USE OF SEMI-EMPIRICAL PRESSURE RELIEF VALVE MODEL IN FREQUENCY DOMAIN ANALYSIS OF FLUID POWER CIRCUITS

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Abstract

The semi-empirical method for modeling various fluid power components has appeared to be very effective and thus it is used in many time-domain simulation programs. In studies carried out early 90's effective non-linear time-domain semi-empirical models for pressure relief valves were proposed. In this method an analytical model is brought into a form in which its parameters can be identified by using measured characteristic curves. This paper shows that this model can also be used in frequency-domain analysis of fluid power circuits including short or long pipelines. This model provides similar advantages to as it provides in time-domain analysis, i.e. it is not required to dismantle the valve to identify the parameters. The model is also in a compact form in which the number of parameters that can be identified from measured characteristic curves is the lowest possible. The applicability of the linearized semi-empirical model is demonstrated in analyzing dynamics of fluid power circuits with short and long pipelines. The frequency domain analysis of the fluid power circuits are mainly needed in stability analysis on closed loop systems and in pressure transient analysis of long pipelines. The advantage of the presented model is clearly its usability over the entire operating range of the system. It is because its parameters are firstly identified in non-linear time-domain and then the model is linearized in the vicinity of the selected operating point.

Keywords: pressure relief valve, modeling, semi-empirical, frequency domain

Presenting Author's biography

Heikki Handroos has been Prof. of Machine Automation and Head of Institute of Mechatronics and Virtual Engineering in Lappeenranta University of Technology since 1993. He earned his M.Sc and D.Sc degrees in Tampere University of Technology, 1985 and 1991. His research interests range from modeling, simulation and control of servodrives to serial and parallel robotics. He has published about 150 scientific journal and conference papers.

1 Introduction

An analytical model for a fluid power component uses physical equations that describe the behaviour of the internal elements and their connections. Its parameters consists of dimensions of the internal elements spring constants etc. The major drawback of such a model is that the component has to be dismantled to determine their values. In most cases commercially available components are used in the circuit that is to be analysed. It was shown in [1] and [3] that the analytical model of a pressure relief valve can be brought into a form in which the parameters can be identified from measured characteristic curves. This information is partly available in manufacturers catalogues. It can also quite easily be measured in a simple laboratory rigs. The frequency-domain analysis of hydraulic circuits is less popular than time-domain analysis because the stability and resonance behaviour is normally important only in special cases such as servo-systems, long pipelines, load-sensing pump circuits, counter balance valve circuits etc. The fluid power circuits are also quite non-linear systems and thus their dynamics is highly dependent on the operating point. To get a complete understanding of its behaviour system should be studied in several operating points. Despite this the frequency–domain analysis can be valuable in special cases. In long pipeline dynamics analysis pressure relief valves are often modelled roughly using intuitive approach [5]. This paper proposes systematic approach by means of which relief valve models can quite easily be postulated in frequency-domain.

2 Semi-empirical frequency-domain models for a relief valve

2.1 Nonlinear time-domain model

For a commonly used relief valve construction shown in Fig. 1 the following non-linear analytical model can be derived [1]. The force balance for the poppet is

$$m\ddot{x} + k_m x = F_p - F_{f1} + F_{f2} - F_o \qquad (1)$$

In (1) F_p is the pressure force that can be expressed as

$$F_p = p_2 A \qquad (2)$$

The reaction and impulse flow forces respectively can be written as follows

$$F_{f1} = 2 \cdot C_d \cdot p_1 \pi D_1 x \sin\alpha \cos\alpha \qquad (3)$$

$$F_{f2} = \beta p_1 x \qquad (4)$$

The continuity equations for the main flow and leakage flow respectively are

$$Q_1 = C_d \pi D x \sin\alpha \sqrt{\frac{2 \cdot p_1}{\rho}} = K\sqrt{p_1} \qquad (5)$$

$$Q_2 = k_c (p_1 - p_2) = A\dot{x} \qquad (6)$$

The model Eqs.(1)-(6) has nine parameters that compose of physical constants, geometrical dimensions etc. that cannot be identified without dismantling the valve.

