DYNAMIC MODEL OF ELECTRICAL WHEELCHAIR WITH SLIPPING DETECTION

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Abstract

A power wheelchair was designed and built to faciliate the mobility for disabled people. Past few decades, the number of electric wheelchair users has increased as a result of an aging poplations, health care and technological advances. A dynamic model for power wheelchair taking into account the slipping effect that takes place while driving under non-normal conditions is presented in this paper. The parameters of the slipping will be also estimated. The wheelchair that considered in this paper is composed of two independently driven rear wheels and two castors. Electric wheelchairs have always very heavy weight. This makes them easy to slip under unfavorable road conditions. It is estimated that there are many serious accidents annually because of this problem. A method for detecting wheelchair slipping is also introduced. Simulated results are given demonstrating the applicability of the proposed method. Unlike other methods, the proposed scheme is based on only wheel speed measurements and does not require any information on the work space or wheelchair acceleration. Another advantage of this method is that its implementation cost is low.

Keywords: dynamic modeling, slipping, acceleration, wheelchair.

Presenting Author's Biography

Hamed Emam is a PhD student at the University of Versailles Saint Quentin in Yvelines, Systems Engineering Laboratory of Versailles (LISV). His thesis subject is"Dynamic modeling and control design for electrical wheelchair". This work is registered within the Assistance and Handicap (AH) team of LISV. This team aims to develop the evaluation methodology and adaptation processing of assistive technology for disabled people.

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1 Introduction

The assistance devices enable disabled people to perform many activities of daily living. An example of these devices is electric power wheelchair, which provides means for independent mobility to individuals with disability or who do not possess the physical capacity to propel a manual wheelchair. The number of individuals that will be need the electric power wheelchair is increasing every year due to the improved health care and reduction of the mortality rates. Around the world, millions of disabled young and elderly people rely on wheelchairs in their daily lives [1]. During last decade, many researchers have oriented their work towards this field [2, 3, 4]. Several research groups have developed many modifications on power wheelchairs such as avoiding obstacles, passage through doors, and wall following [2, 5, 6]. Very few address the dynamic problem.

The study of the slipping problem for power wheelchairs is important for many reasons. Firstly, today, power wheelchairs are used under unfavorable road conditions. Due to snow, ice or leaves, on the line of the wheelchair, the slipping often occurs. Secondly, the weight of a wheelchair with a person is relatively high, and under abnormal driving conditions, it is found that the probability of slipping is high. Also, it is estimated that there are many number of serious accidents annually because of this problem. The authors believe that improving the physical safety and operation robustness necessitates the detection of slipping. Furthermore, the first step toward producing an effective control is determining the model taking into account this parameter

This paper is organized as follows. In section 2, the dynamic model of the wheelchair is giving taking into account the slipping phenomenon. In section 3, the a slipping detection method is presented. In section 4, simulation results are given. Finally, in section 5, we give some concluding remarks.

2 Dynamic Modeling

The structure of wheelchairs differs depending on their intended use. The wheelchair that considered in this paper is composed of two independently driven rear wheels and two castors, freewheeling, at the front, as shown in Fig.1. Two identical electric motors produce torques influencing the rotation angle of the two drives. The difference in angular velocity defines the direction of the wheelchair movement. The derivation of the dynamic model relies on Lagrange formalism [7]. In this method, the difference between the kinetic and potential energy of the system is used. Because of the assumption that the wheelchair is moving only horizontally, the potential energy is constant. The kinetic energy of the entire wheelchair is a combination of the kinetic energy derived from individual parts of the wheelchair. Components of the coordinates vector are shown in Fig.1. The vector is:

$$q = [x_{\mathrm{G}} \quad y_{\mathrm{G}} \quad \phi \quad \theta_d \quad \theta_g]^{\mathrm{T}}$$

Where:
x_{G}, y_{G} are the coordinates of the wheelchair center
ϕ is the orientation angle of the wheel chair axis, and
θ_g, θ_d, are the angular positions of the left and right rear wheels, respectively.

