB}$

.

Output flow of a pump is an ideal (theoretical) flow reduced by flow losses due to internal leakage (denoted by index i) and external leakage (denoted by index e)[1] and so-called compressibility flow losses

[1] The external leakage, as well as the compressibility flow losses, is different at different sides (ports A or B) of pump or motor due to different pressure (high pressure and low pressure side). On the other hand, the internal leakage is the same, but it has an opposite direction. Namely, what goes out from the high-pressure chambers goes into the low-pressure chambers.

(denoted by index c). The compressibility flow losses express that the actual flow rate in each control volume is modified when pressurized. Note that the compressibility flow losses appear in equations only in [1] and elsewhere are omitted. Motor flows have different orientations. The pump and motor flows are shown in Fig. 3.

It is possible to extract all the losses, and express them as the mean flow equations:

$$\frac{Q_{1i}+Q_{2i}}{2}=\frac{p_h-p_l}{R_i} \tag{3}$$

$$\frac{Q_{1eA}+Q_{2eA}+Q_{1cA}+Q_{2cA}}{2}=\frac{p_h}{R_e} \tag{4}$$

$$\frac{Q_{1eB}+Q_{2eB}+Q_{1cB}+Q_{2cB}}{2}=\frac{p_l}{R_e} \tag{5}$$

These flows are expressed now via "resistances", one of internal losses flow R_i ; and the other of external losses and compressibility losses flow R_e. Unit of the resistances are [Pa s/m^3]. Otherwise flow losses can be given as leakage coefficients, which are the inverse of resistances [m^3/Pa s]. Internal losses in (3) are caused by the pressure difference between high and low pressure lines, while the external losses are caused by high (in (4)) or low pressure (in (5)) and the pressure in the chassis, which is 0. For laminar flows, which de facto exist in narrow passages, such a linear expression is valid. The resistance dependency on rotational speed is relatively minor, so it is usually neglected. That is discussed in [1]. The resistances should be estimated experimentally. Somewhere both resistances are considered as one, which can be more convenient for estimation (for example in [2], [5] or [6]).

Note that the flow losses have the most important influence on system's damping factor, and omitting them in simulations leads to unrealistically large oscillations.

The ideal flow of an axial piston unit (pump or motor) is caused purely by piston displacement:

$$Q_{\mathrm{i}\_ideal}=\omega_{\mathrm{i}}\cdot D_{\mathrm{i}} \tag{6}$$

Where are:
ω_{i} – unit rotational speed [rad/s]
D_{i} – unit displacement [m^3/rad]

In the case of variable displacement pump, the flow rate is not proportional to the swash-plate angle ([6]). The relation for ideal flow can be given by following:

$$Q_{1\_ideal}=\omega_1\, N_1\, A_1\, R_1\, \tan\theta\,/\,\pi \tag{7}$$

Due to small possible swash-plate angle (max. about 20º), the linear approximation is quite correct:

$$Q_{1\_approx}=\omega_1\cdot D_{1\_\max}\cdot\theta/\theta_{\max} \tag{8}$$

where:
$D_{i\_\max}$ – maximal unit displacement (at max swashplate angle) [m^3/rad]
θ – angle of the swash-plate [rad]
$\theta_{\max}$ – maximal angle of the swash-plate [rad]
N_1 - number of pistons
A_1 - area of the piston [m^2]
R_1 - radius of the piston pitch [m]

Inserting equation (3)-(8) into (1) and (2) the flow equations become:

$$\omega_1\cdot D_{1\_\max}\cdot\theta/\theta_{\max}=\omega_2\cdot D_2+\frac{2(p_h-p_l)}{R_i}+\frac{2p_h}{R_e}+\frac{V_h}{\beta}\frac{dp_h}{dt} \tag{9}$$

$$\omega_1\cdot D_{1\_\max}\cdot\theta/\theta_{\max}=\omega_2\cdot D_2+\frac{2(p_h-p_l)}{R_i}-\frac{2p_l}{R_e}-\frac{V_l}{\beta}\frac{dp_l}{dt}+Q_b \tag{10}$$

If (9) and (10) equalize, it can be concluded that the flow from the boost pump has to replace the external leakages and flow missed due to compression:

$$Q_b=\frac{2(p_h+p_l)}{R_e}+\frac{V_h}{\beta}\frac{dp_h}{dt}+\frac{V_l}{\beta}\frac{dp_h}{dt} \tag{11}$$

The pressure in low-pressure line of the closed loop hydrostatic transmission, if system is properly designed[2], should not fall below the pressure of the boost pump pressure p_b. In that case, the low pressure in the system will be constant and will equal p_b. The pressure can rise above p_b if the load torque supports the motion of the hydraulic motor.

