# IDENTIFICATION OF EXTERNAL LOAD MODELS TO SYSTEMS WITH INEXACT DESCRIPTION

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**Abstract**

The problem of correct choice of mathematical model of external load to dynamic system arises often at mathematical modeling of real motion of open dynamic systems. Direct measurements of these external loads are impossible as a rule. However an important question remains open if the function of external load obtained by experimental way is going to be the best model of external load. Besides that the goals of mathematical modelling are different: the modelling of selected motion of dynamic system, different evaluations of response of dynamic system (from below, from above), the modelling of the best forecast of motion of dynamic system with account of inaccuracy of the mathematical description, the modelling of the guaranteed evaluations of system responses, the maximal stable model to small changes of experimental conditions and so on. The choice of model of external load depends from goals of the use in the future.

In the given work the problem of construction (synthesis) of mathematical model of unknown external load to open dynamic system for different goals of the use at mathematical modeling in the future by the identification method is considered [1,2,3,4]. These problems are ill-posed by their nature and so the method of Tikhonov's regularization is used for its solution. For increase of exactness of problem solution of synthesis for models class the method of choice of special mathematical models is offered.

**Keywords: external load, mathematical model, identification, regularization.**

## Presenting Author's biography

Yuri Menshikov. He is working under incorrect problems of identification of external loads on dynamic systems since 1975 year. He has a scientific degree of the Dr. of Science. He is published about 200 scientific works. The monograph "Identification of Models of External Load" (together with Prof. Polyakov N.V.) was prepared for printing by Dr. Menshikov Yu.

 

# 1 Introduction

One of the important problems of mathematical modelling of dynamic systems is the coincidence of modelling results with experimental measurements. Such coincidence is being attained by construction of "correct" mathematical model (MM) of the dynamical system and the choice of "good" model of external load. MM of object the motion of which coincides with experimental measurements with acceptable accuracy under action of model of EL (or external impact) which corresponds to real EL ("good" model) is understood as "correct" model. Thus the degree of "correctness" of MM depends directly on the chosen model of EL and required accuracy of the coincidence with experiment. It is formally possible to write as an inequality for models with the concentrated parameters

$$F(A_p z, u_\delta) \le \varepsilon, \qquad (1)$$

where $A_p$ is an operator of the certain structure which is carrying out the connection of EL $z$ and the response of MM $u$ ($A_p z = u$) and which depends on vectors - parameters $p$; $u_\delta$ is experimentally obtained reaction of real dynamic system on real EL; $\varepsilon = Const > 0$ is a required accuracy of the coincidence of experiment with results of mathematical modelling; $z$ is the function of external impact, $z \in Z$; $u$, $u_\delta$ are the vector-functions of the response of researched object on external load, $u \in U$, $u_\delta \in U$. One of possible variants of an inequality (1) can be the following inequality

$$\| A_p z - u_\delta \|_U \le \varepsilon, \qquad (2)$$

where $\| \cdot \|_U$ there is a norm in functional space $U$.
Characteristic feature for problems of a considered type is that the operator $A_p$ is compact operator [5].
The value $\varepsilon$ is set a priori and characterizes desirable quality of mathematical modelling. The vector-function $u_\delta$ is obtained from experiment with a known error $\delta$:

$$\| u_T - u_\delta \|_U \le \delta, \qquad (3)$$

where $u_T$ is an exact response of object on real EL.
It is obvious that in the case of performance of an inequality (2) operators $A_p$ and function $z$ is connected. It is easy to show, that at the fixed operator $A_p$ in (2) exists infinite set of various among themselves functions $z$, which are satisfy to an inequality (2) [5]. And, on the contrary, at the fixed function $z$ there are infinite many various operators for which an inequality (2) is valid [5].
As a rule the check of an inequality (2) is not being executed in the practice of mathematical modelling, but its performance is meant. The error of the measuring equipment $\delta$ is contained in value $\varepsilon$ as obligatory component and therefore the inequality $\delta \le \varepsilon$ is always taken place. It occurs for the reason, that the accuracy of experimental measurements is higher as required accuracy of modelling as a rule. Frequently only qualitative coincidence of results of mathematical modelling with experiment is content us.