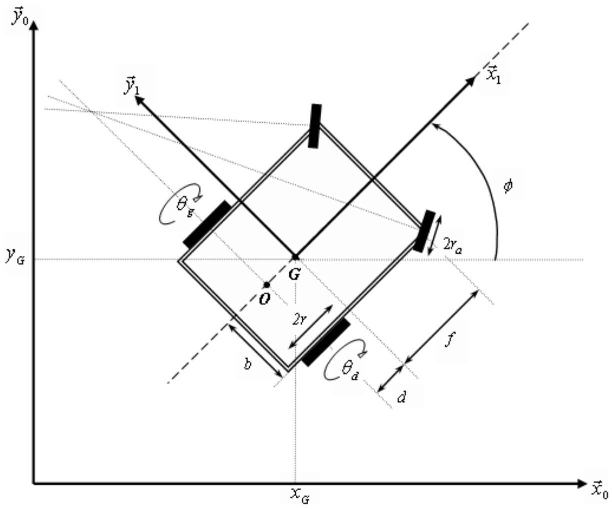

Fig. 1

The entire kinetic energy of the system is:

$$T = \frac{1}{2}M(\dot{x}_{\mathsf{G}}^2 + \dot{y}_{\mathsf{G}}^2) + \frac{1}{2}I\dot{\phi}^2 + M_p d\phi(\dot{y}_{\mathsf{G}}\cos\phi - \dot{x}_{\mathsf{G}}\sin\phi) + \frac{1}{2}I_t(\dot{\theta}_d^2 + \dot{\theta}_g^2) \tag{1}$$

Where:
$I = I_p + 2m_r(d^2 + b^2) + 2I_m$
$M = M_p + 2m_r$
M_p : Mass of wheelchair without the driving wheels and the rotor of its motors;
m_r : Mass of each driving wheels plus the rotor of its motor;
I_p : Moment of inertia of the wheelchair without the driving wheels about a vertical axis throug point O;
I_t : Moment inertia of the motrized wheel about axes;
I_m : Moment of inertia of each wheel and the motor rotor about the wheel diameter.

Using the Lagrange formalism [7], the following vector matrix form of the equation of wheelchair dynamics is obtained:

$$M(q)\ddot{q} + C(q,\dot{q}) = E\tau - A^{\mathrm{T}}\lambda \tag{2}$$

Where:$M(q)$is inertia matrix,
$C(q,\dot{q})$,$\mathbf{A}(q)$ and $E(q)$ are given by:

$$C(q,\dot q) = \begin{bmatrix} -M_p d\dot\phi^2 \cos\phi \\ -M_p d\dot\phi^2 \sin\phi \\ 0 \\ 0 \\ 0 \end{bmatrix}, E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A(q) = \begin{bmatrix} -\sin\phi & \cos\phi & -d & 0 & 0 \\ -\cos\phi & -\sin\phi & -b & r & 0 \\ -\cos\phi & -\sin\phi & b & 0 & r \end{bmatrix}$$

Where:
$Q = E\boldsymbol{\tau}$ is the matrix the couples motors;
$\boldsymbol{\tau} = [\tau_d \quad \tau_g]^{\mathrm{T}}$ are the torques acting on the wheel axis generated by the right and left motors;
$\boldsymbol{\lambda}_{3\times1} = [\lambda_1 \quad \lambda_2 \quad \lambda_3]^{\mathsf{T}}$are the Lagrange multipliers.
Eq.(2) gives the dynamic model of the wheelchair without slipping.

The model with slipping is derived as fallows: first eliminate the Lagrange multipliers then add the defaults of the slipping. It must be noted that the matrix of fraction coefficient is adjoined to the dynamic model as in [8, 9].After eliminations the Lagrange multipliers, we calculate the matrix $S(q)$ that satisfies the following relations:

$$\begin{cases} A(q)S(q) = 0 \\ \dot q = S(q)v \end{cases} \tag{3}$$

where:
$v = \begin{bmatrix}\dot\theta_d & \dot\theta_g\end{bmatrix}^{\mathsf{T}}$ is new vector coordinates and $S(q)$:

$$S(q) = \begin{bmatrix} \frac{r}{2}(\cos\phi - \frac{d}{b}\sin\phi) & \frac{r}{2}(\cos\phi + \frac{d}{b}\sin\phi) \\ \frac{r}{2}(\sin\phi + \frac{d}{b}\cos\phi) & \frac{r}{2}(\sin\phi - \frac{d}{b}\cos\phi) \\ \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We introduce the slipping parameters in the general coordinate, as the slipping occurred at the powered wheels also the center of mass very near to these wheels. So, the slipping parameters will be introduced as in the following formula:

$$\dot q = S(q)v + \boldsymbol{\delta} \tag{4}$$

Where:
$\boldsymbol{\delta} = \begin{bmatrix}\dot{\tilde x}_{\mathrm{G}} & \dot{\tilde y}_{\mathrm{G}} & \dot{\tilde\phi} & 0 & 0\end{bmatrix}^{\mathrm{T}}$:Parameters of slipping that can be determined by encoder on the front wheels of the wheelchair.
$\dot{\tilde x}_{\mathrm{G}}$:The speed of longitude slipping,
$\dot{\tilde y}_{\mathrm{G}}$: The speed of lateral slipping
$\dot{\tilde\phi}$ Deviation of orientation
to get $\ddot q$ Eq.(4) is differentiated with respect to q:

$$\ddot q = S(q)\dot v + \dot S(q)v + \dot{\boldsymbol{\delta}} \tag{5}$$

By adding the matrix of fraction coefficient and applying the combining Eqs.(5) and (2)the following relationship is obtained:

$$\begin{aligned} MS\dot v + M\dot S v + M\dot{\boldsymbol{\delta}} + F(Sv + \boldsymbol{\delta}) \\ +C = E\boldsymbol{\tau} - A^{\mathrm{T}}\boldsymbol{\lambda} \end{aligned} \tag{6}$$

The matrix F as in [10]. Multiplying Eq.(6) in the transport of $S(q)$, gives:

$$\begin{aligned} S^T MS\dot v + S^T M\dot S v + S^T M\dot{\boldsymbol{\delta}} + S^T FSv + \\ S^T F\boldsymbol{\delta} + S^T C = S^T E\boldsymbol{\tau} \end{aligned} \tag{7}$$

Where:
Eq.(7) represents the dynamic model with slipping. For simplification purposes, the dynamic model is written in the following form:

$$\dot v = [H]v + [B]\boldsymbol{\tau} + [C] \tag{8}$$

Where:
$[H] = -(S^T MS)^{-1}(S^T M\dot S + S^T F)$

$[B] = -(S^T MS)^{-1}$

$[C] = -(S^T MS)^{-1}(S^T M\dot{\boldsymbol{\delta}} + S^T F\boldsymbol{\delta} + S^T C)$

2.1 Determining the slipping parameters

Determining the parameters of slipping is performed using the absolute and relative measurements. The principal idea is using encoders to determine the real velocity and orientation (absolute) of the wheelchair. This is achieved by using two encoders on the front wheels (caster wheels). It is assumed that the speed of these wheels reflect the real situation of the wheelchair (position and speed). This due to the force effect on the driving wheels (rear wheels). Also, the center of the mass is very near to these wheels. Measuring the relative velocity and orientation is achieved by using odometer on the motorised wheel. In addition, one encoder is needed to measure the rotation angle. This encoder is placed on the free wheels.

The structure consists of an encoder on every wheel to measure the velocity and on one of the caster wheels to determine the orientation of the wheelchair (see Fig. 2.)
Using Fig.3, and after mathematic derivation, the real velocities and the orientation of the wheelchair could be estimated as in the following relations:

$$\begin{aligned} \dot x_{\mathsf{Gr}} = (&\frac{r_a}{2}(\dot\theta_{ad}\cos\alpha_d + \dot\theta_{ag}\cos\alpha_g) \\ &+ \frac{e}{2}(\dot\alpha_d \sin\alpha_d + \dot\alpha_g \sin\alpha_g))\cos\phi_r \end{aligned} \tag{9a}$$