It is shown in [1] that Eqs (1)-(6) can be brought into the semi-empirical form

$$\begin{aligned} &\ddot{K} + 2C_3C_4\dot{K} + C_2^2(C_1 + C_2 p_1)K \\ &= C_3^2(p_1 - p_{ref}) \\ &Q_1 = K\sqrt{p_1} \end{aligned} \qquad (7)$$

in which there are only four parameters whose physical equivalents are as follows

$$C_1 = \frac{k_m}{AC_d\pi D\sin\alpha\sqrt{\frac{2}{\rho}}} \qquad C_2 = \frac{\cos\alpha}{C_d A\sqrt{\frac{2}{\rho}}}$$

$$C_3 = \sqrt{\frac{A\,C_d\pi D\sin\alpha\sqrt{\frac{2}{\rho}}}{m}} \qquad C_4 = \frac{A^2}{2 \cdot m \cdot k_d C_3} \qquad (8)$$

K in Eq.(7) is a variable including cross section area of valve orifice, fluid density and discharge coefficient.

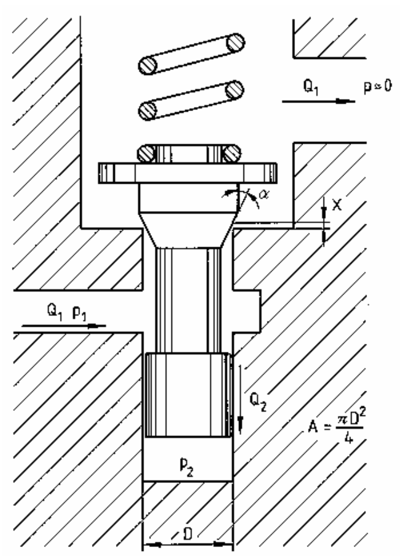

Fig. 1 Commonly used relief valve construction

It is shown in [1] that the values for the parameters $C_1 \ldots C_4$ parameters can be identified from the measured characteristic curves of the valve to be modelled. The method has two important advantages: firstly, it is unnecessary to dismantle the valve to identify the parameters and secondly, a single valve model can describe various valve constructions as shown in [4].

2.2 Relief valve with compressible volume

Fig. 2 shows the test circuit in which a step response on the valve can be measured. If the pump in Fig. 2 is assumed to be an ideal flow source, the relief valve with the compressible oil volume can be described by Eq.1 and the following compressible flow Eq.

$$\dot{p}_1 = \frac{B_e}{V}(Q_p - Q_1 - Q_e) \tag{9}$$

By neglecting the second order term in Eq.(7) and selecting an operating point p_{10} , Q_{10} and K_0 as well as assuming Q_e=0, the transfer function (between pump flow and pressure) of relief valve with compressible volume can be obtained as follows

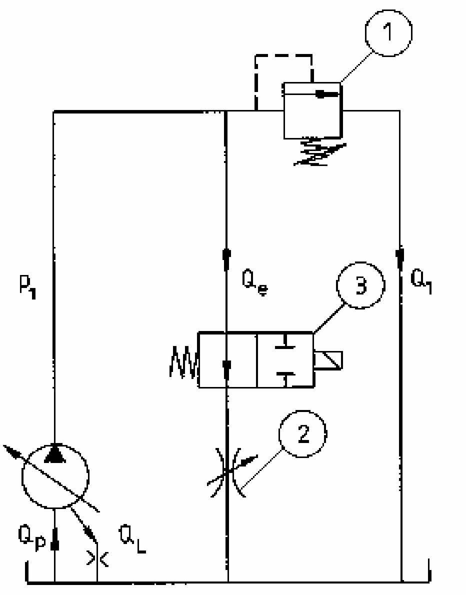

Fig. 2 Test circuit for relief valves

$$\frac{p_1}{Q_p}(s) = \frac{a_1 s + a_0}{\dfrac{s^2}{\omega_0^2} + \dfrac{2\delta_0 s}{\omega_0} + 1} \tag{10}$$

where

$$\omega_0 = \sqrt{\frac{\dfrac{B_e}{V}K_3 + \dfrac{B_e}{V}K_1K_3 - \dfrac{B_e}{V}K_4C_1 - \dfrac{B_e}{V}K_2K_4}{K_5}}$$

