A TWO-DIMENSIONAL SNOW PACK MODEL

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Abstract

The investigation of snow pack properties aims to one of the main topics of snow science. Consequently a number of different snow pack models have been developed, world wide. The most sophisticated models are able to calculate snow temperatures, settlement, densification, snow metamorphism and weak layers within the snow pack. Among all these models there exists no two-dimensional approach for arbitrary shaped cross sections of a slope. SnowSim, developed by the Institute of Mountain Risk Engineering, is the first model which allows the calculation of two-dimensional temperature profiles on arbitrary cross sections of the snow pack. Additionally to the calculation of snow temperatures the model handles snow settlement and densification. Snow settlement, densification and temperature calculation are associated very close with each other. Therefore an accurate modeling of these properties is essential for receiving reliable simulation results. The required input data for the model are measured by automatic snow and weather measurement sites which are placed on representative locations in the alpine area. Two different exposed measurement sites in about 2000 meter sea level are collecting all necessary data for operating the snow pack model. An extrapolation of the measured values to the area allows the expansion to the second dimension. The two-dimensionality helps to localize small scaled temperature deviations along possible avalanche tracks and respites the discovery of weak zones along endangered zones. Furthermore the modeling of snow settlement can be enhanced to calculate occurring forces within the snow pack and between obstacles but latter will not be dealt in this paper.

Keywords: snow pack modeling, Finite Element Method, heat transfer, snow settlement

Presenting Author's biography

Harald Teufelsbauer studied technical mathematics at the Vienna University of Technology and did his PhD in January 2007. Till now he is employed as Postdoc at the University of Natural Resources and Applied Life Sciences. The main topics of his research are focused on snow cover modeling and avalanche dynamics.

1 Introduction

Modeling of physical processes within the snow pack is counting to the major goals of snow science. A number of one dimensional snow pack models like SNOWPACK [2,4,5], CROCUS [6,7,8] and SNTHERM [9] already exists. A two-dimensional approach of snow cover modeling is given by [2,12] with the program Heafeli which operates with simplified rectangular snow blocks on unified inclined slopes. In Austria physical snow pack modeling has started a couple of years ago. SnowSim is the first snow cover model which handles real digital elevation models, measured either by airborne laser scans or by terrestrial laser scan technique. An arbitrary chosen intersection along the slope allows generating a two-dimensional cross section of the snow cover. This cross section has to be continuously adapted to the measured snow depth of the slope. Therefore the model has to process time-variable two-dimensional geometries. The governing partial differential equations are solved by the Finite Element Method which is implemented in Matlab. The work was focused on the calculation of snow temperatures and settlement, including the variation of density caused by mechanical snow pack deformations. For the parameterization of the model, automatic weather and snow measuring sites provide continuous measurements of atmosphere and snow properties, which are used to parameterize the snow cover model. SnowSim includes error correcting algorithms for preparing the measurements to use them as input data of the model.

In comparison to the one-dimensional Swiss snow cover model Snowpack, the partitioning of the snow cover in different layers of different grain types and sizes is not implemented in SnowSim because it would yield to a very high number of finite elements which leads to an explosion of computing time. Even though the snow pack is simplified to a homogeneous continuum with spatial and time-changing snow densities, the calculation of settlement and temperature deviation within the snow cover is approximated very well. The two-dimensionality offers the opportunity to locate small scaled temperature differences along an arbitrary chosen intersection of a slope which gives information of slop parallel varieties of the snow cover.

The implementation was realized in Matlab, which gives the advantage of a well-founded mathematical background and therefore the possibility of a rapid implementation. Furthermore Matlab provides good facilities for the visualization of datasets and the creation of graphical user interfaces.

The two-dimensional snow cover modeling provides a wide range of applications. The calculation of snow temperatures can be used to determine snow metamorphism within the snow cover and to predict surface hoar whereas the simulation of snow settlement and densification can be used to derive the acting forces between the snow cover and obstacles. The goal of this paper is a detailed presentation of the temperature and settlement simulation of the snow cover.

2 Generation of two-dimensional cross sections of the snow pack

Digital elevation models generated by air borne or terrestrial laser scans provide the basis for the geometry definition of a two-dimensional snow cover model. The preprocessing GUI (graphical user interface) of the simulation program allows the definition of an arbitrary chosen intersection of a slope (Fig. 1).