$$\begin{aligned} \dot y_{\mathsf{Gr}} = (&\frac{r_a}{2}(\dot\theta_{ad}\cos\alpha_d + \dot\theta_{ag}\cos\alpha_g) \\ &+ \frac{e}{2}(\dot\alpha_d \sin\alpha_d + \dot\alpha_g \sin\alpha_g))\sin\phi_r \end{aligned} \tag{9b}$$

$$\begin{aligned} \dot\phi_r = &\frac{r_a}{2(f+d)}(\dot\theta_{ad}\sin\alpha_d + \dot\theta_{ag}\sin\alpha_g) \\ &- \frac{e}{2(f+d)}(\dot\alpha_d\cos\alpha_d + \dot\alpha_g\cos\alpha_g) \end{aligned} \tag{9c}$$

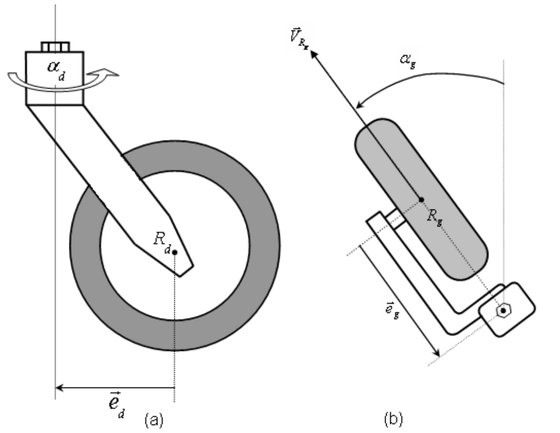

Fig. 2 (a) Schema of the side view of the right front Wheel (b) Schema of the high view of the left front

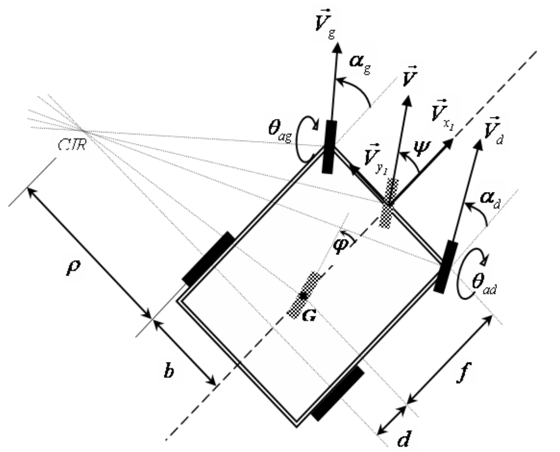

Fig. 3 Geometric Schematics for different parameters

The slipping parameters may then be determined by comparing the measurements of the speed of the active wheels with the measurements of passive wheels as in the following relationships:

$$\tilde{\dot{x}}_{\mathrm{G}} = \dot{x}_{\mathtt{Gr}} - \dot{x}_{\mathtt{Gt}} \quad (10a)$$

$$\tilde{\dot{y}}_{\mathrm{G}} = \dot{y}_{\mathtt{Gr}} - \dot{y}_{\mathtt{Gt}} \quad (10b)$$

$$\tilde{\dot{\phi}} = \dot{\phi}_r - \dot{\phi}_t \quad (10c)$$

where:
$\dot{x}_{\mathtt{Gt}}$, $\dot{y}_{\mathtt{Gt}}$ and $\dot{\phi}_t$ are obtained by using the speed of motorized wheels.

3 Slipping detection

The wheel slip is usually defined as the relative difference of a driven wheel circumferential velocity and its absolute velocity [11, 12]. Slip can also be considered as the reduction in actual vehicle travel speed when compared to the theoretical speed that should be attained from the speed of wheels. This problem occurs when the acceleration of the wheel exceeds the maximum level that could be applied under certain situations (this value depends on friction coefficients, mechanical loads ...). Under normal driving conditions, the wheelchair's velocity is almost the same as wheel velocity. However, when a wheel slips, the wheelchair and wheel velocities (driving wheels) may be quite different. According to this fact, a method is proposed for detecting wheelchair slipping. This is based wheel speed measurements only without any additional information on the environment or wheelchair acceleration.