$$\delta_0 = \frac{\omega_0}{2}\left(\frac{C_1 + K_2 + \dfrac{B_e}{V}K_4K_5}{\dfrac{B_e}{V}K_3 - \dfrac{B_e}{V}K_1K_3 - \dfrac{B_e}{V}K_4C_1 - \dfrac{B_e}{V}K_2K_4}\right)$$

$$a_0 = \frac{C_1 + K_2}{K_3 - K_1K_3 - K_4C_1 - K_2K_4}$$

$$a_1 = \frac{K_5}{K_3 - K_1K_3 - K_4C_1 - K_2K_4}$$

and

$$K_1 = C_2K_0$$

$$K_2 = C_2 p_{10}$$

$$K_3 = \sqrt{p_{10}}$$

$$K_4 = \frac{K_0}{2\sqrt{p_{10}}}$$

$$K_5 = 2\frac{C_4}{C_3}$$

Neglecting the second order dynamics of the valve can be justified, since the inertial force is normally much smaller than the spring force and hydraulic forces due to small poppet mass. At this point, it must be noted that constants $K_1…K_5$ consist of the original parameters $C_2…C_4$ and operating point data K_0, p_{10} and Q_{10}. Value of K at the operating point can easily be found as

$$K_0 = \frac{Q_{10}}{\sqrt{p_{10}}} \tag{11}$$

The parameters can easily be identified from static and dynamic characteristic curves of the valve as shown in [1]. This feature makes this method superior also in the frequency domain analysis. Fig. 3 shows comparison of measured step responses and those obtained by using the non-linear model Eq.(7) [1],[3] in case of commercial relief valve. By using the same parameter data and corresponding operating points three frequency responses are calculated by Eq.(10). Fig. 4 shows the results. It can be concluded that transfer function Eq.(10) corresponds well the dynamics of real valve (when comparing overshoot, resonance peak and resonance frequency). Tab. 1 shows the used parameter values.

2.3 Relief valve with long pipeline

In long pipeline dynamics analysis [5] the transfer function of relief valve must be postulated between valve flow Q_1 and pressure p_1.
Again by neglecting the second order term in Eq.(7) and selecting an operating point p_{10}, Q_{10} and K_0 the transfer function can be derived as follows

$$\frac{Q_1}{p_1}(s) = \frac{K_4 K_5 s + K_4 (C_1 + K_2) + K_3 (1 - K_1)}{K_5 s + C_1 + K_2} \tag{12}$$

where $K_1 \ldots K_5$ are as shown in Eq.(10). Now by calculating the frequency responses by Eq.(12) in the same operating points as before the result Fig. 5 are obtained. It can be concluded from the Fig. that relief valve works in a long pipeline similarly to a first order filter as concluded in [5]. In addition to this from Eq.(10) it can also be concluded that the model has also a zero that decreases the phase lag in larger frequencies. This is an interesting finding although it hardly has important effect into the frequency response of a long pipeline in the normal frequency range.

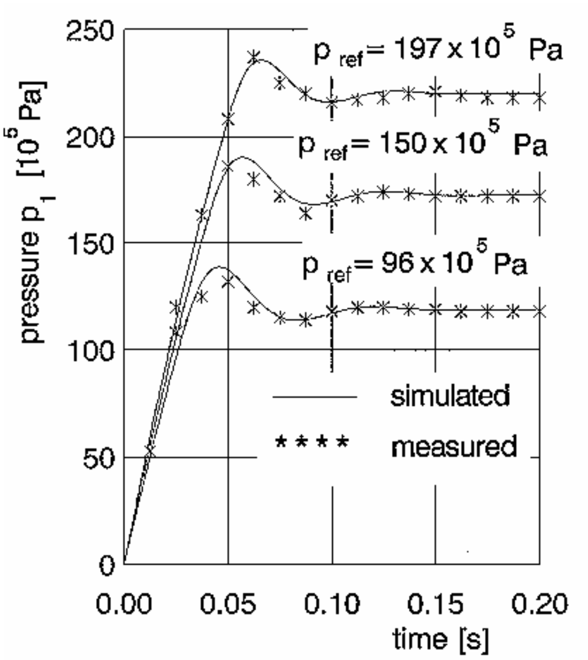

Fig. 3 Experimental verification of non-linear model

3 Numerical Examples

3.1 Circuit with short pipelines

Fig. 6 shows a simple fluid power circuit including constant displacement pump, pressure relief valve, 3/3-directional valve and single-acting cylinder with mass load.