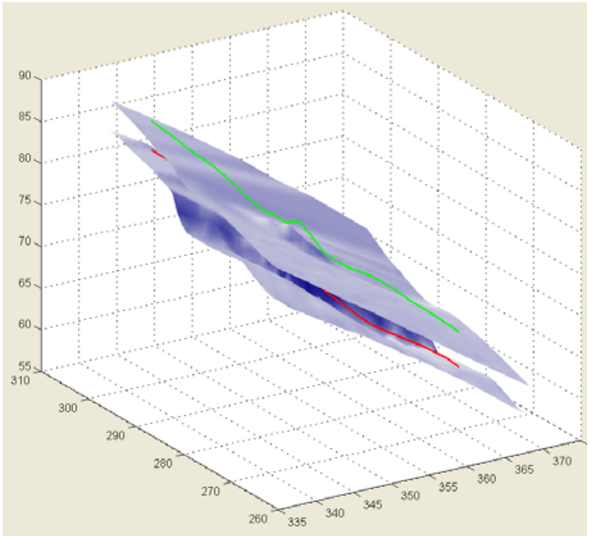

Fig. 1 Intersection between slope and snow surface

To generate a two-dimensional cross section two laser scans – one of the soil and one of the snow surface – have to be laid on top of each other. Snow depths along the intersection can be derived as distances between the both elevation models. The occurring cross section of the snow cover has to be triangulated in order to solve the governing model equations via the Finite Element Method. Therefore a delaunay-algorithm is used. SnowSim allows the definition of the maximum element size and the desired mesh size at the snow surface and the ground. A mesh growth rate factor defines how fast the element size, defined at the boundaries is converging to the maximum element size inside the cross section. The quality of the Finite Element Method solution depends strongly on the configuration of the triangulation. Penetrating short wave radiation has a big influence on snow temperatures of the upper 40 cm of the snow cover. These fast and big temperature deviations enforce a fine mesh generation near the snow surface. In order to reduce computing time it is suitable to generate a coarser mesh near the ground. An example of a cross section triangulation is shown in Fig. 2.

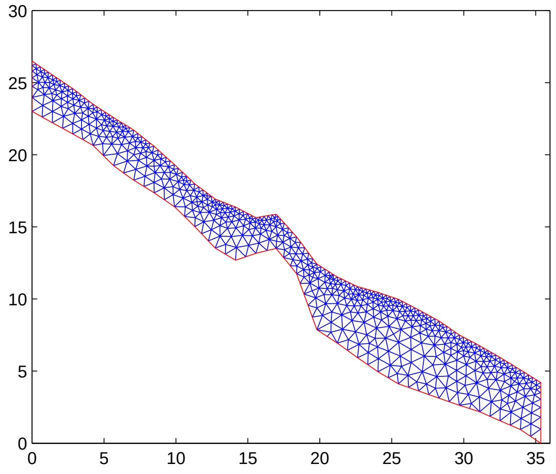

Fig. 2: Triangulation of the cross section

Additionally to the three-dimensional terrestrial laser scans, snow depths are measured punctually but therefore continuously by dint of supersonic measurement devices. The continuous recording of total snow depth combined with a settlement calculation of the snow cover allows the calculation of the amount of fresh-fallen snow and wind drift. These data are used for updating the cross section geometry continuously. Consequently the triangulation of the cross section must be adapted during the simulation. Therefore it has to be differentiated between pure snow settlement and mass change caused by fresh-snow, wind drift and melting. Snow settlement describes the deformation of the snow cover caused by its own weight. A detailed description of the settlement calculation is given in section 4. While the pure settlement calculation conserves the total mass of the snow cover the addition of snow drift, melting and fresh-snow leads to a change of total mass. The differentiation between a mass conserving and a mass changing description of the snow covers' deformation leads to the following rule for mesh generation. The pure settlement is defined by a lagrangian description which means that each node of the finite element mesh moves identically to the displacement of the material points of the snow cover. Consequently no remeshing for pure settlement is required. Contrary to the pure settling the addition or subtraction of snow masses enforces a remeshing of the geometry. After the remeshing process a mapping between the two different meshes has to be performed. If the amount of total mass changes between time step t_n and t_{n+1} the following procedure describes the mapping process. If Ω_n defines the geometry at time step t_n and Ω_{n+1} defines the deformed geometry in the next time step t_{n+1}, the mapping between Ω_n and Ω_{n+1} is performed just on the intersection of the both geometries $\Omega_{mapping} = \Omega_1 \cap \Omega_2$. The new added areas $\Omega_{new} = \Omega_2 \setminus \Omega_1$ are representing fresh-snow, consequently temperatures and densities are derived empirically from atmospheric condition. The area $\Omega_{drift} = \Omega_1 \setminus \Omega_2$ represents a reduction of snow depth due to snow drift. Hence elements within this area are deleted. The data mapping of the area $\Omega_{mapping}$ is performed by a two-dimensional linear interpolation within each triangle using the same linear basis functions as used for the Finite Element Method. All partial differential equations for describing settlement, densification and temperatures of the snow cover can be solved on the same triangulation. Furthermore the mapping process effects an automatic re-initialization after each geometry change which allows solving the differential equations on time-variable geometries.

