

# GREY-BOX MODELING OF AN OCEAN VESSEL FOR OPERATIONAL OPTIMIZATION

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## Abstract

Operational optimization of ocean vessels, both off-line and in real-time, is becoming increasingly important due to rising fuel cost and added environmental constraints. Accurate and efficient simulation models are needed to achieve maximum energy efficiency. In this paper a grey-box modeling approach for the simulation of ocean vessels is presented. The modeling approach combines conventional analysis models based on physical principles (a white-box model) with a feed forward neural-network (a black-box model). Two different ways of combining these models are presented, in series and in parallel. The results of simulating several trips of a medium sized container vessel show that the grey-box modeling approach, both serial and parallel approaches, can improve the prediction of the vessel fuel consumption significantly compared to a white-box model. However, a prediction of the vessel speed is only improved slightly. Furthermore, the results give an indication of the potential advantages of grey-box models, which is extrapolation beyond a given training data set and the incorporation of physical phenomena which is not modeled in the white-box models. Finally, included is a discussion on how to enhance the predictability of the grey-box models as well as updating the neural-network in real-time.

**Keywords:** ocean vessel, operational optimization, grey-box modeling, serial approach, parallel approach.

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Dr. Leifur Þ. Leifsson is a faculty member at Reykjavik University, Reykjavik, Iceland. He graduated with a Ph.D. in Aerospace Engineering from Virginia Tech, Blacksburg, USA in the year 2006. His speciality is in the field of aerodynamics and multidisciplinary design optimization. In the year 1999 he graduated with a C.Sc. in Mechanical and Industrial Engineering from the University of Iceland, Reykjavik, Iceland and a M.Sc. in Mechanical Engineering in the year 2000. Leifur has worked on several projects in the industry, which include design and development of autonomous underwater vehicles, aircraft design and operational optimization of marine vessels.



## Nomenclature

### Symbols

|             |  |
|-------------|--|
| $a$         | Parameters ( $a = (a_1, \dots, a_N)$ )       |
| $F$         | A function                                   |
| $f_c$       | Fuel consumption (liters per nautical mile)  |
| $H$         | Wave-height                                  |
| $M$         | Number of measurements                       |
| $n$         | Engine rotational speed                      |
| $N$         | Number of parameters                         |
| $p$         | Propeller pitch                              |
| $P$         | Propulsion power                             |
| $P_G$       | Electrical consumption                       |
| $r$         | Residual function                            |
| $R$         | Vessel resistance                            |
| $R_{AW}$    | Added resistance due to surface waves        |
| $t$         | Time   |
| $T_A$       | Aft draft                                    |
| $T_F$       | Forward draft                                |
| $V$         | Vessel speed (relative to water)             |
| $V_{cu}$    | Sea current speed in direction of the vessel |
| $V_w$       | Wind speed (relative to vessel)              |
| $\dot{v}_f$ | Fuel flow rate (liters per hour)             |

### Greek symbols

|            |                                     |
|------------|-------------------------------------|
| $\lambda$  | Wave-length                         |
| $\mu$      | Wave-direction                      |
| $\nabla$   | Vessel displacement                 |
| $\theta_w$ | Wind direction (relative to vessel) |

### Subscripts

|     |                     |
|-----|---------------------|
| $c$ | Calculated variable |
| $m$ | Measured variable   |

### Acronyms

|      |                             |
|------|-----------------------------|
| BB   | Black-Box                   |
| FFNN | Feed Forward Neural-Network |
| GB   | Grey-Box                    |
| RMSE | Root-Mean-Squared Error     |
| WB   | White-Box                   |

## 1 Introduction

Achieving maximum energy efficiency of transport and cargo vessels is a goal from the standpoint of both the vessel designer and operator. With rising oil prices and an increase in emission restrictions, this becomes increasingly important. The operational cost of an ocean vessel is high and depends on several factors. Presently, the largest operational cost factor is the fuel cost, or 40% of the overall operational cost for a typical container vessel as shown in Figure 1. The annual fuel cost for a cargo vessel can be in the order of millions of dollars, therefore, a few percent off the vehicle fuel consumption can lead to considerable annual savings. This gives great incentive to minimize the vessel fuel consumption.