If it is needed to study cylinder speed v as a function of pump flow Q_p the following linearized equations can be derived as follows: By using the relief valve model with the compressible volume Eq.(10) pressure p_1 becomes

$$p_1 = \frac{a_1 s + a_0}{\frac{s^2}{\omega_0^2} + \frac{2\delta_0 s}{\omega_0} + 1}(Q_p - Q_v) \tag{13}$$

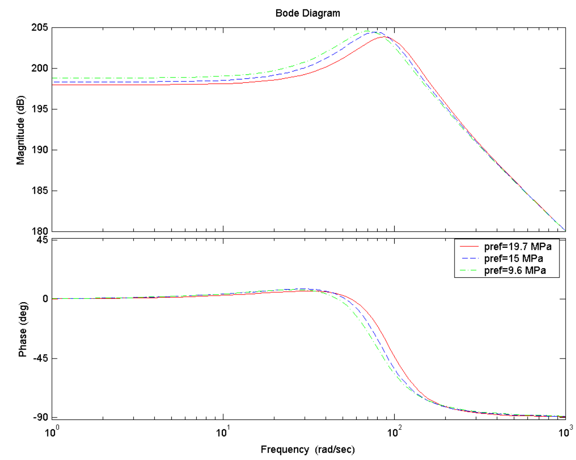

Fig. 4 Frequency responses of relief valve with compressible volume

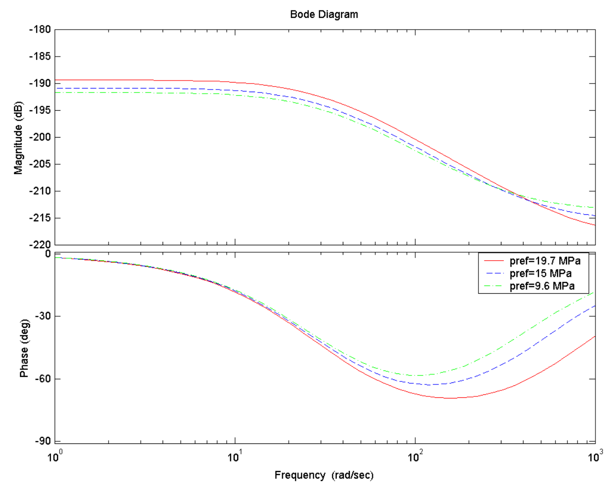

Fig. 5 Frequency responses of relief valve without compressible volume

Directional valve flow Q_v is

$$Q_v = K_v p_1 - K_v p_2 \tag{14}$$

where $K_v = \frac{c_v}{2\sqrt{p_{10} - p_{20}}}$

Pressure p_2 becomes

$$p_2 = \frac{B_{e2}}{Vs}(Q_v - vA) \tag{15}$$

The cylinder speed is

$$v = \frac{A}{ms + b} p_2 \tag{16}$$

Fig 7. shows the block diagram of the system. From Eqs.(13…16) the transfer function between the pump flow Q_p and the cylinder speed v can be derived as follows

$$\frac{v}{Q_p}(s) = \frac{a_1 s + a_0}{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{17}$$

where

$$b_4 = \frac{mV_2}{K_v B_{e2} A \omega_0^2}$$

$$b_3 = \frac{2mV_2\delta_0}{K_v B_{e2} A \omega_0} + \frac{bV_2}{K_v B_{e2} A \omega_0^2} + \frac{mV_2 a_1}{B_{e2} A}$$

$$b_2 = \frac{mV_2}{K_v B_{e2} A} + \frac{2bV_2\delta_0}{K_v B_{e2} A \omega_0} + \frac{A}{K_v \omega_0{}^2} + \frac{mV_2 a_0}{B_{e2} A} + \frac{bV_2 a_1}{B_{e2} A}$$