The principal idea is using encoders to determine the real velocity and orientation (absolute) of the wheelchair, this is achieved by using two encoders on the front wheels (caster wheels). It is found that the speed of theses wheels reflects the real situation of the wheelchair (position and speed). This due to the fact that the orientation wheels are those on the front side but those on the rear side are near the center of gravity. Measuring the relative velocity and orientation is achieved by using odometer on the motorized wheel (in the case of none slipping). For simplification purposes, in order to detect the wheelchair slipping, the expression (11) is used. this depends on the speed of the front (real velocity) and driving (motors velocity) wheels. A value of $\mathbf{S}$ different from zero indicates slipping. It should be noted that the value of $\mathbf{S}$ is proportional to the slipping. In the free rolling case (no slipping) the value of $\mathbf{S}$ is equal or very near to zero. Generally, the absolute value of $\mathbf{S}$ in located in the interval [0, 1]. The parameter $\mathbf{S}$ can be calculated as follows:

$$\mathbf{S} = \frac{\mathbf{h} - \mathbf{g}}{max(\mathbf{h}, \mathbf{g})} \quad (11)$$

where $\mathbf{h}$ is the real velocity of the wheelchair that is obtained by measuring the velocity of front wheels and $\mathbf{g}$ is theoretical velocity that could be obtained by using the angular velocity of the rear wheels (driving wheel).

4 Simulation result

In this section, simulation results are presented. The entire dynamic model of wheelchair, the joystick model, and the control system of the power wheelchair are described by using software tool Matlab/Simulink [13] with its standard blocks, in addition to these; we used m-function for complex calculations. Figs. 4, 5, and 6 give the results at different values of the maximum acceleration (after this value the slipping occurred). The values of the maximum acceleration are 5, 10, and 15. The first curve in every plot represents the speed of the powered wheels (motors speed - theoretical speed), the second represents the real speed (the speed of the front wheels), and the third represents the applied acceleration.

From Figs. 4, 5 and 6, it may be observed that when the applied acceleration is more than the threshold value slipping occurred. In this case, there are differences between the real velocity (the velocity calculated by using the speed of the front wheels) and theoretical velocity of the wheelchair (the velocity calculated by using the speed of the motored wheels). We used an arrow to indicate to this period, it may be noticed that there is a large perturbation in this area. On the other hand, it may be noticed that the real and theoretical velocities are equal when the applied acceleration is less than the threshold acceleration. It must also be noted that the value of the maximum acceleration depends on the environmental conditions. There are many parameters that may effect this value such as the friction coefficient of the wheelchair and the nature of the floor, the mass of wheelchair with person, etc.