Maximizing the energy efficiency of an ocean vessel needs to be done in both design and operation. The vessel designer can maximize the vehicle energy efficiency by designing the hull for minimum resistance

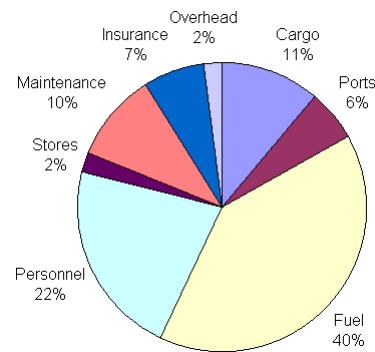


Fig. 1 A breakdown of the operational cost for a typical medium-sized container vessel [1].

and maximizing the propeller efficiency, and selecting appropriate sub-systems, such as the main and auxiliary engines, for a given operating profile. During operation the energy efficiency can be maximized by controlling the vessel speed and selecting the optimal route, as well as maximizing the efficiency of the sub-systems. Since this is a complicated process, both off-line and real-time simulation and optimization can play an important part in operational optimization of marine vessels.

### 1.1 White-Box Modeling

A performance analysis model of a given ocean vessel that is to be used for operational optimization must be able to account for the ship dynamics, as well as the effects of the environment on the vehicle, such as wind, currents and surface waves. Furthermore, a real-time performance analysis requires a computationally efficient model. The common approach to model the powering requirement of ships involves applying physical principles and results from model and full-scale experiments. This is referred to as *white-box modeling*. Usually the ship resistance is divided into three main components: (1) still-water resistance (including added resistance due to hull fouling), (2) added resistance due to surface waves, and (3) wind resistance. A semi-empirical approach based on results of systematic model experiments has been successful in modeling of still-water resistance [2, 3]. These methods are intended for use during conceptual design and can provide an approximate value of ships powering requirement in calm weather and still water. These methods also give a good indication of the changes in ship powering requirements related to any changes in the hull dimensions and configuration, and are therefore a valuable tool in the design process. Models that account for the effects of wind are also semi-empirical and based on systematic model experiments [4]. An accurate prediction of the wind effects on vessel resistance is difficult to obtain using semi-empirical models since flow past the superstructure of ships is complicated and quite different between ships. Calculating the added resistance due to surface waves is also difficult and depends on information of the state of the surface waves, such as wave-height, wave-length and wave-direction. Theoretical methods for the estimation of the time-averaged

added resistance due to surface waves are available [5]. It should be noted that white-box models are normally based on first principles and/or empirical data. Therefore, the associated assumptions and uncertainties that are implicit will affect the applicability and predictability of the models.

### 1.2 Black-Box Modeling

Another approach to model the ship powering requirement is to use *black-box methods* [6, 7, 8]. A black-box model is a mathematical model describing relations between input and output data for a given process or a system. In contrast to white-box modeling, the black-box modeling approach does not need any prior knowledge or theoretical consideration about the modeled process. The relations between input and output data are modeled building only on experimental data to forecast the system behavior. A black-box modeling approach is useful when the behavior of a process is not fully understood or indeed when an available white-box model estimating the process lacks predictability. The main disadvantage of the black-box modeling method is the dependence on the data used to model the process which can result in limited extrapolation properties beyond the data that it is derived from.

### 1.3 Grey-Box Modeling

The approach termed as *grey-box modeling* can be found in the literature [8, 9, 10]. The grey-box modeling method is a combination of white-box and black-box modeling methods. Several terms are used in the literature referring to grey-box modeling approach, e.g. semi-physical modeling [9, 11], hybrid modeling [10, 12] and semi-mechanistic modeling [13]. The definitions of these approaches differ slightly, but they all aim at bringing different advantages of white-box and black-box modeling together in one model.

The objective of the present research is to apply a grey-box modeling approach to model the performance of a given ocean vessel. More specifically, the objective is to model the fuel consumption of a container vessel using a grey-box modeling approach. The conjecture is that the white-box model will retain the physical behavior of a ship with respect to its speed and state, and the black-box model will scale the output from the white-box model to fit operational data of a given ship, as well as attaining any phenomena which is not modeled in the white-box model. The grey-box model should therefore yield an accurate model of the ship performance which is suitable for use in off-line and real-time operational optimization.

