SENSITIVITY ANALYSIS IN MOSFET ANALOG INTEGRATED CIRCUITS

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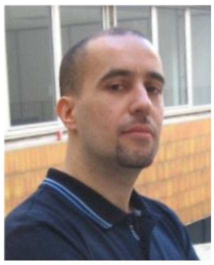
Abstract

Generally in the optimization phase of analog integrated circuits, sensitivity analysis is performed in order to reduce the high number of iterations needed to reach the best solution and equally to ensure that possible fluctuations of the parameters of the circuit won't affect the good behavior of the performance. A major design goal is to establish the degree of influence of each parameter on the target performance and to detect sensitive parameters, i.e. those whom fluctuations around their nominal value will modify in an unacceptable way the performance. The computational effort has to be focused only on this reduced set of parameters in order to optimize the solution. Traditionally, sensitivity analysis has been performed with respect to electrical parameters (resistance, inductance, capacitance etc.). In the context of MOSFET integrated circuits, these parameters belong either to the equivalent circuit of transistors or to interconnection paths and consequently the designer has no direct access to them. For this reason, sensitivity analysis with respect to technological or geometrical parameters of the components is certainly more useful. In this paper a method to compute performance sensitivities with respect to the dimensions and the technological parameters of various MOSFETs is presented. Some results are shown and we discuss how this approach can be extended to tolerance analysis, in order to estimate the maximum fluctuations of the target performance, when the tolerances on each parameter are imposed. This is expected to improve fabrication yield efficiency.

Keywords: Analog integrated circuits, MOSFET, Modeling, Sensitivity, Tolerance

Presenting Author's biography

Farouk Vallette obtained his PHD in applied physic in 1993. He's working on applications of sensitivities to tolerance analysis and circuit design in the SYEL (Systèmes Electroniques) team in University Pierre & Marie Curie, Paris 6, since 1996.



1 Introduction

Generally, the process of design and optimization of analog linear integrated circuits involves an important amount of iterative simulations. Finally, one or several solutions are reached. For each solution, the values of various circuit parameters are well known. However, as it is difficult to accurately control every step during fabrication, the real values of the parameters are deviating from their nominal values, in a more or less important way. This may induce, from sample to sample, unacceptable variations of the target performance, and as a result the manufacturing yield is drastically lowered. In order to choose the best solution and to ensure that possible differences between real and nominal values of parameters will produce negligible variations of the performance, sensitivity analysis is a necessary step. Indeed, a multi-order sensitivity analysis allows the characterization of each parameter influence over the performance [1] and is more efficient than the time-consuming Monte-Carlo method.

If the interest of sensitivity analysis is well understood, the problem that the designer faces is to accede all hot parameters, i.e. those whose influence on the performance value is significant. In all existing software packages, sensitivities are computed only with respect to electrical parameters (belonging either to discrete elements or to the small signal equivalent circuit of various transistors). However, for a selected topology, only the geometrical parameters are under the designer control and the link between electrical parameters and the geometrical and technological parameters is missing, even if sometimes it is more or less intuitive. Undoubtedly, it is more interesting to perform a sensitivity analysis with respect to the dimensions of the components, and this is the goal of our work.

SAMI (Sensitivity Analysis of Microwave Circuits) is a software developed at University Pierre & Marie Curie, which performs sensitivity analysis up to the order 5, for linear analog circuits with respect to all electrical parameters of the circuit. A module has been added to SAMI to generate from the values of sensitivities with respect to the internal electrical parameters of the MOSFET (capacitances C_{gs} and C_{gd}, output conductance r_0, and transconductance g_m) the sensitivities with respect to both the dimensions of MOSFET (the gate width W, the gate channel length L, the overlap gate length L_{ov}) and the gate capacitance C_{ox}. This paper presents a sample of obtained results and their applications to tolerance analysis of analog integrated circuits.