3 Modeling of heat transfer within the snow pack

The calculation of the heat transfer within the snow cover can be described by dint of the heat equation.

$$\rho_s C_s \frac{\partial T(x,y,t)}{\partial t} = \nabla \cdot (k_s(x,y,t) \nabla T(x,y,t)) + Q(x,y,t)$$

This parabolic partial differential equation is solved by dint of the Finite Element Method on any two-dimensional cross section of the snow cover. This equation requires the knowledge of snow density ρ_s, specific heat capacity C_s, effective thermal conductivity k_s and a source term Q, which describes the influence of short wave radiation (0.3 – 3 µm) penetrating the upper 30 to 40 cm of the snow pack. The modeling of the source term is described in the following subsection. Different approaches for the calculation of the thermal conductivity and heat capacity is given in [1,2,4,10,8,9,11,15].

3.1 The source term Q

The amount of short wave radiation penetrating the snow cover is dependent on the impact angle between sun rays and snow surface. Therefore the position of the sun has to be derived for each time step of the simulation [14]. The inclination and exposition of each grid point of the digital elevation model has to be determined in order to calculate the impact angle φ between the solar irradiation and the terrain's normal vector. The intensity I defines a value between zero and one, related to the portion of short wave radiation impacting the inclined area:

$$I(\varphi) = \cos(\varphi)$$

The short wave radiation is measured by a pyranometer which determines the amount of incoming short wave radiation impacting a horizontal area. In order to extrapolate the measuring data to the area, the radiation measured by the pyranometer has to be converted to a reference value kw_{ref} which represents the measuring of a pyranometer which would be permanently directed normal to the incoming sun rays. The angle φ_{ref} defines the angle between the normal vector of a horizontal plain and the incoming solar radiation. The amount of energy E_{in} which finally arrives the snow surface is calculated as follows:

$$kw_{ref} = \frac{kw_{in}}{\cos(\varphi_{ref})}$$

$$E_{in} = (1-a) \cdot I(\varphi) \cdot kw_{ref}$$

a is the albedo which is defined as ratio between reflected and incoming radiation and depends on the properties of the snow surface. The albedo can be set empirically to values between 0.7 and 0.9. The amount of reflected radiation is strongly dependent on the age of the snow cover whereas old snow has a lower albedo than fresh snow.

The calculation of kw_{ref} provides reliable results, except for the time of sunrise and sunset (division by zero). This time span is approximated by a bell-shaped curve. The intensity of radiation within the snow pack decreases exponentially with penetration depth *d*. Hence the source term of the heat equation *Q* is a function of penetration depth *d* and incoming energy E_{in} on the snow surface:

$$Q(d) = u_1 \cdot E_{in} \cdot \exp(-u_2 \cdot d)$$

u_1 and u_2 are empirically determined constants.

3.2 Long wave radiation

The long wave radiation (3 – 60 µm) is measured by two no ventilated pyrradiometers divided in the measurement of incoming and emitted radiation. Pyrradiometers are very sensitive to weather conditions. During the winter above all they should be maintained in intervals of one to two weeks. A practical use of this measuring device in alpine terrain combined with rough weather conditions is not efficient. Therefore the measurement was replaced by a calculation of long wave emission and irradiation. The emission lw_{out} is derived by the Stefan Boltzmann Equation:

$$lw_{out} = \sigma \cdot T_s^4 \quad with \quad \sigma = 5.669 \cdot 10^{-8} [\tfrac{W}{m^2}]$$

This equation relates the surface temperature T_s [°*K*] to the emitted radiation energy lw_{out}. Contrary to a quite accurate calculation of the emission stands a more empirical calculation of the long wave irradiation lw_{in}. The irradiation is not only dependent on air temperature but also on relative humidity, cloudiness, air pollution and much more. An easy correlation between air temperature *LT*, relative humidity *rh* and long wave irradiation is given by the author as follows:

$$lw_{in} = 0.8 \cdot \frac{rh}{100} \cdot \sigma \cdot LT^4 + 90$$

The net long wave radiation $lw = lw_{in} - lw_{out}$ is an important factor for describing the energy balance between snow pack and atmosphere. Therefore a reliable calculation is essential for the snow cover model. The comparison between measurement of net long wave radiation and calculation shows quite good correlations (Fig. 3).