# 2 General

Let us considered some dynamic system $\Sigma$ the motion of which is being described by ordinary differential equations of n-order. It is suggested that the records of all external loads $f_2(t), f_3(t), ..., f_m(t)$ (except only one $f_1(t)$) and one state variable, for example $x_1(t)$, are obtained by experimental way during motion of system for some interval of time $t \in [0, T]$.
The model $z(t)$ of external load $f_1(t)$ is necessary to find after the action of which the mathematical model of system $\Sigma$ (MM$\Sigma$) moves such way that the state variable $x_1(t)$ coincides with experimental record $\tilde{x}_1(t)$ of $x_1(t)$. The rest external loads are coincided with external loads $f_2(t), f_3(t), ..., f_m(t)$ known from experiment. The problems such type was named the problems of external loads identification [1,2,3,4].
The model $z(t)$ which was obtained such method depends from chosen MM$\Sigma$ and from goals of the use at mathematical modeling in future.
If the initial dynamic system do not satisfy the condition as have been specified above then this system can be reduce to system $\Sigma$ with help additional measurements [4,6].
Let us assume that the MM$\Sigma$ is linear and that the connection between unknown function $z(t)$ and functions $f_2(t), f_3(t), ..., f_m(t)$, $x_1(t)$ has the form:

$$A_p z = B_p x, \qquad (4)$$

where $A_p$ is linear integral operator ($A_p : Z \to U$) which depends continuously on vector parameters $p$ of mathematical model of system (MM$\Sigma$), $p = (p_1, p_2, ..., p_N)^T, (\cdot)^T$ is the sign of transposition, $p \in R^N, R^N$ is the Euclidean vector space with norm $\|p\| = (p, p)$; $B_p$ is linear bounded operator ($B_p : X \to U$) which depends continuously on vector parameters $p$; $x = (x_1(t), f_2(t), . . . , f_m(t))^T$; $z \in Z, x \in X$; $Z, X, U$ are Gilbert spaces. The functions $x_1(t), f_2(t), ..., f_m(t)$ are given with known inaccuracy $\tilde{x} = (\tilde{x}_1(t), \tilde{f}_2(t), . \quad \tilde{f}_m(t))^T$ as these functions had been obtained from experimental measurements:

$$\| x(t) - \tilde{x}(t) \|_X \le \delta, \qquad (5)$$

where $x(t)$ is the exact vector function of initial data, $\delta$ – given value.

 

Besides it is suppose that the vector parameters $p$ is given inexactly. So vector $p$ can has values in some closed domain $D$: $p \in D \subset R^N$. Two operators $A_p, B_p$ correspond to each vector from $D$. The set of possible operators $A_p$ has been denoted as class of operators $K_A$, the set of possible operators $B_p$ has been denoted as class of operators $K_B$. So we have $A_p \in K_A, B_p \in K_B$. The maximal deviations of operators $A_p$ from class $K_A$ and operators $B_p$ from class $K_B$ are equal:

$$\| A_{p_\alpha} - A_{p_\beta} \|_{Z \to U} \le h \,, \ \| B_{p_\eta} - B_{p_\gamma} \|_{X \to U} \le d \,.$$

Denote by $Q_{\delta,p}$ the set of the possible solutions of equation (1) with account of experimental measurements inaccuracy only:

$$Q_{\delta,p} = \{z : z \in Z, A_p z \in U_{\delta,p}, p \in D\} ,$$

wher $U_{\delta,p} = \{u = B_p x : u \in U, \ x \in X_\delta, p \in D\}$ ,

$$X_\delta = \{x : x \in X, \ \| x - \widetilde{x} \|_X \le \delta\} .$$

The operator $A_p$ in equation (4) is a completely continuous operator for overwhelming majority of cases and so the set $Q_{\delta,p}$ is unbounded set in space $Z$ as a rule (ill-posed problem) [6,7].