$$b_1 = \frac{bV_2}{K_v B_{e2} A} + \frac{2\delta_0 A}{K_v \omega_0} + \frac{bV_2 a_0}{B_{e2} A} + a_1 A$$

$$b_0 = \frac{A}{K_v} + a_0 A$$

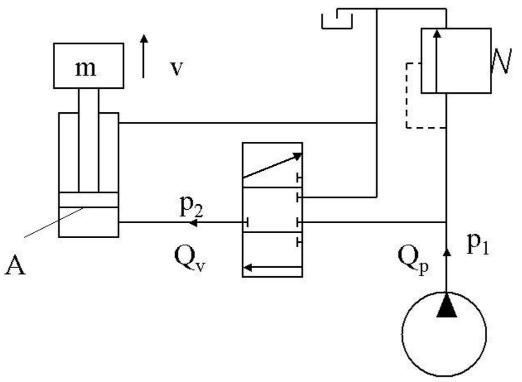

Fig. 6 Example circuit with short pipelines

Fig. 8 shows the frequency responses of Eq.10 with two different values for parameter C_4 describing the relief valve dynamics. Tab. 2 shows the other parameter values used in the calculations.

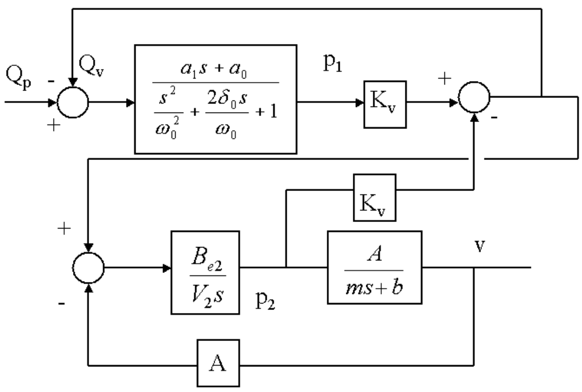

Fig. 7 Block diagram of test system

It can clearly be concluded from Fig. 8 that the resonance frequency of the cylinder-mass system is dominating the system behaviour. In spite of this with about five times larger value for relief valve parameter C_3 highlights the importance of relief valve dynamics that can clearly be seen from the blue curve in Fig. 8.

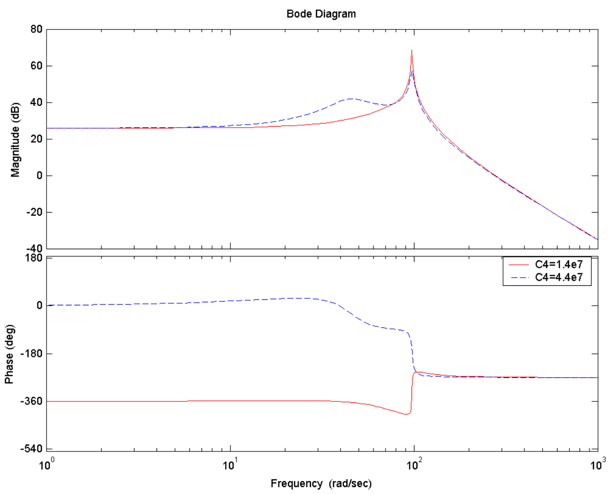

Fig. 8 Frequency response of test system with two values of the dynamic parameter of relief valve

This example has shown that the semi-empirical pressure relief valve model is effective tool in frequency domain analysis of hydraulic circuits with short pipelines especially when relief valve dynamics has important effect into the system behavior.

3.2 Circuit with long pipelines

In [5] a systematic approach is developed for analyzing fluid power circuits with long pipelines in frequency domain. Also inclusion of such components as a relief valve, pressure compensated pump, accumulator and various kinds of passive filters is described. The proposed pressure relief valve model is a very simple and by means of it factors like flow force and effect of operating point cannot be taken into account. By using the semi-empirical model Eq.(12) proposed in this study the operating point effects can easily be studied. Fig.9 shows the circuit to be modeled. It includes a constant displacement pump, relief valve two 6.17 m long pipelines and an orifice.