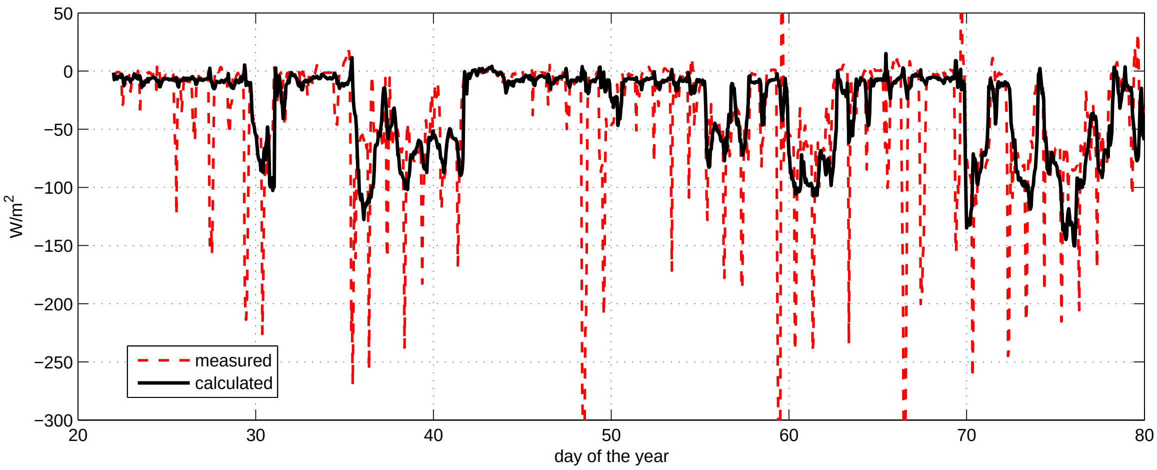

Fig. 3: Comparison between the measured and calculated net long wave radiation *lw*

Outliers are not captured by the calculation of the balance between long wave irradiation and emission. The simulation of snow surface temperatures has shown that the usage of calculated long wave radiations yields to better results than the usage of measured values. Furthermore it could be observed that the outliers of the measurement are a result of erroneous measurements.

3.3 The boundary conditions of the heat equation

Fig. 2 shows a cross section of the snow cover. The geometry is bounded by four sections, the boundary between soil and snow, the boundary between snow and atmosphere and the left and right boundaries. On each of this boundary sections, different conditions can be defined. The boundary between ground and snow is defined by a Dirichlet boundary condition Γ_D which enforces a measured temperature on the bottom of the cross section. Homogeneous Neumann boundary conditions Γ_N are associated to the left and right margin of the cross section. This condition represents symmetry because the heat flux g normal to the surface is set to zero. The boundary of the snow surface can be set either to a Dirichlet condition if measurements of the snow surface temperature are available or a hybrid boundary condition Γ_H which is an enhancement of the Neumann boundary condition. By means of this boundary condition influences of convective fluxes and thermal fluxes can be modeled in addition to the simple heat flux g.

$$T = T_D \qquad on\ \Gamma_D$$

$$k_s \frac{\partial T}{\partial \vec{n}} = g \qquad on\ \Gamma_N$$

$$k_s \frac{\partial T}{\partial \vec{n}} = g + v \cdot (T_{ext} - T) \qquad on\ \Gamma_H$$

The accuracy of the snow temperature calculation depends strongly on the precision of the description of heat fluxes between snow pack and atmosphere. The energy exchange has a number of different causes, including sensible $q_{sensible}$ and latent heat flux q_{latent}, long wave solar radiation lw, wind flow v_{wind} and different temperatures between snow surface T_s and ambient air T_{ext}. These energy fluxes can be summarized to the hybrid boundary condition Γ_H as follows.