## 2 Methodology

Grey-box modeling methods can be distinguished into two main categories depending on how they are applied. These modeling approaches are called *serial modeling* and *parallel modeling* [7, 11, 14]. Figure 2 emphasizes the composition of white-box and black-box modeling into grey-box modeling and its division into serial and parallel modeling.

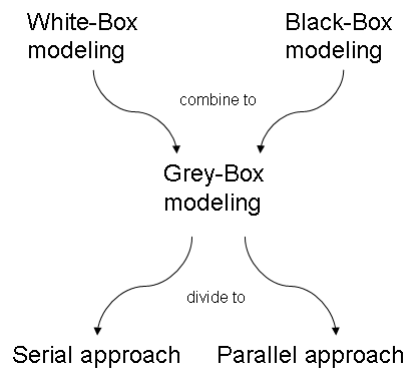


Fig. 2 An overview of the combination of the grey-box modeling method.

The serial approach involves modeling a process by configuring two or more models in a series [14]. Whereas the approach as a whole is classified as a grey-box modeling method, at least one model is a white-box and one is a black-box. An example of this approach would be to configure two models in series where the first model relates input data to a particular value by some black-box methodology, and feeds this value as an input parameter to the second model which is based on a physical principle. Here the role of the first sub-model can be considered as preprocessing of data [14]. The serial approach is indeed applicable when some parameter values have to be estimated for a given process by regressing them to some value [7]. The approach is applicable when some part of a process is not known or needs to be improved.

The parallel modeling approach involves: (1) modeling a white-box model, (2) 'training' a black-box model by minimizing the difference between the white-box model output and the desired output, and (3) combining the white-box and black-box models in parallel. The white-box model provides an output for a given set of input parameters and the black-box model 'corrects' the estimated output. Hence, the black-box model works as an algorithm that forecasts and corrects the residuals between the white-box model output and the desired output [7, 11, 14].

In the present research, the white-box model models the resistance of the hull, the performance of the propeller and the performance of the main and the auxiliary engines. The hull resistance is estimated with two models, one for the still-water resistance and the other for the wind resistance. The still-water resistance is calculated using a semi-empirical model by Holtrop et al. [3], which is based on systematic model experiments. Included is a model for added resistance due to hull fouling [1]. The wind resistance is calculated using Isherwood's semi-empirical model for merchant ships [4]. The resistance model neglects any other resistance components, such as added resistance due to surface waves and added resistance due to steering. The propeller performance is calculated using a semi-empirical model based on the Wageningen B-screw se-

ries [15]. The performance of the main and auxiliary engines is modeled according to a typical Diesel engine cycle [16] and available operational data from the manufacturers. Properties of other mechanical devices are also included in the model, such as the gear between the main engine and the propeller and shaft between the gear and the propeller.

An important feature of this white-box model is that it is implicit in the sense that the vessel speed relative to the water,  $V$ , is determined by a non-linear equation, balancing the vessel resistance,  $R$ , with the propulsion power,  $P$ ,

$$V \cdot R(\nabla, T_F, T_A, V_w, \theta_w, V) = P(n, p, V), \quad (1)$$

where the resistance depends on vessel displacement ( $\nabla$ ), fore and aft draft ( $T_F, T_A$ ) and wind speed and direction ( $V_w, \theta_w$ ), apart from the vessel speed, whereas the propulsion power depends on the rotational speed of the main engine ( $n$ ), and the propeller pitch ( $p$ ), apart from the vessel speed. This equation is solved by the Newton-Raphson iterative method. Defining for fixed values of all the parameters except the speed

$$F(V) = V \cdot R(V) - P(V), \quad (2)$$

the iterative step is

$$V^{(i+1)} = V^{(i)} - \frac{F(V^{(i)})}{\frac{d}{dV} F(V^{(i)})} \quad (3)$$

where  $V^{(i)}$  denotes the  $i$ -th iterative value of the speed, and the derivative can be approximated numerically. Knowing the value of  $V$ , the fuel flow rate,  $\dot{v}_f$  in litres per hour, can be calculated explicitly from engine information on how the specific fuel consumption depends on engine-load and rotational speed. In terms of energy optimization, however, the fuel consumption in litres per sailed nautical mile, relative to the ground, is usually of more interest, in which case  $\dot{v}_f$  has to be divided by  $V + V_{cu}$ , where  $V_{cu}$  denotes an estimate of the sea current speed in the direction of motion of the vessel.