2 Background

Considering F as the performance of interest, and h_i as one of the n electrical parameters of the circuit, the first-order relative sensitivity of F with respect to h_i, denoted S_{h_i}, is:

$$S_{h_i} = \frac{h_i}{F} \cdot \frac{\partial F}{\partial h_i} \tag{1}$$

However, the value of first-order sensitivity may be not sufficient. A typical situation where this is true is the following. Consider the case of a circuit whose performance F with respect to the parameter h_i it happens to be close to an extremum (maximum or minimum value). Since the first order sensitivity is proportional to the slope, it is obvious that its value results very low, but this does not mean that the parameter in question is not influent! In this case, it's necessary to perform a second-order sensitivity computation, which also gives an information of the degree of influence of h_i over F. Furthermore mixed sensitivities (i.e. sensitivities with respect to several parameters) can provide information on possible correlation between the parameters in question.

The second-order sensitivity is either a single second-order relative sensitivity (2) or a mixed second-order relative sensitivity (3).

$$S^2_{h_i} = \frac{1}{2} \cdot \frac{h_i^2}{F} \cdot \frac{\partial^2 F}{\partial h_i^2} \tag{2}$$

$$S^2_{h_i h_j} = \frac{h_i . h_j}{F} \cdot \frac{\partial^2 F}{\partial h_i \partial h_j} \tag{3}$$

The general form of the n-order relative sensitivity is:

$$S^n_{h_i^\alpha h_j^\beta .. h_k^\gamma} = \frac{1}{\alpha!\beta!...\gamma!} \frac{h_i^\alpha . h_j^\beta ... h_k^\gamma}{F} \cdot \frac{\partial^n F}{\partial h_i^\alpha \, \partial h_j^\beta \, ... \, \partial h_k^\gamma} \tag{4}$$

with $n = \alpha+\beta+...+\gamma$.

3 MOSFET Small-Signal Model

The NMOS traditional structure is presented in Fig.1. The component is described by its dimensions (the length L, the width W of the gate, the overlap length L_d), and its technological parameters (the oxide capacitance C_{ox}, and the mobility μ_n).

A simple equivalent circuit of the MOSFET has been chosen and is presented in Fig.2. The influence of the bulk has been neglected and the threshold voltage V_{Tn} is supposed constant.

Considering the MOSFET in the saturation mode, the current in the transistor is given by equation (3):

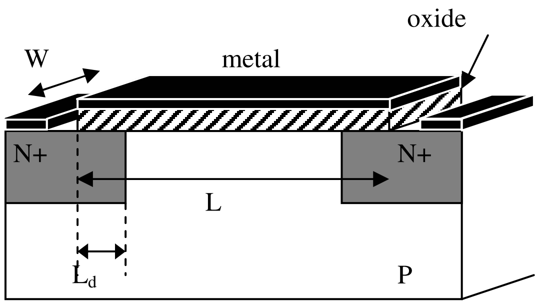


Fig. 1 The NMOS structure

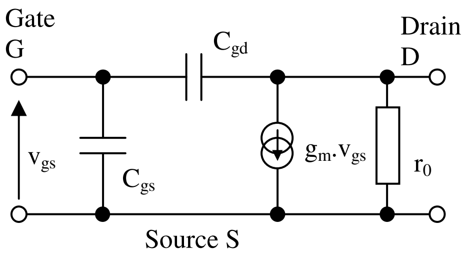


Fig. 2 The simplified MOSFET small-signal model

$$I_D = \frac{1}{2}.\frac{W}{L}.K_{Pn}.(V_{GS} - V_{Tn})^2.(1+\lambda.V_{DS}) = I_{Dsat}.(1+\lambda.V_{DS}) \tag{3}$$

with $K_{Pn} = \mu_n.C_{ox}$, I_{Dsat} is the saturation current, and λ is the channel-length modulation parameter.

The parameters of the small-signal equivalent circuit of the transistor are given by equations (4) to (7).

$$C_{gs} = \frac{2}{3}.W.(L - L_d).C_{ox} + 2.W.L_{ov}.C_{ox} \tag{4}$$

$$C_{gd} = W.L_d.C_{ox} \tag{5}$$

$$r_0 = \frac{1}{\lambda}.\frac{1}{I_{Dsat}} = \frac{1}{\lambda}.\frac{1}{\frac{1}{2}.\frac{W}{L}.K_{Pn}.(V_{GS} - V_{Tn})^2} \tag{6}$$

$$g_m = \frac{W}{L}.K_{Pn}.(V_{GS} - V_{Tn}) \tag{7}$$

4 Sensitivities Computation

SAMI performs analysis with respect to passive elements (R, L, C, impedance Z, admittance Y), transconductances (for current voltage sources), S-parameters of active devices, and ideal or microstrip transmission line parameters. The performance of interest may be node voltage, voltage gain, transducer power gain, or specific microwave performances like reflection or transmission coefficients. The method of computation is based on the adjoint network approach and Tellegen's theorem [2].