The set of possible solutions of equation (4) $Q_{\delta,p}$ has to expand to the set $Q_{\delta,D}$ if additionally the inaccuracy of operators $A_p$, $B_p$ take into account:

$$Q_{\delta,D} = \{z : z \in Z, A_b z \in \bigcup_{p \in D} Q^*_{\delta,p}, b \in D\} ,$$

where $Q^*_{\delta,\mathrm{p}} = \{ z, \ z \in Z, \ A_p z \in U^*_{\delta,D}, \ p \in D\}$ ,

$$U^*_{\delta,D} = \bigcup_{a \in D} U_{\delta,a} \,.$$

Let us formulated some problems of synthesis of external models for different cases of the use at mathematical modeling in the future.

Any function $z$ from set $Q_{\delta,p}$ is simulating of the motion of dynamic system MM$\Sigma$ with the inaccuracy of experimental measurements only.

Any function $z$ from set $Q_{\delta,D}$ is simulating of the motion of dynamic system MM$\Sigma$ with the inaccuracy of experimental measurements and inaccuracy of operators $A_p$, $B_p$.

## 2 Formulation of Problems of Models Synthesis

As the sets $Q_{\delta,p}$, $Q_{\delta,D}$ are unbounded for any $\delta > 0, D \subset R^N$ so the functions $z_1$, $z_2$, are not stable to small change of initial data [5,7].

The regularization method for equations with inexact given operators were used for an obtaining of stable solutions of denoted above problems [5,7].

Let us consider the stabilizing functional $\Omega[z]$ which has been defined on set $Z_1$, where $Z_1$ is everywhere dense in $Z$ [5,7]. Consider now the extreme problem I:

$$\Omega[z_p] = \inf_{z \in Q_{\delta,p} \cap Z_1} \Omega[z], \ p \in D. \qquad (6)$$

It was shown that by certain conditions the solution of the extreme problem exist, unique and stable to small change of initial data $\widetilde{x}_1(t), \delta, A_p, B_p$ [5,7].

The function $z_p$ is the stable solution of EL after account of experimental measurements inaccuracy only.

Consider now the extreme problem II:

$$\Omega[\widetilde{z}_p] = \inf_{z \in Q_{\delta,D} \cap Z_1} \Omega[z], \ p \in D. \qquad (7)$$

In complete analogy the function $\widetilde{z}_p$ is the stable solution of EL after account of experimental measurements inaccuracy and inaccuracy of operators $A_p$, $B_p$.

Then the stable model $z_{\min}$ which gives the evaluation from below of the selected response $B_p x_1 = A_p z$ of system MM$\Sigma$ can been defined as result of the solution of the following extreme problem III:

$$\| A_{b_{\min}} z_{\min} \|^2_U = \inf_{A_b \in K_A, B_b \in K_B} \inf_{z_p} \| A_b z_p \|^2_U \,, \ b \in D \,,$$

where $z_p$ is the solution of extreme problem (6) on set $Q_{\delta,p}$.

The stable model $z_{\max}$ which gives the evaluation from above can been defined as result of the solution of the following extreme problem IV:

$$\| A_{b_{\max}} z_{\max} \|^2_U = \sup_{A_b \in K_A, B_b \in K_B} \sup_{z_p} \| A_b z_p \|^2_U \,, \ b \in D \,.$$

The stable model $z_{un}$ of external load which gives the best result of motion of system MM$\Sigma$ with guarantee as the solution of the following extreme problem V:

$$\| A_{b_{un}} z_{un} - \widetilde{x}_1 \|^2_U = \inf_{p \in D} \sup_{c \in D} \| A_c z_p - B_c \widetilde{x}_1 \|^2_U = d_T \,, \quad (8)$$

$b_{un} \in D$.

Consider now the following extreme problem:

 

$$\inf_{z\in Q_T}\sup_{p\in D}\| A_p z - B_p x_T \|_U = \| A_{p_0} z_0 - B_{p_0} x_T \|_U \,. \quad (9)$$

**Theorem.** Function $z_{un,\delta} \in Q_{D,\delta} \subset Q^*_\delta$ exist and is stable to small change of initial data (function $x_\delta$), if the functional $\Omega[z]$ is stabilizing functional and the function $z_o$ is defined unique from (9).