$$k_s \frac{\partial T}{\partial \vec{n}} = lw + q_{latent} + q_{sensible} \qquad on\ \Gamma_H$$

$$lw = lw_{out} - lw_{in}$$

$$q_{sensible} = f(v_{wind}) \cdot (T_{air} - T_s)$$

$$q_{latent} = n \cdot v_{wind} \frac{L^{i/w} \cdot \rho_{air}}{p_{air}} (e_s^w(T_{ext}) \cdot rh - e_s^i(T_s))$$

$$e_s^{i/w}(T) = p_t \cdot \exp(\frac{L^{i/w}(T - T_t)}{R_v \cdot T_t \cdot T})$$

$L^i = 2838$ *[kJ/kg]*..........latent heat of sublimation
$L^w = 2256$ *[kJ/kg]*.........latent heat of vaporization
rh..................................relative humidity
v_{wind} ....wind speed
$p_t = 610{,}5$ *[Pa]* ....triple point pressure
$T_t = 273{,}16$ *[K]* ....triple point temperature
$R_v = 461{,}9$ *[J/(kg K)]*.....specific gas constant
n....................................empirical constant

The hybrid boundary condition includes the difference between snow surface and air temperature. If the boundary condition is assembled implicitly to the Finite Element Method, the snow surface temperature T_s is simultaneous input value and solution of the calculation. The calculation of the long wave emission requires the knowledge of the surface temperature too. In order to conserve the linearity of the FEM system of equations, the long wave radiation is derived explicitly with surface temperatures of the time step before.

4 Modeling of snow settlement and densification

It is assumed that the settlement is caused by the snow pack's own weight. Therefore the creeping of the snow pack can be modeled by the equilibrium condition of the plane strain.

$$\nabla \cdot \sigma_{ij} = -\rho \begin{pmatrix} g_x \\ g_y \end{pmatrix}$$

The stress – strain rate relation is defined as

$$\sigma = \eta\, \dot{\varepsilon}$$

Using an empirical derived viscosity

$$\eta = \begin{cases} (h_1 \cdot \rho)^{(h_2 - h_3 \cdot T)} & \text{für } \rho < \rho_g \\ h_4 \cdot (h_1 \cdot \rho)^{(h_2 - h_3 \cdot T)} & \text{für } \rho \geq \rho_g \end{cases}$$

Finally the following relation between strain rate and displacement velocity allows the calculation of the deformation (u_x, u_y) of the snow pack.

$$\begin{pmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ 2\dot{\varepsilon}_{xy} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \cdot \begin{pmatrix} \dot{u}_x \\ \dot{u}_y \end{pmatrix} \quad with \quad \begin{pmatrix} \frac{\partial u_x}{\partial t} \\ \frac{\partial u_y}{\partial t} \end{pmatrix} = \begin{pmatrix} \dot{u}_x \\ \dot{u}_y \end{pmatrix}$$

Attention is invited to the different notation of the stress and strain tensor, respectively σ and ε:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \quad \sigma = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix}, \quad \dot{\varepsilon} = \begin{pmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{xy} \end{pmatrix}$$

The influence of snow metamorphism is not modeled explicitly, but it is implicated indirectly at the definition of the snow viscosity. The viscosity is derived empirically, depending on snow density and snow temperature. As a consequence of the settlement snow densities arise. The densification of the snow pack can be calculated by dint of the mass conservation equation applied on compressible materials.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \dot{\vec{u}}) = 0$$

The settlement calculation is combined with an empirical snow drift model. The comparison between measured and simulated snow depth shows that the snow drift model works quite well, but evaluations are only done punctually at the spatial points of the automatic gauging stations.

5 Evaluation

In the following section simulation results are compared with measurements recorded by automatic gauging stations. The evaluation is mainly based on one-dimensional analyses and comparisons related to the measuring sites. The stations are placed on a north-exposed slope and a south-east exposed slope. A data logger saves long- and shortwave radiation, air temperature, relative humidity, wind speed, snow depth, soil temperature, snow surface temperature and temperatures within the snow pack in a time interval of 10 minutes. The continuous measurements of atmosphere and snow pack conditions allow the evaluation of simulated snow temperatures within the snow pack and on its surface. The evaluation of the snow settlement and snow drift compares continuous measured snow depths punctually at the meteorological measuring sites with the simulation. The only way to evaluate snow densities is the verification by manually digged out snow profiles. The two-dimensional snow pack model is evaluated with data of the winters 2004/05 and 2005/06.