The choice of a feed forward neural-network [17] as a black-box model is made since neural networks are considered to be good and versatile function approximators [18]. In relation to this it is emphasized that any other black-box model could have been chosen in this study, e.g. some other method from the machine learning class, like support vector machines [19] or a nonlinear multiple regression model.

The fact that the underlying white-box model is implicit, presents difficulties in using a black-box model as a data preprocessing tool in a serial grey-box approach as described above. Thus, in this research a serial approach was applied with the white-box sub-model preceding the black-box sub-model as depicted

in Figure 3. However, it is also considered how black-box data preprocessing can be implemented in the implicit setting, where the role of the black-box model can, e.g. be to improve empirical predictions of the resistance effect of wind or waves.

In the serial grey-box model the input to the white-box sub-model is propeller pitch, engine speed, vessel displacement, fore and aft draft, electrical consumption, and the relative wind speed and direction. The white-box feeds an estimate of the fuel flow rate ( $\dot{v}_f'$ ) and the vessel speed through the water ( $V'$ ) to the black-box model, which provides a forecast of the actual fuel flow rate ( $\dot{v}_f$ ) and vessel speed ( $V$ ).

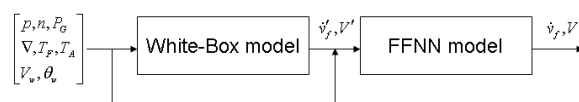


Fig. 3 A serial grey-box model of a container vessel.

In this research a parallel grey-box model was also constructed as shown in Figure 4, with the black-box modeling the residual of the measured and calculated fuel flow rate ( $r(\dot{v}_f)$ ) and vessel speed ( $r(V)$ ). The input to both the white-box model and black-box model are the same as the input to the white-box model in the serial model.

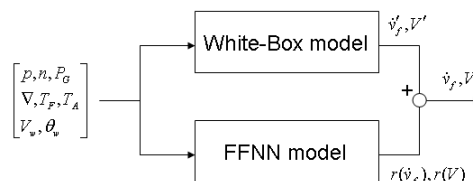


Fig. 4 A parallel grey-box model of a container vessel.

### 3 Results

The container vessel is called Dettifoss and was built in 1995. The vessel is 166 m long and 29 m wide. Its deadweight is 17 thousand metric tons, of which the cargo capacity is 10 thousand metric tons. The main engine is a diesel engine and the maximum brake-power is 20 thousand horsepowers (14.8 MW) and the propeller is a controllable pitch propeller with a diameter of 6.5 m. There are three auxiliary engines with a total of 3.38 MW capacity. The design cruise speed is 20 knots and the average fuel consumption is 56.5 metric tons/24h.

#### 3.1 Data

The duration of one voyage of Dettifoss is approximately two weeks. The voyage is divided into eleven legs, each leg lasting between two consecutive destinations. Here, each leg is divided further into three stages according to the operation of the vessel: (1) leaving harbor, (2) cruising, and (3) steaming to harbor. The operation of the engines and the nature of the sailing pro-

cess differs significantly between these stages. As an example, starting the engine and accelerating the vessel requires different operation and power than maintaining the vessel speed while cruising. The present study is restricted to cruising (stage 2). The operational data is gathered from March to August in 2006, when Detti-foss was operating in the North-Atlantic sea. The data was resampled, but otherwise there was no preprocessing. Figures 5 to 7 show typical operational data of the vessel speed, the propulsion system, and the weather for two legs and four different trips.

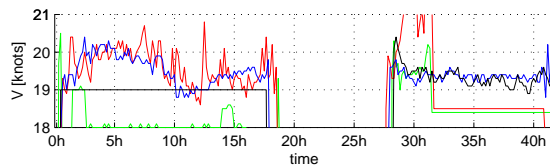


Fig. 5 Typical operational data for the vessel speed. Shown is data for two legs and four different trips.