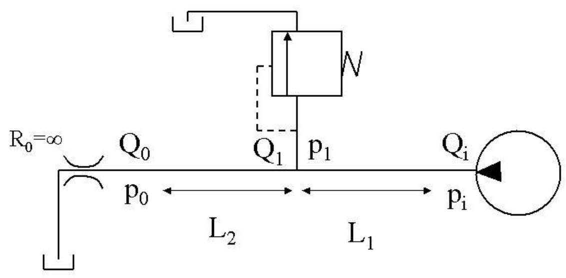

Fig. 9 Circuit with long pipelines

By using the chain rule the dependency of flow and pressure (p_0,Q_0) in the end of the second pipe on the input pressure and flow (Q_i ,p_i) of the first pipe can be derived as follows

$$\begin{bmatrix} A_{L2} & B_{L2} \\ C_{L2} & D_{L2} \end{bmatrix} \begin{bmatrix} 1 & -G \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_{L1} & B_{L1} \\ C_{L1} & D_{L1} \end{bmatrix} \begin{bmatrix} Q_i \\ p_i \end{bmatrix} = \begin{bmatrix} Q_0 \\ p_0 \end{bmatrix} \qquad (18)$$

where G is the transfer function of relief valve. By using the semi-empirical model Eq.(7) for relief valve, the boundary condition

$$p_0 = R_0 Q_0 \qquad (19)$$

and the following expressions for $A_{L1}...D_{L2}$

$$\begin{aligned} A_{L1} &= A_{L2} = \cosh T\sqrt{Ns} \\ B_{L1} &= B_{L2} = \frac{-\sinh T\sqrt{Ns}}{Z_c\sqrt{N}} \\ C_{L1} &= C_{L2} = -Z_c\sqrt{N}\sinh T\sqrt{Ns} \\ D_{L1} &= D_{L2} = \cosh T\sqrt{Ns} \end{aligned} \qquad (20)$$

where

$$N = \frac{\alpha_v}{s} + 1 + \frac{0.1515}{1+0.3030\dfrac{s}{\alpha_v}} + \frac{0.1620}{1+0.04\dfrac{s}{\alpha_v}} + \frac{0.020}{1+0.001\dfrac{s}{\alpha_v}}$$

The amplitude $\frac{p_0}{p_i}$ can be calculated as a function of natural angular velocity. It must be noted that in this particular example two pipes with equal dimensions are used to simplify the model Eq.(18). The results are shown in Fig. 10. To highlight the capability of semi-empirical model to take into account the operating point the results are calculated in three operating points that are exactly similar to those in Fig.3. Tab. 3 shows the parameter values used in calculating the responses in Fig.10. Also to show the influence of valve parameter on the response dynamic parameter C_4 is made four times larger. Fig. 11 shows the dramatic effect of the valve dynamics into the first resonance amplitude.

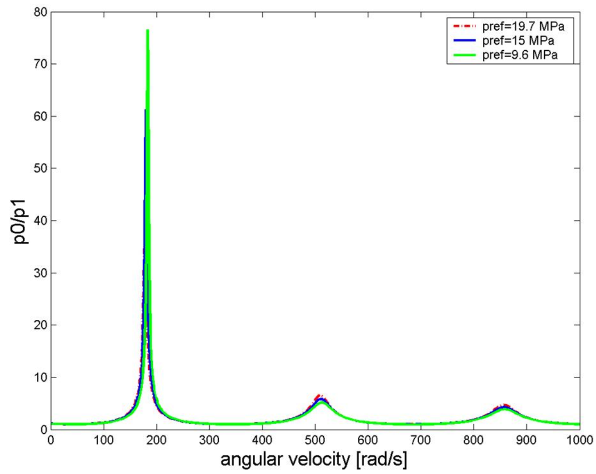

Fig. 10 Dynamics of test system with long pipelines

4 Conclusions

The present paper shows how the semi-empirical non-linear pressure relief valve model can effectively used in frequency domain analysis of fluid power circuits including short or long pipelines. The model is based on physical analytical model and its parameter values can be identified from the measured characteristic curves provided by manufacturer or can easily be measured in a simple test rig. It is shown how the original time-domain model can easily be transformed into frequency domain and expressed in the forms normally used in circuit analysis both in case of short pipelines and long pipelines. Numerical examples are given in analyzing two circuits; one with short and other with long pipelines. The results show how the operating point or parameter influences can easily be studied by means of the proposed model.