The following two graphics show the comparison between measured and simulated snow surface temperatures. Fig. 4 shows surface temperature simulated with measured long wave radiation whereas Fig. 5 presents temperatures derived under consideration of calculated long wave radiation. The comparison implicates that the calculation of long wave radiation leads to a more reliable simulation of the surface temperature than the usage of measured radiation.

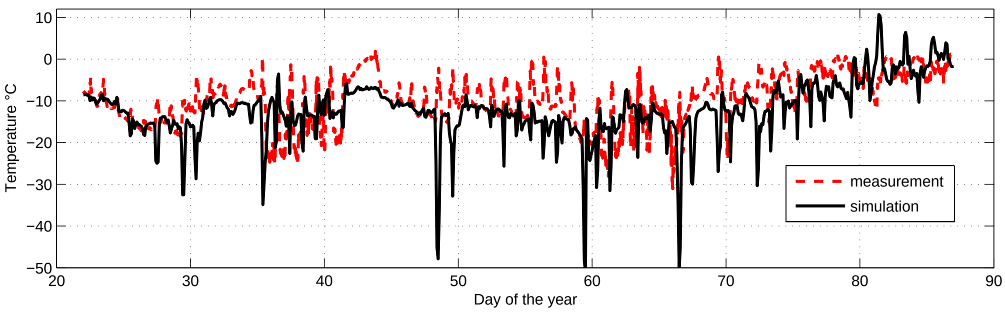

Fig. 4: Comparison between simulated and measured surface temperatures;
Measured long wave radiation was used to derive the energy balance

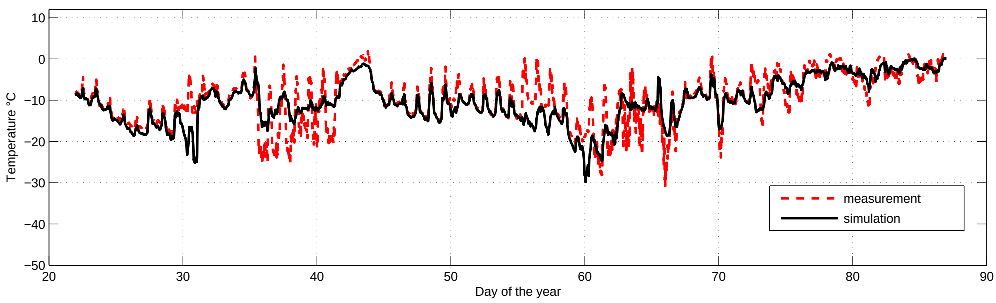

Fig. 5: Comparison between simulated and measured surface temperatures;
Calculated long wave radiation was used to derive the energy balance

The temperatures within the snow pack are evaluated by means of temperature sensors placed in a vertical distance of about 20 cm. Fig. 6 shows a schematic representation of the temperature measurement. The positions of temperature sensors are measured between soil and head of the sensors.

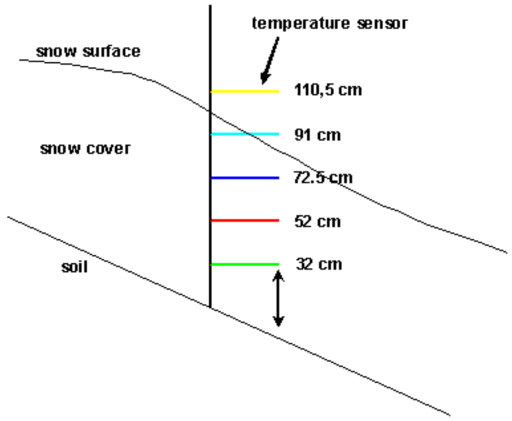

Fig. 6: Temperature measurement within the snow pack.

The evaluation of the simulated and measured snow temperatures is represented in the following Fig. 7. The evaluation displays only snowed in temperature sensors. The colors of the evaluation plot and the temperature sensor positions are correlating between Fig. 6 and Fig. 7. The analysis shows that simulated temperatures near the snow surface do not correlate exactly to the measurements. The reasons of this deviation are partly caused by penetrating short wave radiation heating the upper temperature sensors. Other uncertainties occur at the determination of the amount of penetrating short wave radiation, the definition of the albedo, measurement of the wind speed above the snow surface et cetera.

Fig. 8 gives a comparison between the simulation and the measurement of snow settlement. The snow depths are evaluated at the south-east exposed measuring site. The evaluation shows that snow depths are over estimated at the end of the winter because the model has implemented not yet a calculation of snow melting.