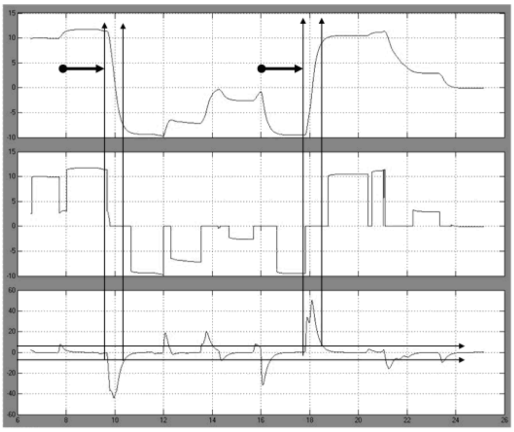

Fig. 4 Slipping with threshold acceleration equal to 5

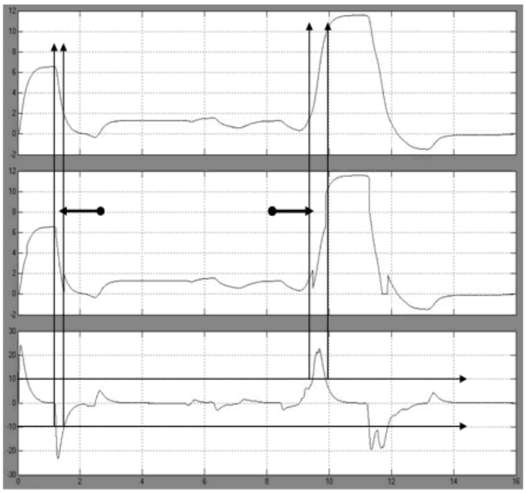

Fig. 5 Slipping with threshold acceleration equal to 10

5 Conclusion

This paper gives kinetic and dynamic models for electric wheelchair taking into account the slipping phenomenon. A new method for detecting the wheelchair slipping is presented. The goal of detecting the slipping is to improve the safety of the electric wheelchair. The simulation results have validated this simple and cost effective method. Unlike other proposals, this method does not require prior information concerning the environment

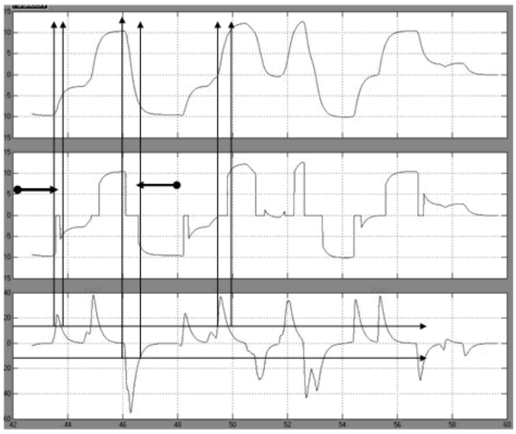

Fig. 6 Slipping with threshold acceleration equal to 15

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A Appendix

Derivation of the slipping parameters

In order to calculate the slipping parameters, first calculate the real orientation and velocity. First apply the Eq.(10) as explained in section 2.1.

For the orientation:

From Fig.3, the real direction of the wheelchair could be calculated as follows:

$$\alpha_d = \arctan\left[\frac{\sin\alpha_g}{\cos\alpha_g + \frac{2b}{(f+d)}\sin\alpha_g}\right] \tag{12}$$

$$\psi = \arctan\left[\frac{\sin\alpha_g}{\cos\alpha_g + \frac{b}{(f+d)}\sin\alpha_g}\right] \tag{13}$$

$$\phi = \arctan\left[\frac{d}{(f+d)}\tan\psi\right] \tag{14}$$

From Eqs. (13) and (14), using Fig. 3. the following expression is obtained:

$$\phi = \arctan\left[\frac{d\sin\alpha_g}{(f+d)\cos\alpha_g + b\sin\alpha_g}\right] \tag{15}$$

$$\dot{\phi}_r = \frac{\mathbf{v}_{y1}}{(f+d)} = \frac{\mathbf{v}}{(f+d)}\sin\psi \tag{16}$$

For the velocity, we have the following relation:

$$\begin{cases} \dot{\mathsf{x}}_{\mathsf{G}} = \mathbf{v}_{x1}\cos\phi \Rightarrow \dot{x}_{\mathsf{G}} = \mathbf{v}\cos\psi\cos\phi \\ \dot{\mathsf{y}}_{\mathsf{G}} = \mathbf{v}_{x1}\sin\phi \Rightarrow \dot{x}_{\mathsf{G}} = \mathbf{v}\cos\psi\sin\phi \end{cases} \tag{17}$$

From Fig. 3, and the geometrical relations, we have the values of $\vec{\mathbf{v}}_d$ and $\vec{\mathbf{v}}_g$ as follows:

$$\begin{cases} \vec{\mathbf{v}}_d = \vec{\mathbf{v}}_{Rd} + \vec{\dot{\alpha}}_d \wedge \vec{e_d} \\ \vec{\mathbf{v}}_g = \vec{\mathbf{v}}_{Rg} + \vec{\dot{\alpha}}_g \wedge \vec{e_g} \end{cases}$$

with:

$$\vec{\mathbf{v}}_{Rd} = \begin{bmatrix} r_a\dot{\theta}_{ad}\cos\alpha_d \\ r_a\dot{\theta}_{ad}\sin\alpha_d \\ 0 \end{bmatrix}, \vec{\mathbf{v}}_{Rg} = \begin{bmatrix} r_a\dot{\theta}_{ag}\cos\alpha_g \\ r_a\dot{\theta}_{ag}\sin\alpha_g \\ 0 \end{bmatrix}, \vec{\dot{\alpha}}_d = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha}_d \end{bmatrix},$$