**Prov.** Let the function $z_{un,\delta}$ is solution of problem (12) with function $x_\delta$, for which inequality is valid

$$\| x_T(t) - \tilde{x}_1(t) \|_X \le \delta \,.$$

We can now show that $z_{un,\delta}$ is stable to small change of function $x_\delta$. It means that for any $\varepsilon > 0$ exists the value $\delta(\varepsilon) > 0$ for which from inequality $\| x_T - x_\delta \|_X \le \delta \le \delta(\varepsilon)$ follow the inequality $\| z_{un,\delta} - z_o \|_Z \le \varepsilon$.

The function which satisfies the condition

$$\sup_{z\in Q_T}\Omega[z]=\Omega[z^M]$$

we denote as $z^M$.

It is assumed that function $z^M$ exists and is bounded. Evidently that for any $\delta > 0$ for function $z_{un,\delta}$ the inequality is valid

$$\Omega[z_{un,\delta}] \le \Omega[z\colon z \in Q_T] \le \Omega[z^M] = c_0\,,$$

where $Q_T = \{z : z \in Z_1; A_p z = B_p x_T, p \in D\}$.

The function $z_{un,\delta}$ belongs to compact set

$$Z_0 = \{z : z \in Z_1, \Omega[z] \le c_0\}. \quad (10)$$

from the definition of functional $\Omega[z]$ [12].

Let us choose the subsequence of positive numbers $\delta_k$ which converge to zero $\{\delta_k\} \underset{k\to\infty}{\to} 0$. The function $z_{un,\delta_k} \in Z_0$ which is the solution of extreme problem V is assigned to each $\delta_k$. We can extract converging subsequence $\{z_{un,\delta_{k_i}}\} \underset{i\to\infty}{\to} \hat{z}_0 \in Z_0$ from sequence $\{z_{un,\delta_k}\}$. The sequence $\{x_{\delta_{k_i}}\}$ converges to $x_T$ at $k\to\infty$ as $\{\delta_{k_i}\} \underset{i\to\infty}{\to} 0$. Let us change of indexing of this sequence as $\{x_{\delta_n}\}$.

Fixed vector $p \in D$, operators $A_p \in K_A$, $B_p \in K_B$ and value of inaccuracy of initial data $\delta_n$. Let $z_{p,\delta_n}$ is the solution of extreme problem (6). It follows from theory of ill-posed problems that $z_{p,\delta_n} \underset{n\to\infty}{\to} z_{p,T}$ by $\{\delta_n\} \underset{n\to\infty}{\to} 0$. The functions $z_{p,\delta_n}$ have to belong to the set $Q_{\delta_n,p}$ which is the subset of $Q_{\delta_{n-1},p}$ by any $n$. Function $z_{un,\delta_n}$ will be pertain to set $Q_{D,\delta_n}$ by any $n$. On the basis of of this the function $\hat{z}_0$ must belong to the set $Q_T$, $\hat{z}_0 \in Q_T$. We can now show that the equality

$$\| A_{p_o}\hat{z}_0 - u_{p,T} \|_U = \inf_{z\in Q_T}\sup_{p\in D}\| A_p z - u_{p,T} \|_U = d_T$$

for function $\hat{z}_0$ with some vector $p\in D$ is valid. So function $\hat{z}_0$ is the solution of extreme problem V.

Each function $z_{un,\delta_n}$ satisfies condition

$$\inf_{z\in Q_{D,\delta_n}}\sup_{p\in D}\| A_p z - B_p x_{\delta_n} \|_U = \| A_{p_{un,n}} z_{un,\delta_n} - B_p x_{\delta_n} \|_U \,,$$

where

$$Q_{D,\delta_n} = \{z_\alpha : \Omega[z_\alpha] = \inf_{z\in Q_{\delta_n,p}\cap Z_1}\Omega[z]\},$$

$$Q_{\delta_n,p} = \{z : \| A_p z - B_p z_{\delta_n} \|_U \le \delta_n\} \,.$$

So we have on the definition of operator $A_{p_{un,n}}$

$$\inf_{z\in Q_{D,\delta_n}}\sup_{p\in D}\| A_p z - B_p x_{\delta_n} \|_U = \| A_{p_{un,n}} z_{un,\delta_n} - B_p x_{\delta_n} \|_U \ge$$

$$\ge \| A_{p_0} z_{un,\delta_n} - B_p x_{\delta_n} \|_U \,.$$

Approaching the limit as $n \to \infty$ in last inequality we obtain

$$\inf_{z\in Q_T}\sup_{p\in D}\| A_p z - B_p x_T \|_U = \| A_{p_0}\hat{z}_0 - B_p x_T \|_U \,.$$