The MOSFET has been added in the repertoire of SAMI that computes relative sensitivities with respect to the parameters C_{gs}, C_{gd}, g_m and r_0 of each MOSFET. A module associated to SAMI transfers these results from the output file of SAMI and process them to compute sensitivities with respect to the parameters L, W, L_d and C_{ox}, of each MOSFET.

For instance, consider the relative sensitivities of the performance F with respect to the parameter W. Its expression is depending on the first-order relative sensitivities of F with respect to C_{gs}, C_{gd}, g_m and r_0, as in relation (8).

$$\frac{W}{F}.\frac{\partial F}{\partial W} = \frac{W}{F}.\left(\frac{\partial F}{\partial C_{gs}}.\frac{\partial C_{gs}}{\partial W} + \frac{\partial F}{\partial C_{gs}}.\frac{\partial C_{gs}}{\partial W} + \frac{\partial F}{\partial r_0}.\frac{\partial r_0}{\partial W} + \frac{\partial F}{\partial C_{gs}}.\frac{\partial C_{gs}}{\partial W}\right) \tag{8}$$

Introducing relative sensitivities, we have:

$$\frac{W}{F}.\frac{\partial F}{\partial W} = \frac{C_{gs}}{F}.\frac{\partial F}{\partial C_{gs}}.\frac{W}{C_{gs}}.\frac{\partial C_{gs}}{\partial W} + \frac{C_{gd}}{F}.\frac{\partial F}{\partial C_{gd}}.\frac{W}{C_{gd}}.\frac{\partial C_{gd}}{\partial W} + \frac{r_0}{F}.\frac{\partial F}{\partial r_0}.\frac{W}{r_0}.\frac{\partial r_0}{\partial W} + \frac{g_m}{F}.\frac{\partial F}{\partial g_m}.\frac{W}{g_m}.\frac{\partial g_m}{\partial W} \tag{9}$$

Let the relative sensitivities with respect to the MOSFET small-signal model be denoted $S_{C_{gs}}$, $S_{C_{gd}}$, S_{r_0}, and S_{g_m}. According to relation (1), this can be rewritten like :

$$S_W = S_{C_{gs}}.\frac{W}{C_{gs}}.\frac{\partial C_{gs}}{\partial W} + S_{C_{gd}}.\frac{W}{C_{gd}}.\frac{\partial C_{gd}}{\partial W} + S_{r_0}.\frac{W}{r_0}.\frac{\partial r_0}{\partial W} + S_{g_m}.\frac{W}{g_m}.\frac{\partial g_m}{\partial W} \tag{10}$$

It is necessary to obtain the relative sensitivities of the parameters C_{gs}, C_{gd}, g_m and r_0, with respect to W. We have:

$$\frac{W}{C_{gs}}.\frac{\partial C_{gs}}{\partial W} = 1 \tag{11}$$

$$\frac{W}{C_{gd}}.\frac{\partial C_{gd}}{\partial W} = 1 \tag{12}$$

$$\frac{W}{r_0}.\frac{\partial r_0}{\partial W} = -1 \tag{13}$$

$$\frac{W}{g_m}.\frac{\partial g_m}{\partial W} = 1 \tag{14}$$

Relative sensitivities with respect to L, L_d or C_{ox} are given by equation (10) by replacing W by L, L_d or C_{ox}. To get their value it is necessary to perform the computation of relative sensitivities of C_{gs}, C_{gd}, g_m and r_0, with respect to L, L_d and C_{ox} as it is shown in relations (11) to (14).

By differentiating equation (8) it's possible to get the second-order relative sensitivities S^2_W, S^2_{WL}, $S^2_{WL_d}$, and $S^2_{WC_{ox}}$.