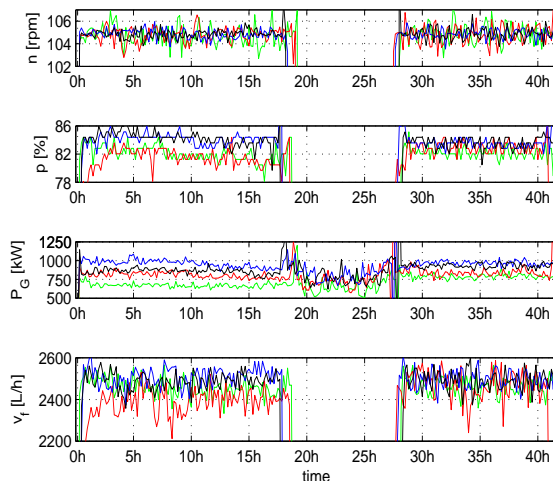


Fig. 6 Typical operational data for the propulsion system and power generation. Shown is data for two legs and four different trips.

### 3.2 Numerical Simulation

A two-layered network was selected as it is commonly viewed as a standard feed-forward neural network [18, 20]. Five neurons were selected for the hidden layer. The transfer function selected for the network layers is the sigmoid function. The objective of the neural network is to minimize the sum of squared error between actual data and network output. The Levenberg-Marquardt algorithm together with Bayesian regulation was chosen to be the training method.

A leave-one-out cross validation is used to validate the models. The Root-Mean-Squared Error (RMSE) was used as the validation metric. The data is divided into four equally sized data sets. The method validates the data-sets where each set is used once in turn as a test set and the remaining three sets as a training set.

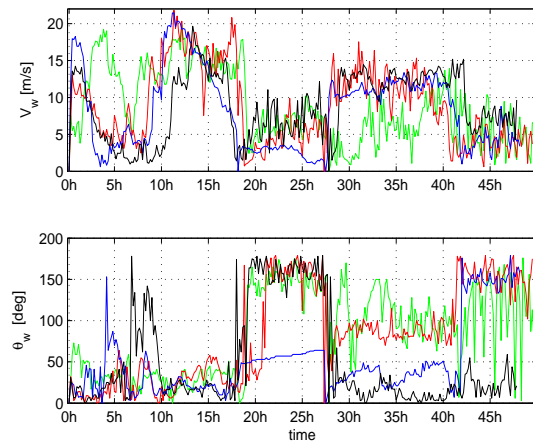


Fig. 7 Typical data for the relative wind speed and wind direction. Shown is data for two legs and four different trips. Note: Head-wind at  $\theta_w = 0$  deg, Tail-wind at  $\theta_w = 180$  deg.

Figure 8 shows a comparison of the grey-box models to the white-box model and the black-box model. Clearly there is a significant improvement in the validation metric for the fuel consumption, or approximately 65% reduction in RMSE, for both the grey-box models and the black-box model compared to the white-box model. However, only a slight reduction in the prediction of the vessel speed is shown by the grey-box and black-box models compared to the white-box models. Note that the RMSE of the vessel speed is on the order of 0.65 knots and the uncertainty in measurements of the vessel speed is approximately 0.5 knots.

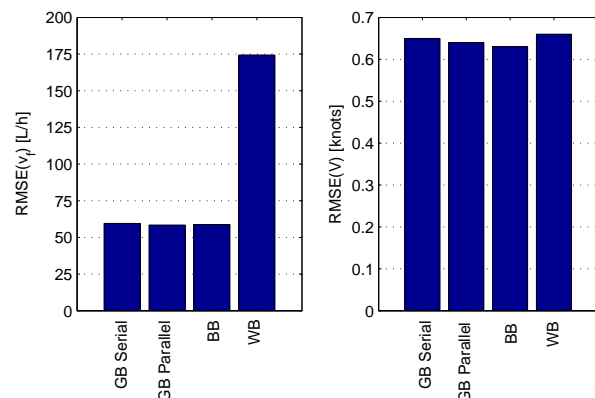


Fig. 8 A comparison of the validation metrics, RMSE of fuel flow rate  $\dot{v}_f$  and vessel speed  $V$ , for the constructed models.

Figures 9 and 10 give an indication of the potential advantages of the grey-box models over the white-box models on one hand and the black-box models on the other. Figure 9 gives an indication of the extrapolation ability of the grey-box model and the inability of the black-box model to extrapolate. Figure 10 indicates that

the grey-box model, as well as the black-box model, was able to follow the operational data relatively well, while the white-box failed. In this example it is believed that environmental components, which are not modeled by the white-box model, were a larger factor than in a normal operation.