The torque equation of a hydraulic motor is obtained from the Newton's II law [1]:

$$(p_h-p_l)\cdot D_2=J_t\frac{d\omega_2}{dt}+B_2\cdot\omega_2+T_L \tag{12}$$

Where are:
J_t - total moment of inertia of motor and load (referred to the motor shaft) [kgm^2]
B_2 – viscous friction damping coefficient [Nms/rad]
T_L – load torque [Nm]

In eq. (12) just the viscous friction term is included. Coulomb friction can be included, as well, as some additional friction losses, which are complicated function of speed and pressure. That type of friction can be considered as a combination of hydrostatic, hydrodynamic and sticking losses, and can be important at low speeds [1]. Relatively low hydro-

[2] The boost pump should be large enough to replace all necessary fluid

mechanical efficiency at low speeds is a consequence of that kind of losses.
Block-diagram of a hydrostatic transmission is given later in Fig. 5 based on equations (9) and (12).

2.3 Swash-plate dynamics

An important part of the hydrostatic transmission dynamics is the dynamics of the control mechanism that adjusts the pump or motor displacement. That is often called the swash-plate dynamics. There are several papers that address the issue. A quite complex generic model of the swash-plate dynamics is developed in [5]. Therein the model is also reduced to a second order system, which approximates swash-plate dynamics fairly well. Probably the first time the numerical solution for the torques acting on a swash-plate was derived in [7]. Improvement of that solution and experimental verification is given in [8]. The swash-plate dynamics were approximated as a first order system in [3]. In these studies a hydraulic piston with a hydraulic or electro-hydraulic control were applied to move the swash-plate. Application of a DC servomotor as a drive option for a swash-plate to improve its dynamics is considered in [6]. More flexible and more advanced control methods can be easier applied using such way of swash-plate control.

The mathematical model of a pump controlled by a DC motor is somewhat simpler, due to absence of terms associated with the hydraulic control. The forces acting on the swash-plate in the case of DC control according to [6] can be arranged as following:

$$T_{em} = J_{sw}\ddot{\theta} + T_f + T_r + T_p \tag{13}$$

where:
T_{em} - torque developed by the drive motor of the swash-plate
J_{sw} - moment of inertia (average) of swash-plate and yoke assembly
T_f – torque due to friction
T_r – torque related to the rotation of barrel
T_p – torque related to the pressure effects

$$T_f = \mathrm{sgn}(\dot{\theta})T_{fC} + B_{sw}\dot{\theta} \tag{14}$$

$$T_r = S_1 + S_2\theta \tag{15}$$

$$T_p = -K_{p1}\,p + K_{p2}\,p_h\,\theta \tag{16}$$

where are:
θ – swash-plate angle
T_{fC} – torque due to Coulomb friction
B_{sw} – viscous damping coefficient [Nms/rad]
S_1 – coefficient [Nm]
S_2 – coefficient [Nm/rad]
K_{p1} – coefficient [Nm/Pa]
K_{p2} – coefficient [Nm/Pa rad]

The relations from equations (14) – (16) come from experimental identification procedure. The coefficients should be determined experimentally, as well. The impact of individual torques from (13) is given in Fig. 4. Data are experimentally determined in [6], where small open loop pump for the open circuit were applied (Vickers PVB5, 12 cm^3/rev). The acceleration term of the swash-plate is excluded. The pressure torque is significant, and it supports the increase of the pump's displacement, while it withstands the decrease of displacement. The time responses can be affected by that asymmetry, and that should be taken into account. Relations in closed loop pump are somewhat different, because the swash-plate moves in both directions, while the rotation of barrel is always in one direction (hence T_r and T_p will be verified in this case of the closed loop pump).

Block-diagram of a hydrostatic system with a variable displacement pump and a fixed-displacement motor, and with a swash-plate dynamics included, is given in Fig. 5. Low-pressure is assumed to be 0 here, but in the final paper the low-pressure side will equal p_b.