An example of relative second-order sensitivity is given in the expression (15) and (16):

$$\frac{W.L}{F}\cdot\frac{\partial^2 F}{\partial W \partial L} = \frac{W.L}{F}\cdot\Bigg(\frac{\partial^2 F}{\partial L \partial C_{gs}}\cdot\frac{\partial C_{gs}}{\partial W} + \frac{\partial F}{\partial C_{gs}}\cdot\frac{\partial^2 C_{gs}}{\partial\, \partial L} + \frac{\partial^2 F}{\partial L \partial C_{gd}}\cdot\frac{\partial C_{gd}}{\partial W} + \frac{\partial F}{\partial C_{gd}}\cdot\frac{\partial^2 C_{gd}}{\partial W \partial L} + \frac{\partial^2 F}{\partial L \partial r_0}\cdot\frac{\partial r_0}{\partial W} + \frac{\partial F}{\partial r_0}\cdot\frac{\partial^2 r_0}{\partial W \partial L} + \frac{\partial^2 F}{\partial L \partial g_m}\cdot\frac{\partial g_m}{\partial W} + \frac{\partial F}{\partial g_m}\cdot\frac{\partial^2 g_m}{\partial W \partial L}\Bigg) \quad (15)$$

with :

$$\frac{\partial^2 F}{\partial L \partial C_{gs}} = \frac{\partial^2 F}{\partial {C_{gs}}^2}\cdot\frac{\partial C_{gs}}{\partial L} + \frac{\partial^2 F}{\partial C_{gs} \partial C_{gd}}\cdot\frac{\partial C_{gd}}{\partial L} + \frac{\partial^2 F}{\partial C_{gs} \partial r_0}\cdot\frac{\partial r_0}{\partial L} + \frac{\partial^2 F}{\partial C_{gs} \partial g_m}\cdot\frac{\partial g_m}{\partial L} \quad (16)$$

5 Application to Tolerances Analysis

Tolerance analysis is a subject neglected by analog circuit designers. Generally, they adopt the actual values of the circuit parameters equal to the nominal values. After optimization, the circuit is fabricated and tested. Either the prototype is satisfactory and can be fabricated on a large scale, or not, and optimization is iteratively applied to find a new solution. However, a tolerance analysis can be useful, by speeding up this process. If the inherent technological deviations of each parameter are known, it is possible to compute the maximum deviations of the target performance with respect to its nominal value. In this way, it is possible to both speed up the process and improve the fabrication yield.

Consider an analog circuit.

- Each of its n parameters, h_i, has a nominal value noted h_{i0}, and an actual value $h_i = h_{i0} + \Delta h_i$.
- The target performance is denoted F.
- $F_0 = F(h_{10}, h_{20}, ..., h_{n0})$ is the nominal value of the performance, and $F(h_1, h_2, .., h_n) = F_0 + \Delta F$ is the effective value.

The goal of tolerance analysis is to establish the maximum and the minimum possible variation of F around F_0, denoted respectively F_{max} and F_{min}. This can be done using sensitivity analysis [3].

6 Results

We have performed sensitivity analysis on two elementary amplification cells.

6.1 Common-Source Amplifier

Consider the single-stage amplification cell shown in Fig.3; where a NMOS common-source transistor Q_N is biased with a PMOS transistor Q_P that has a gate bias potential of 3.85V. Input potential has a bias value of 1.05V. This circuit has a magnitude voltage gain $\frac{V_{out}}{V_s}$ of 347.3 [4].

In Tab.1 are given the first-order relative sensitivities with respect to the electrical parameters (C_{gs}, C_{gd}, r_0, and g_m) and in Tab. 2 the first-order relative sensitivities with respect to the technological parameters (L, W, L_d and C_{ox}) of each MOSFET.