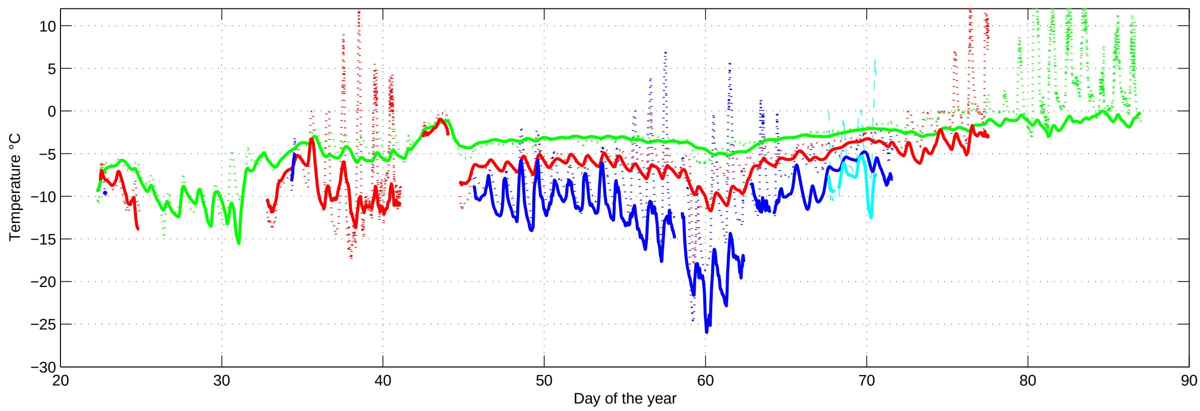

Fig. 7: Comparison between measured and simulated snow temperatures. The colors are related to Fig. 6. Drawn through lines represent simulation results and dashed lines measurements.

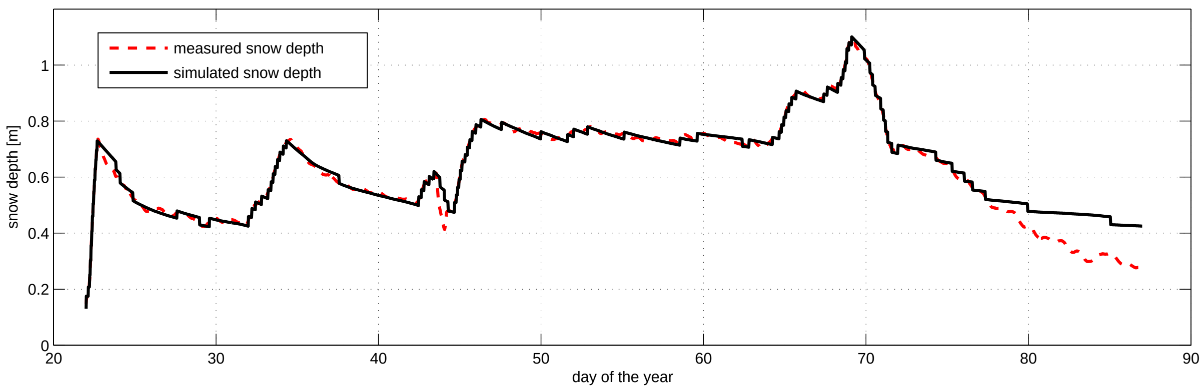

Fig. 8: Comparison between simulated and measured snow depth.

The visualization of two-dimensional simulation results provides an insight into the temperature deviation along intersection of the snow cover. If snow depth, slope inclination and exposition are quite inhomogeneous along the cross section, the analysis of simulation results has shown that temperature deviations in lateral direction are not negligible. The three areas marked in Fig. 9 show substantive different temperature profiles within a distance of about 30 meters.

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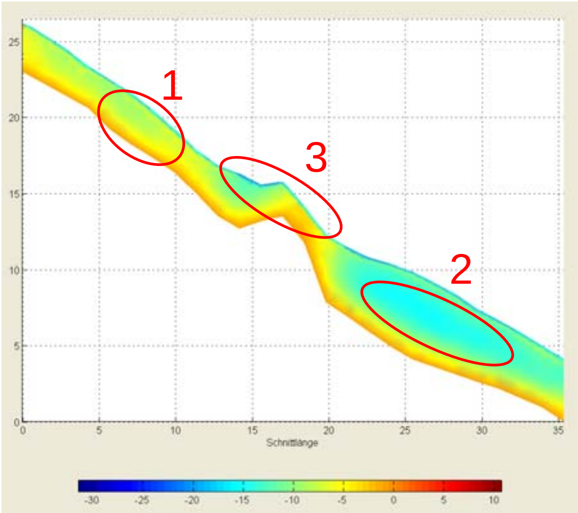

Fig. 9: Snow temperatures along the cross section of the snow pack.