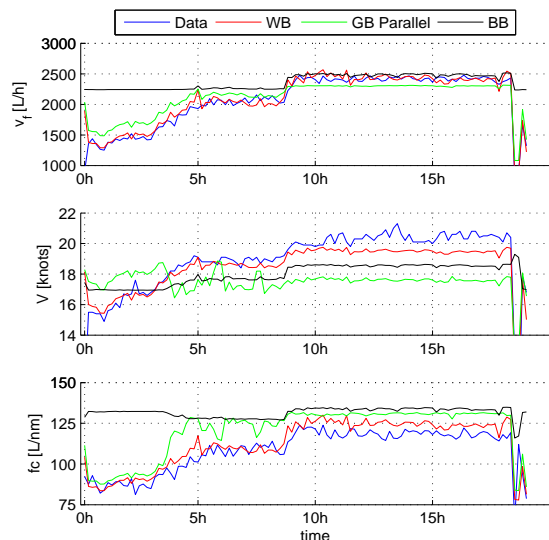


Fig. 9 An indication of the extrapolation properties of a grey-box model.

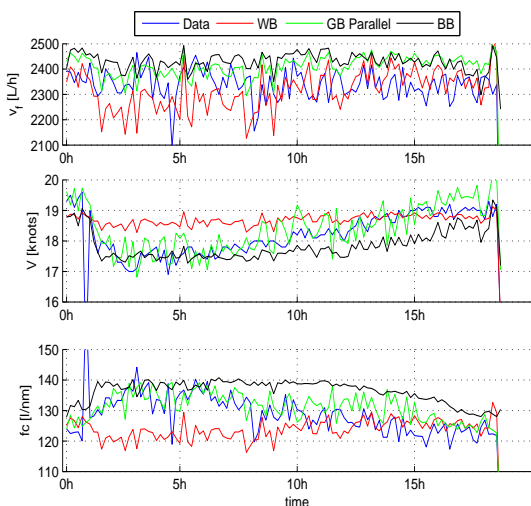


Fig. 10 An indication of a grey-box model gaining advantages of the black-box model.

Thus, while these results are still too limited to be conclusive, they show more promise with regards to grey-box modeling than those of Kristjansson [20]. He applied a black-box method to model the powering requirement of a fishing vessel with a feed-forward neural network (FFNN). Investigated were different modeling approaches of a pure black-box approach and different modeling approaches involving a combination of white-

box models and black-box models. The results showed that a two-layered FFNN-model with three neurons in the hidden layer outperformed the grey-box modeling methods and the forecast of a pure white-box model.

## 4 Discussion

The models discussed in this paper can be used as a tool for energy optimization in two ways. Firstly, calculated estimates of fuel consumption per nautical mile, relative to the ground, are a necessary input into methods for global optimal vessel routing with respect to, e.g. currents and wind [21]. Secondly, the models can be used to control optimal speed locally based on measurements of environmental factors such as sea currents, wind, and sea-state. In this case the fuel cost per nautical mile has, however, to be balanced against an estimate of costs caused by delays of reaching a destination on time [22]. Note also that by fixing the speed of the vessel in equation (1), it can be solved for the pitch,  $p$ , by an iterative approach, similar to that used for solving it with respect to speed. Thus, the optimal cost can be determined not only with respect to speed, but also with respect to pitch.

There are some problems applying grey-box modeling in the present context that have not yet been properly addressed, but are likely to affect the success of this approach. Firstly, the measurements of many of the critical input parameters such as wind speed and wave-height are often not very accurate and other, such as vessel speed, include considerable noise. Secondly, and perhaps more importantly, measurements are collected under operational conditions, which implies in some cases limited variation in the input values, which in turn makes the training of neural networks less reliable.

As already mentioned, it is of particular interest to be able to apply a serial black-box approach when some parameter values have to be estimated for a given sub-process for which the physics are not sufficiently well known. In order to illustrate how that can be handled in the present context, let us assume that one wishes to include the effect of surface waves on the resistance using a neural-network rather than theoretical methods as presented, e.g. in [5]. Thus, we further assume that there are available measurements of the sea-state, such as wave-height  $H_i$ , wave-length  $\lambda_i$ , and wave-direction  $\mu_i$ , in addition to the existing measurements at  $M$  different times,  $t_i$ ,  $i = 1, \dots, M$ . Letting  $a = (a_1, \dots, a_N)$ , denote the parameters of the neural-network, a modified implicit model becomes, cf. equation (1),