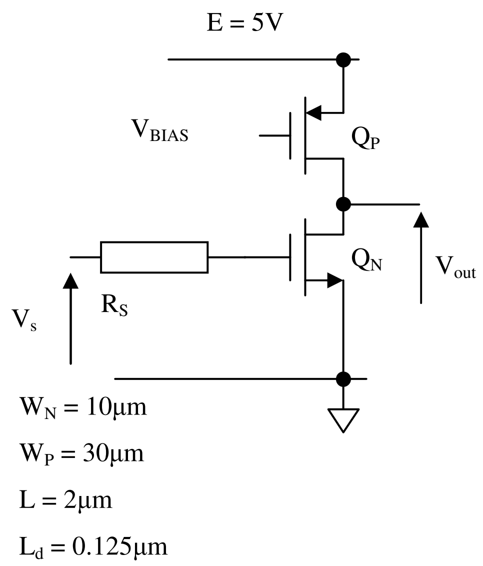
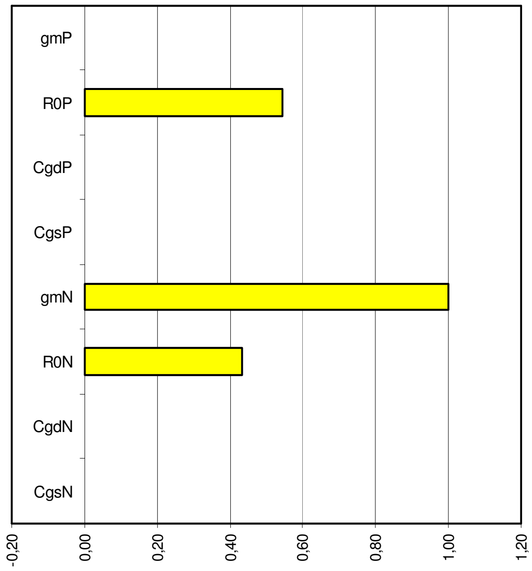
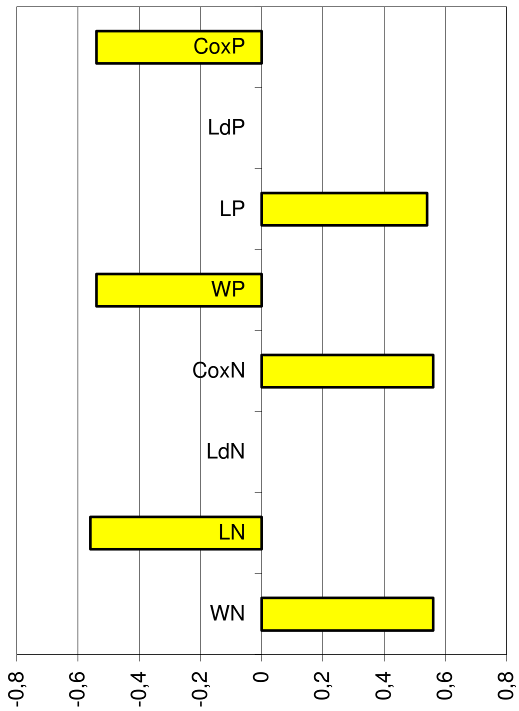


Fig. 3 Common-source amplifier

Tab. 1 First-order sensitivities with respect to electrical parameters



Tab. 2 First-order sensitivities with respect to geometrical and technological parameters



Using both 1st and 2nd order sensitivities, a tolerance analysis has been performed. Considering ±5% tolerances on L, W, L_d and C_{ox} we have established that the value of the gain magnitude is inside the domain [337.67-356.33], which represents a maximum gain deviation of 2.8% from its nominal value.

6.2 Differential Amplifier

Consider the differential amplifier circuit shown in Fig.4 [4]; this amplifier is biased with a current mirror obtained with two PMOS transistors Q_1 and Q_3; the differential cell is formed with two NMOS transistors Q_2 and Q_4. The dimensions are given in Fig. 4. The load is an equivalent resistance R_L.

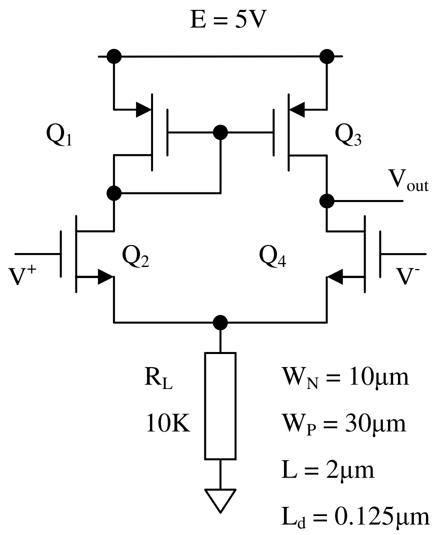


Fig. 4 Differential amplifier

The gate potential of Q_3 and Q_4 is 3.85V. The inputs V^+ and V^- of the differential amplifier are biased with 2.5V.