Snow temperatures from area *1* differ from the temperatures in area *2* about 5 to 10 degrees. The main influence of this deviation is given by the different snow depths. The heat energy dissipated by the soil can not heat up the center of the snow cover in the area *2*. Another difference can be obtained at the snow surface in area *3*. The surface temperature is dependent on the inclination and exposition of the snow surface. The analysis of this simple cross section demonstrates clearly the influence of different slope expositions, inclinations and snow depths of the snow pack.

6 Outlook

The two-dimensional snow cover modeling opens new vistas for snow temperature analysis and capturing forces within the snow cover. SnowSim offers the basis for following up simulations of surface hoar and melting zones within the snow pack. The locating of isothermal areas is made a lot easier by the two-dimensional simulation. The calculation of settlement and densification can be enhanced for the simulation of forces appearing within the snow pack and acting on obstacles.

Improvements are expected for the calculation of the energy balance and wind drift by incorporating reliably simulated wind speeds in alpine terrain.

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References

[1] Adams E. Edward, Sato Atsushi, 1993. Model for effective thermal conductivity of a dry snow cover composed of uniform ice spheres. *Annals of Glaciology* 18 1993, 300-304

[2] Bartelt P., Christen M., Witter S., 2000. Program Haefeli – Two-dimensional numerical simulation of the creeping deformation and temperature distribution in a phase changing snowpack. *Snow Engineering Recent Advances & Development, Trondheim Norway June 19 –21, Balkema, Rotterdam*. 13 – 22.

[3] Bartelt P., Lehning M., 2002. A physical Snowpack model for the Swiss avalanche warning, Part I: numerical model. *Cold Reg. Sci. Technol*. 35, 123 – 145

[4] Bartelt P., Lehning M., 2002. A physical Snowpack model for the Swiss avalanche warning, Part II: snow microstructure. *Cold Reg. Sci. Technol*. 35, 147-167

[5] Bartelt P., Lehning M., 2002. A physical Snowpack model for the Swiss avalanche warning, Part III: meteorological forcing, thin layer formation and evaluation. *Cold Reg. Sci. Technol*. 35, 169-184

[6] Brun E., David P., Sudul M., Brugnot G., 1992. A numerical model to simulate snowcover stratigraphy for operational avalanche forecasting. *Journal of Glaciology*, Vol. 38, No. 128, 13–22.

[7] Brun E., Martin E., Simon V., Gendre C., Coleou C., 1989. An energy and mass model of snow cover suitable for operational avalanche forecasting. *Journal of Glaciology*, Vol. 35, No. 121, 333-342

[8] Durand Y., Giraud G., Brun E., Merindol L., Martin E., 1999. A computer-based system simulating snowpack structures as a tool for regional avalanche forecast. *Journal of Glaciology*, Vol. 45, No. 151, 469–484.

[9] Jordan R., 1991. A one-dimensional temperature model for snow cover. *Special Report 91-6, US Army Corps of Engineers, Cold Regions Research and Engineering Laboratory (CRREL)*, Hanover, NH, 49 pp.

[10] McComb T.J.L., Rimmer A.B., Rodgers M.L.B. Turver K.E., Vivkers A.F., 1992. A mathematical model for the prediction of temperature in a dry snow layer. *Cold Reg. Sci. Technol*. 20, 247-259

[11] Morris E. M., 1983. Modelling the flow of mass and energy within a snowpack for hydrological forecasting. *Annals of Glaciology* 4, 198-203

[12] Phillips M., Bartelt P., Christen M., 2000: Influence of avalanche defence snow supporting structures on ground temperatures in Alpine permafrost terrain, *Annals of Glaciology*, 31, 422 - 426.

[13] Prokop A., 2005, Hangbezogene Ermittlung der flächigen Schneehöhenverteilung mittels Laserscanners, *Journal of Torrent, Avalanche, Landslide and Rock Fall Engineering*, Nr. 154

[14] Winter C.-J., Sizmann R.L., Vant-Hull L.L., 1991. *Solar Power Plants*, Springer Verlag. 17-27