But there are not functions $z_\varepsilon$ and operators $A_p \in K_A$, $B_p \in K_B$, which given the value of (8) the less than $d_T$. From this it follows that

$$\| A_{p_0}\hat{z}_0 - B_p x_T \|_U = d_T = \| A_{p_0} z_0 - B_p x_T \|_U \,.$$

In view of solution uniqueness of extreme problem V, we have

$$\hat{z}_0 = z_0 \,.$$

Theorem is proved.

If the classes $K_A$, $K_B$ consists from the limited number of operators $K_A = \{A_1, A_2, \ldots A_N\} = \{A_i\}$, $K_B = \{B_1, B_2, \ldots B_N\} = \{B_i\}$, $i = 1,N$, then the algorithm of finding of the best unitary model of external load $z_{un}$ has the form

$$\inf_{z\in Q_{D,\delta_n}}\sup_{p\in D}\| A_p z - B_p x_{\delta_n} \|_U = \| A_{p_{un}} z_{un} - B_{p_{un}} x_{\delta_n} \|_U =$$

$$= \min_j \max_i \| A_i z_j - B_i x_\delta \|_U \,,$$

where

$$Q_{D,\delta} = Q_{j,\delta} = \{z_i : \| A_j z_i - B_j \tilde{x}_1 \|_U = \delta, j = 1,2\} \,.$$

## 3 Identification of model of external load on rolling mills

One of the important characteristics of rolling process is the moment of technological resistance (MTR) arising at the result of plastic deformation of metal in the center of deformation. Size and character of change of this moment define loadings on the main mechanical line of the rolling mill. However complexity of processes in the center of deformation does not allow to construct authentic mathematical model of MTR by usual methods. In most cases at research of dynamics of the main mechanical lines of rolling mills MTR is being created on basis of hypothesis and it is being imitated as piecewise smooth linear function of time or corner of turn of the working barrels [4,8]. The results of mathematical modeling of dynamics of the main mechanical lines of rolling mills with such model MTR are different among themselves [4].

In work the problem of construction of models of technological resistance on the rolling mill is considered on the basis of experimental measurements of the responses of the main mechanical system of the rolling mill under real EL [4,8]. Such approach allows carrying out in a consequence mathematical modeling of dynamics of the main mechanical lines of rolling mills with a high degree of reliability and on this basis to develop optimum technological modes.

The kinematics scheme of the main mechanical line of rolling mill was presented on Fig.1. (a).

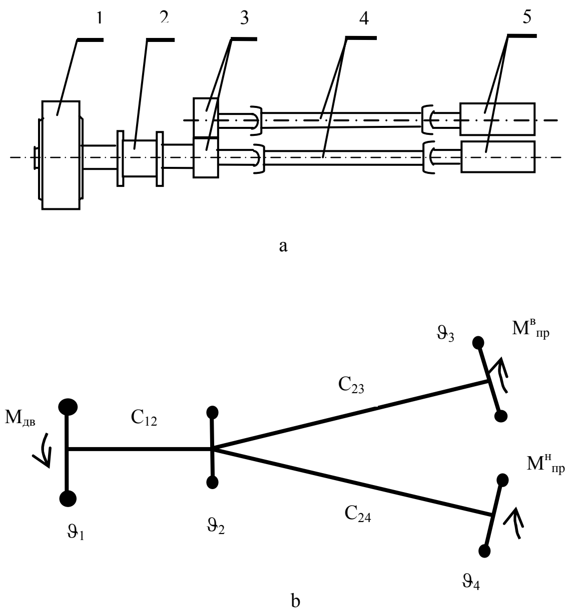


Fig.1. Kinematics scheme of the main mechanical line of rolling mill.