5 Future extensions

This approach can also be applied in cases of semi-empirical models for pressure reducing valve, two-way flow control valve, three-way flow control valve and a counter balance valve. Semi-empirical models for these are already derived in [1],[2],[3] and [4].

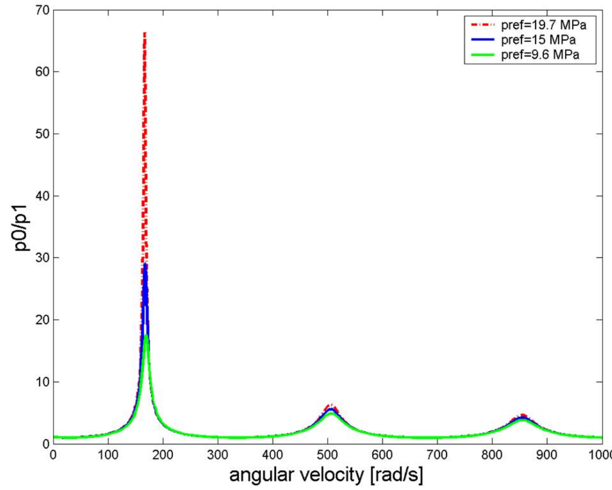

Fig. 11 Influence of valve parameter C_4

Tab. 1 Values of parameters and other constants used in relief valve simulation (Figs. 3-5)

$p_{ref}[10^5Pa]$	96	150	197
B_e [Mpa]	1500	1500	1500
$V [10^{-3}m^3]$	1.5	1.5	1.5
$Q_{10}[dm^3/min]$	30	30	30
$p_{10} [10^5Pa]$	120	170	220

$C_1=1.36*10^{13}$

$C_2=8.56*10^5$

$C_3=6.0*10^{-5}$

$C_4=1.4*10^7$

Tab. 2 Parameters and other constants used in simulating circuit Fig.6

Be_2[Mpa]	1000
b [Ns/m]	3500
M[kg]	1000
$V_2[10^{-3}m^3]$	1
K_v[m3/sPa]	$8.82*10^{-12}$
$P_{10}[10^5Pa]$	220
$p_{20}[10^5Pa]$	33
$Q_{10}[dm^3/min]$	30
$A[10^{-4} m^3]$	31.3

Tab. 3 Parameters and other constants used in simulating circuit Fig.9

T[s]	0.0045
α_v[1/s]	14.2
$Z_c[Ns/m^5]$	$0.922*10^{10}$

Nomenclature

A	piston area, poppet area	$[m^2]$
a_0,a_1	transfer function parameters	
$b_0 \ldots b_4$	transfer function parameters	
b	viscous friction coefficient	[Ns/m]
B_e, B_{e2}	effective bulk modulus,	[Pa]
C_d	discharge coefficient	
$C_1 \ldots C_4$	semi-empirical valve parameters	
D_1	poppet diameter	[m]
F_{f1}, F_{f2},	flow force	[N]
F_p	Pressure force	[N]

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F_0	Preload spring force	[N]
G	transfer function	
$K_1 \ldots K_5$	linearization coefficients	
K	variable including cross section area of valve orifice, discharge coefficient and oil density	[m3/s√Pa]
k_m	spring constant	[N/m]
K_v, k_e	flow-pressure coefficient	[m^3/sPa]
m	mass, poppet mass	[kg]
p_0,p_1,p_2	pressure	[Pa]
p_{ref}	set pressure of relief valve	[Pa]
Q_p	pump flow,	[m^3/s]
Q_e , Q_v	directional valve flow	[m^3/s]
R_0	pressure-flow coefficient	[Pas/m^3]
T	wavepropagation time	[s]
V,V_2	oil volume,	[m^3]
x	poppet position	[m]
Z_c	charateristic impedance	[Ns/m^5]
α	poppet angle	[rad]
α_v	viscosity factor	[1/s]
β	relative effect factor of impulse flow force	
ω_0	natural angular velocity of relief valve	[rad/s]
δ_0	damping factor of relief valve	
ρ	oil density	[kg/m^3]

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