$$\vec{\dot{\alpha}}_g = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha}_g \end{bmatrix}, \vec{\mathbf{e}}_d = \begin{bmatrix} -e\cos\alpha_d \\ -e\cos\alpha_d \\ 0 \end{bmatrix}, \vec{\mathbf{e}}_g = \begin{bmatrix} -e\cos\alpha_g \\ -e\cos\alpha_g \\ 0 \end{bmatrix}$$

So the result may be written in the following form:

$$\begin{cases} \vec{\mathbf{v}}_d = \begin{bmatrix} r_a\dot{\theta}_{ad}\cos\alpha_d + \dot{\alpha}_d e\sin\alpha_d \\ r_a\dot{\theta}_{ad}\sin\alpha_d - \dot{\alpha}_d e\cos\alpha_d \\ 0 \end{bmatrix} \\ \\ \vec{\mathbf{v}}_g = \begin{bmatrix} r_a\dot{\theta}_{ag}\cos\alpha_g + \dot{\alpha}_g e\sin\alpha_g \\ r_a\dot{\theta}_{ag}\sin\alpha_g - \dot{\alpha}_g e\cos\alpha_g \\ 0 \end{bmatrix} \end{cases}$$

where, the velocity of the active wheel is given by:

$$\vec{\mathbf{v}} = \frac{\vec{\mathbf{v}}_d + \vec{\mathbf{v}}_g}{2} \tag{18}$$

and

$$\vec{\mathbf{v}} = \begin{bmatrix} \mathrm{v}_{x1} \\ \mathrm{v}_{y1} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{r_a}{2}(\dot{\theta}_{ad}\cos\alpha_d + \dot{\theta}_{ag}\cos\alpha_g) + \frac{e}{2}(\dot{\alpha}_d\sin\alpha_d + \dot{\alpha}_g\sin\alpha_g) \\ \frac{r_a}{2}(\dot{\theta}_{ad}\sin\alpha_d + \dot{\theta}_{ag}\sin\alpha_g) - \frac{e}{2}(\dot{\alpha}_d\cos\alpha_d + \dot{\alpha}_g\cos\alpha_g) \\ 0 \end{bmatrix}$$

From Eqs. (16), (17) and (18) and the real velocities, the position of the centre of gravity of the wheelchair may be estimated. The orientation $\dot{x}_{\mathsf{Gr}}$,$\dot{y}_{\mathsf{Gr}}$ and $\dot{\phi}_r$are:

$$\begin{aligned} \dot{x}_{\mathsf{Gr}} &= (\frac{r_a}{2}(\dot{\theta}_{ad}\cos\alpha_d + \dot{\theta}_{ag}\cos\alpha_g) \\ &\quad + \frac{e}{2}(\dot{\alpha}_d\sin\alpha_d + \dot{\alpha}_g\sin\alpha_g))\cos\phi_r \\ \dot{y}_{\mathsf{Gr}} &= (\frac{r_a}{2}(\dot{\theta}_{ad}\cos\alpha_d + \dot{\theta}_{ag}\cos\alpha_g) \\ &\quad + \frac{e}{2}(\dot{\alpha}_d\sin\alpha_d + \dot{\alpha}_g\sin\alpha_g))\sin\phi_r \\ \dot{\phi}_r &= \frac{r_a}{2(f+d)}(\dot{\theta}_{ad}\sin\alpha_d + \dot{\theta}_{ag}\sin\alpha_g) \\ &\quad - \frac{e}{2(f+d)}(\dot{\alpha}_d\cos\alpha_d + \dot{\alpha}_g\cos\alpha_g) \end{aligned}$$

After determining the real velocity and orientation, the slipping parameters may be estimated by comparison of the measurements of the speed of the active wheels with the measurements of passive wheels as described in Eq.(10) (see section 2.1).