The four-mass model with weightless elastic connections is chosen as MM of dynamic system of the main mechanical line of the rolling mill (see Fig.1.(b)) [8]:

$$\ddot{M}_{12} + \omega_{12}^2 M_{12} - \frac{c_{12}}{\vartheta_2} M_{23} - \frac{c_{12}}{\vartheta_2} M_{24} = \frac{c_{12}}{\vartheta_1} M_{eng}(t);$$

$$\ddot{M}_{23} + \omega_{23}^2 M_{23} - \frac{c_{23}}{\vartheta_2} M_{12} + \frac{c_{23}}{\vartheta_2} M_{24} = \frac{c_{23}}{\vartheta_3} M^{U}{}_{rol}(t); \quad (11)$$

$$\ddot{M}_{24} + \omega_{24}^2 M_{24} - \frac{c_{24}}{\vartheta_2} M_{12} + \frac{c_{24}}{\vartheta_4} M_{23} = \frac{c_{24}}{\vartheta_4} M^{L}{}_{rol}(t);$$

where $\omega^2{}_{ik} = \frac{c_{ik}(\vartheta_i + \vartheta_k)}{\vartheta_i \vartheta_k}$, $\vartheta_k$ are the moments of inertia of the concentrated weights, $c_{ik}$ are the rigidity of the appropriate elastic connection, $M^U{}_{rol}$, $M^L{}_{rol}$ are the moments of technological resistance put to the upper and lower worker barrel accordingly, $M_{eng}(t)$ is the moment of the engine.

The problem of synthesis of mathematical model of EL can be formulated so: it is necessary to define such models of technological resistance on the part of metal which would cause in elastic connections of model of fluctuations identical experimental (in points of measurements) taking into account of an error of measurements for chosen MM of the main mechanical line of rolling mill.

The information on the real motion of the main mechanical line of rolling mill is received by an experimental way [4,6,8]. Such information is being understood as presence of functions $M_{12}(t)$, $M_{23}(t)$, $M_{24}(t)$.

The records of functions $M_{12}(t)$, $M_{23}(t)$, $M_{24}(t)$ by rolling process are shown on Fig.2.

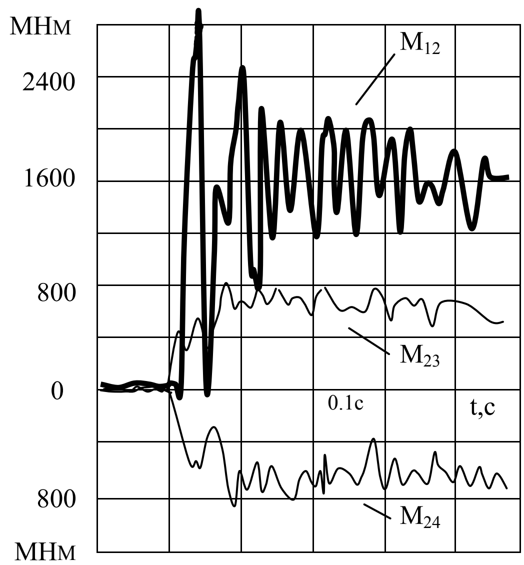


Fig.2. Records of functions $M_{12}(t)$, $M_{23}(t)$, $M_{24}(t)$.

Let's consider a problem of construction of models of EL to the upper working barrel. On the lower working barrel all calculations will be carried out similarly. From system (11) the equation concerning required model $M^U{}_{rol}$ can be received

$$\int_0^t \sin \omega_{23}(t-\tau) M^U_{rol}(\tau) d\tau = u_\delta(t) \text{ or } A_p z = u_\delta, \quad (12)$$

where $z = M^U{}_{rol}(\tau)$, $A_p$ is a linear integral operator.

The maximal deviation of the operators $A_p \in K_A$ one from another is defined by an error of parameters of mathematical model of the rolling mill. The error of definition of values of discrete weights is being accepted as 8 %, the error of stiffness values - 5 %, error of values of damping factors - 30 %. The size of the maximal deviation of the operators $A_p \in K_A$ was defined by numerical methods and it equal $h$ = 0.121.