$$\begin{aligned} V \cdot (R(\nabla, T_F, T_A, V_w, \theta_w, V) + R_{AW}(a, H, \lambda, \mu, V)) \\ = P(n, p, V), \end{aligned} \quad (4)$$

where  $R$  is the total vessel resistance excluding  $R_{AW}$ , which is the added resistance due to surface waves.  $R$  is calculated using a white-box model and  $R_{AW}$  is calculated using a black-box model, a neural-network in this case. In training the neural-network one does, however,



not have any direct measurements of its output,  $R_{AW}$ , but only of the vessel speed,  $V$  (or possibly only some quantity derived from the speed such as the fuel consumption rate) and the sea-state parameters  $H$ ,  $\lambda$ , and  $\mu$ . Thus, the training of the network has to be based on the sum of the squared error between the measured speed,  $V_{i,m}$ , and the calculated speed,  $V_{i,c}$ , at the  $M$  times of measurement, i.e., on

$$E = \sum_{i=1}^M (V_{i,c} - V_{i,m})^2. \quad (5)$$

Most network training algorithms rely on the gradient of this error with respect to the network parameters. Thus, in making use of these algorithms

$$\frac{\partial}{\partial a_j} E = 2 \sum_{i=1}^M (V_{i,c} - V_{i,m}) \frac{\partial}{\partial a_j} V_{i,c} \quad (6)$$

has to be related to the corresponding derivatives of the network output,  $R_{AW}$ , with respect to the parameters. Defining

$$F_i(V, a) = V \cdot (R_i(V) + R_{AW,i}(a, V)) - P_i(V) \quad (7)$$

we have that

$$F_i(V(a), a) = 0 \quad (8)$$

and hence that

$$V \frac{\partial R_{AW,i}}{\partial a_j} + \frac{\partial F_i}{\partial V} \cdot \frac{\partial V}{\partial a_j} = 0, \quad (9)$$

so,

$$\frac{\partial V_{c,j}}{\partial a_j} = - \frac{V_{c,i}}{\frac{\partial F_i}{\partial V} |_{V=V_{c,j}}} \frac{\partial R_{AW,j}}{\partial a_j}, \quad (10)$$

giving the necessary relationship. It should be noted that the derivative  $\frac{\partial F_i}{\partial V} |_{V=V_{c,j}}$  is the same one that is required when solving equation (4) for  $V$  by the Newton-Raphson iterative method, cf. equation (3). The full training process involves a double iteration. After each update of the network parameters by the network training algorithm, the corresponding  $V_i$ -values are calculated by the Newton-Raphson method for  $i = 1, \dots, M$ .

## 5 Conclusions

The grey-box models, both serial and parallel, improve the predictability of the vessel fuel consumption significantly compared to a pure white-box model. One of the reasons is that the white-box model only accounts for part of the vehicle resistance components and neglects important parts, such as added resistance due to surface

waves. The black-box is able to improve the prediction by accounting for any physical phenomena that is not modelled in the white-box model, as well as enhancing the prediction of the white-box model. However, only a slight improvement in the prediction of the vessel speed is shown by the grey-box model compared to the white-box model. The reason for this is not fully understood.

The potential advantages of grey-box models over white-box and black-box models were indicated in two different simulations. One simulation gave an indication of the extrapolation ability of the grey-box model, while another simulation gave an indication on how a grey-box model is able to incorporate the effects of physical phenomena which has been neglected in the white-box model. These properties are not obtained by using a pure black-box model. However, these results are as yet not extensive enough to fully assess the potential advantages of grey-box modeling and therefore further research is required.

It is clear that grey-box modeling could be used to improve the performance analysis of an ocean vessel and therefore has the potential to be used in off-line and real-time operational optimization. However, further improvements to the grey-box modeling approach present in this paper are necessary. For example, information about the state of the surface waves, such as wave-height, wave-length and wave-direction, are necessary to improve the prediction of the vessel speed. Furthermore, the ability to update the black-box in real-time would allow the grey-box model to continuously improve by taking into account new data and any changes in the vessel behavior. Using a black-model to improve, e.g. empirical predictions of the resistance effect of waves, within the framework of the overall implicit white-box model, as outlined in the discussion above, offers a promising approach to deal with both these issues.

## 6 Acknowledgement

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