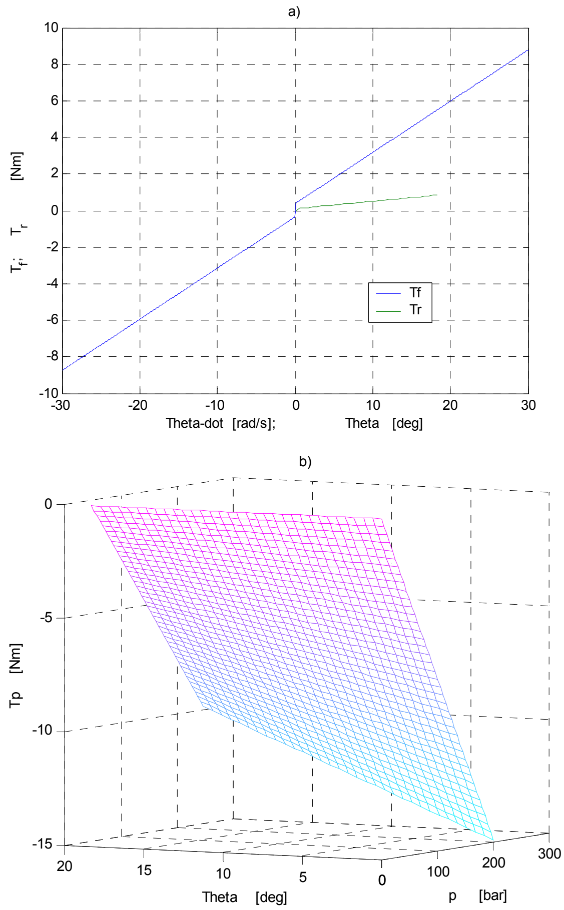

Fig. 4 Torques on the swash-plate due to:
a) friction and barrel; b) pressure

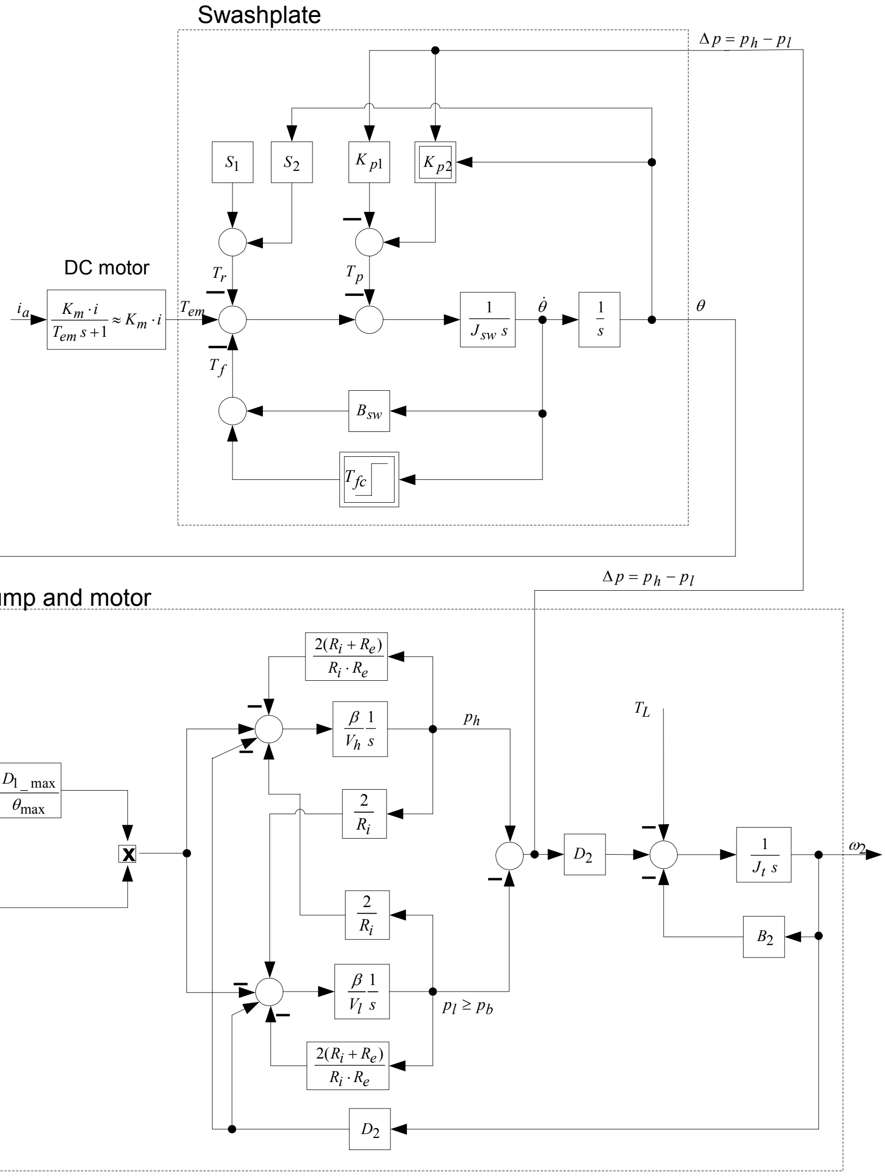

Fig. 5 Block - diagram of hydrostatic transmission

3 Simulations

The mathematical model of the hydrostatic transmission, given in block-diagram in Fig. 5 with the variable displacement pump and the fixed variable displacement motor is simulated using Matlab/Simulink. An example of the simulation is given in Fig 6 in order to demonstrate the functionality of the mathematical model. In the example the variation of speed of rotation of the hydraulic motor (in revolute per minute) and the pressure difference ($p_h - p_l$) variations are presented. The load torque and the swash-plate angle of the hydraulic pump have been considered as input variables, so their history is given in the Fig. 6, as well.

Data that were employed during the simulations are:

$\beta = 145 \cdot 10^7$ Pa

$B_2 = 0.05$ Nms/rad

$V_h = V_l = 0.0006$ m^3

$J_t = 0.01$ kgm^2

$D_1 = D_2 = 2.87 \cdot 10^{-6}$ m^3/rad

$p_b = 15 \cdot 10^5$ Pa

$\omega_1 = 209$ rad/s

Data of the swash-plate dynamics are from [6]:

$J_{sw} = 0.001$ kgm^2

$T_{fC} = 0.36$ Nm

$B_{sw} = 0.28$ Nms/rad

$S_1 = 0.096$ Nm

$S_2 = 2.36$ Nm/rad

$K_{p1} - 746 \cdot 10^{-9}$ Nm/Pa

$K_{p2} - 830 \cdot 10^{-9}$ Nm/Pa rad

The responses from the Fig. 6 are generally in accordance with the expectations.

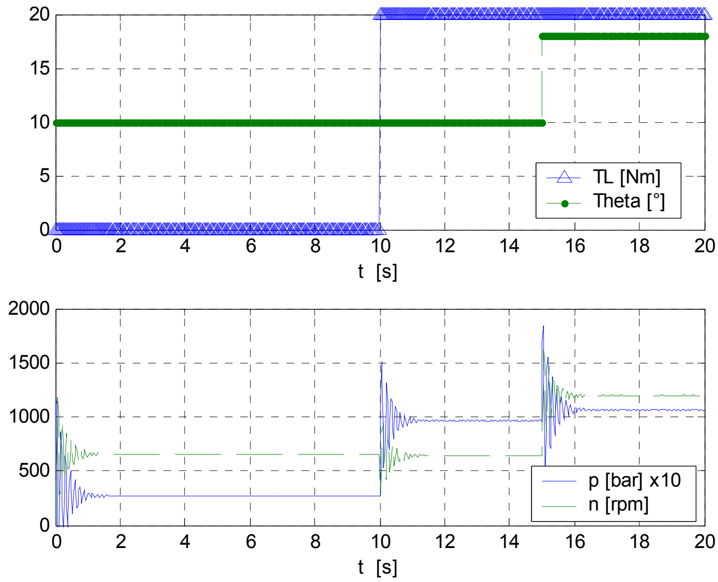

Fig. 6 Input and output variables obtained by simulations of mathematical model

4 Conclusions

Mathematical model of the closed-circuit hydrostatic transmission, with the DC servomotor actuated variable displacement swash-plate pump is given in the paper. The model is in form suitable for the experimental parameterization, since most of the required parameters are unavailable at producers' data. They should be verified experimentally.

The mathematical model will be validated on the experimental setup of the hydrostatic transmission, which is designed at our University. The variable displacement pump (Sauer-Danfoss M25 PV) is connected to the internal combustion gasoline engine, which is the driving motor. The hydraulic motor (Sauer-Danfoss M35 MV) is connected to the AC permanent magnet servo-motor, which is the load motor. The hydraulic motor is also variable displacement, but it can be adjusted just manually and thereafter fixed in a desired position. The pump displacement is controlled by the DC servo drive (Lenze helical geared DC drive GST04-1PVBR-056C21 with gear box (ratio: i = 15.4)). The setup provides precise measurement of all relevant speeds. The engine torque, the fluid flow, pressure and temperature are measured, as well. The experimental recordings on the hydrostatic transmission will include static curves and the dynamic behavior of the transmission, which is important for the validation of the mathematical model.

The presented mathematical model together with experimental validation will be a basis for the development and application of advanced control methods, which can significantly improve dynamic behavior of the hydrostatic transmission.

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