An error initial data for a case $Z = U = L_2[0,T]$ is equal $\delta = 0.0665$ Mнм.
We shall choose the functional $\Omega\,[z]$ as

$$\Omega[z] = \int_0^T (\dot{z}^2 + z^2)dt\,. \qquad (13)$$

The most typical case of rolling on a smooth working barrels was chosen for processing when the upsetting of fluctuations are not observed and when there is not skid [4,8].
In a Figure the diagrams of functions $z_{un}$, $z_{\min}$ for a typical case of rolling on a smooth worker barrel are submitted.

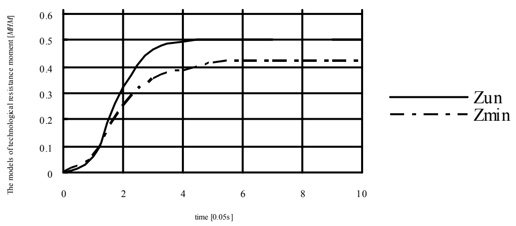


Fig.3. The diagrams of the moment models of technological resistance on the rolling mill.

The results of calculations are showing that the rating from above of accuracy of mathematical modeling with model $z_{un}$ for all $A_p \in K_A$ does not exceed 11 % in the uniform metrics with error of MM parameters of the main mechanical line of rolling mill in average 10 % and errors of experimental measurements 7 % in the uniform metrics.
The calculations of model of EL $\tilde{z}_p$ for a class of models $K_A$, $K_B$ on set of the possible solutions $Q_{\delta,D}$ was executed for comparison. The function which is the solution of a problem of synthesis in this case has the maximal deviation from zero equal 0.01 Mнм. Such model does not represent interest for the purposes of mathematical modeling as it practically coincides with trivial model.
In work [4] the comparative analysis of mathematical modeling with various known models of external load was executed. Thus the model of load $z_{un}$ turn out to be correspond to experimental observations in the greater degree.

## 3 Conclusions

In paper some problems of construction of external load models for dynamic systems with inexact description by help of identification method is considered. The different formulations of such problem is offered: the stable model for obtaining the best results of mathematical modeling with guarantee, the stable model for obtaining evaluation of response from above, stable model for obtaining evaluation of response from below, stable model for mathematical modeling of the selected motion with the fixed model of dynamic system, the stable model for mathematical modeling of the selected motion of system for whole class of mathematical descriptions of system.
The offered approach to synthesis of mathematical models of external loads can find application in cases when the information about external loads is absent or poor and also for check of hypotheses on the basis of which were constructed the known models of external loads.

## 4 References and citations


[1] Ju. Gelfandbein, L. Kolosov. Retrospective identification of perturbations and interferences: Moscow, Science. 1972.

[2] S.Ikeda, S.Migamoto, Y.Sawaragi. Regularization method for identification of distributed systems. Proc. of IY a Symposium IFAC, *Identification and evaluation of parameters of systems*. Tbilisi, USSR, v.3, Preprint. Moscow, 153-162. 1976.

[3] Yu.Menshikov, Identification of external impacts models. *Bulletin of Kherson State Techn. Univ*., Cherson, Ukraine, 2(15): 326-329,2002.

[4] Yu.Menshikov. The Models of External Actions for Mathematical Simulation. *System Analysis and Mathematical Simulation (SAMS*), New-York, v.14, n.2:139-147,1994.

[5] A.Tikhonov, V.Arsenin. Methods of solution of incorrectly problems, Moscow, Science, 1979.

[6] Yu.Menshikov, Identification of external impacts under minimum of a priori information: statement, classification and interpretation. *Bulletin of KNU, Mathematics*, Kiev, Ukraine, 2:310-315,2004.

[7] A.Tikhonov, A.Goncharsky, V.Stepanov and A.Yagola, Numerical methods for the solution of ill-posed problems, Moscow, Science, 1990.

[8] Yu.Menshikov. The synthesis of external impact for the class of models of mechanical objects, *J. of Differential equations and their applications in Physics.* Dnepropetrovsk University, Dnepropetrovsk, Ukraine, 86-91,1985.