The threshold voltage are respectively $V_{Tn} = 0.8V$ for the NMOS transistors and $V_{TP} = -0.9V$ for the PMOS transistors. $K_{PN} = 120\mu A.V^{-2}$ and $K_{PP} = 40\mu A.V^{-2}$. $C_{ox} = 1.75fF.\mu m^{-2}$.

The performance of interest is the voltage gain $\frac{V_{out}}{V^+ - V^-}$, which value is 347, i.e. +50.8dB.

In Tab.3 are given the first-order relative sensitivities with respect to the electrical parameters (C_{gs}, C_{gd}, r_0, and g_m) and in Tab. 4 the first-order relative sensitivities with respect to the technological parameters (L, W, L_d and C_{ox}) of each MOSFET.

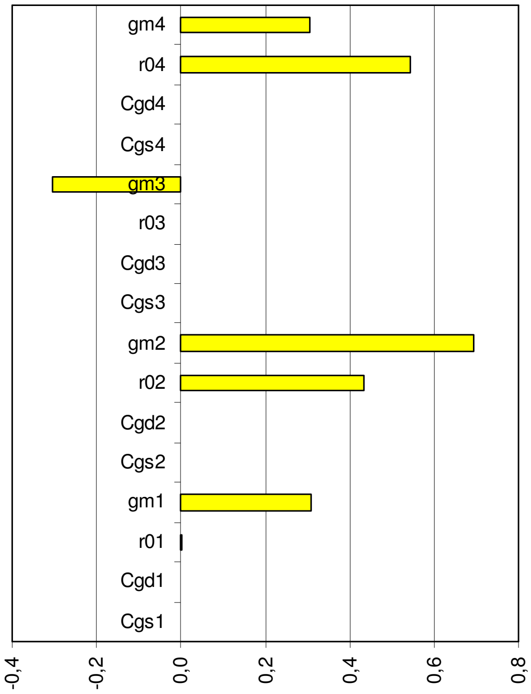
We have considered ±5% tolerances of L, W, L_d and C_{ox}. Using sensitivities we have deduced the maximum fluctuations of the voltage gain, which can vary between 317.35 and 377.78. This represents a [-8.55;+8.86] tolerances domain.

7 Conclusion

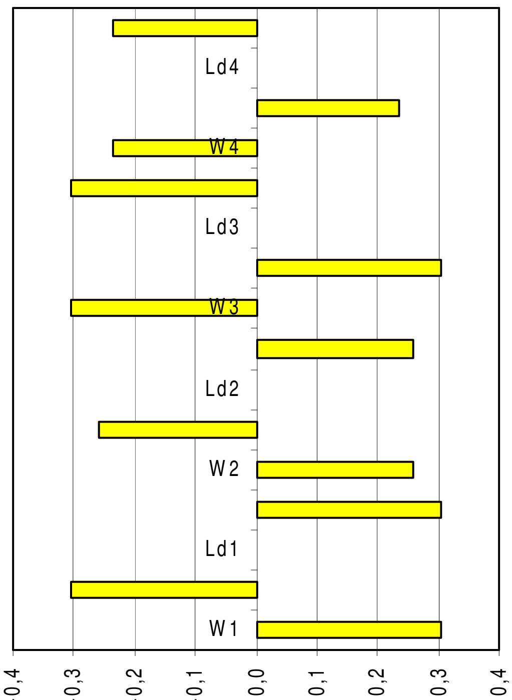
Sensitivity analysis can be applied to more complex analog circuits, containing an increased number of transistors, either considering the global circuit, or by partitioning the circuit in sub-circuits, each one corresponding to an amplifying or filtering stage of the circuit.

The tolerance analysis application shows that it is possible to put in evidence the deviations of the target performance from its nominal value, considering tolerances of technological parameters.

Tab. 3 First-order sensitivities with respect to electrical parameters



Tab. 4 First-order sensitivities with respect to geometrical and technological parameters



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