WRIST EXERCISER - EXERCISING FOR MODELING AND SIMULATION

Boris Curk and Peter Cafuta

University of Maribor , FERI
2000 Maribor, Smetanova 17, Slovenia

peter.cafuta@uni-mb.si *(Peter Cafuta)*

Abstract

Powerball®, Dynabee®, and Gyrotwister® are commercial names of similar gyroscopic devices that are marketed as a wrist exercisers. The device has rotor with two unactuated DOF and can be actuated with suitable motion of additional human or robot wrist axis [1]. After substantial initial rotor's spin, the properly applying torque and motion about wrist axis lead to spin-up of the rotor. Finding this torque intuitively is easy job for most peoples, but not so easy for technical consideration for example in robotics.

In the paper, a modeling of different modes of the device is presented. A dynamic model with nonholonomic rolling connection which appear in normal operate mode is discussed. With rotor and housing connection the additional friction effect is observed . When the nutation reaction torque M_N is to low, only dissipation of energy is observed. But when the reaction torque causes normal forces on a connection in a degree that the friction is high enough, the rotor shaft begins to roll. The connection with the housing takes place in a gap so two connecting pairs are possible. One is up-down for left precession and another is down-up for opposite precession rotation. Simulation results of spin-up considering variable structure dependence from friction effects of device is shown and experimenting with robot is discussed.

Keywords: gyroscopic device, modeling, simulation, robotics.

Presenting Author's biography

Peter Cafuta was born in Maribor, Slovenia, on February 1, 1951. He received the B.S. and M.S. degrees from Faculty of Electrical Engineering, Ljubljana, in 1974 and 1976, respectively, and the Ph.D. degree from Faculty of Technical Sciences, University of Maribor, in 1987. Since 1976, he has been with the Faculty of Electrical Engineering and Computer Sciences of Maribor, in the field of modeling, simulations, servodrives, and robotics.

ISBN 978-3-901608-32-2

1 Introduction

Powerball®, Dynabee®, and Gyrotwister® are gyroscopic devices popular in the 90-ties. They are dedicated to the wrist exercising and are patent pended [1]. Rolling connection of the rotor and the device housing enables the spin-up of the rotor by the appropriate wrist rotations. This movement is accompanied with the torque vector reaction proportional to the square of the rotor spin. The rotor can reach the speeds up to 15000 rpm for the plastic version showed on Fig.1 and an astonishing 20000 rpm for the metal version.

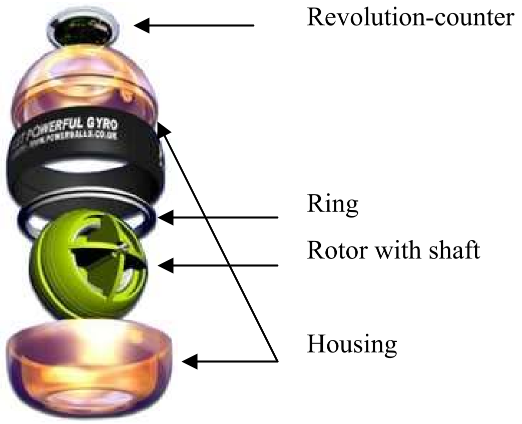

Fig. 1 Powerball® components

Our scenario includes idustrial robot, which should be able to spin-up the Powerball®. This gyroscopic device represents a dynamic load to the robot as is discussed in [2] and is underactuated mechanism extension for the systems cinematic chain [3]. Operation of the wrist exerciser is based on rolling which depends on friction between the rotor shaft and the housing. Friction my be neglected or it is significant. It depends on device state and from the wrist rotation reaction normal force on the connection. The model is of the ODE type and has uncontinuous right side. Therefore it is modeled as a nonholonomic, underactuated, and variable structure system.

The main contribution of the paper in comparison to the results in references [4,5,6] is the dynamic model which replicates all there modes of gyroscopic device: the free rotor, the left, and the right mode of rotor rolling in the housing. This model enables the simulation on precession reversion and also calculates the power dissipation between the model mode changes.

The paper is organized as follows. First the modes of operation and appropriate models are discussed. The variable model structure is developed next. Then the simulation is reported and after that the comparison is made with the experiment. The robotic activation is shortly discussed on the end of the paper.

2 Modes of operation

Wrist exerciser is essentially a gyroscope with three degrees of freedom in rotation. There are spin, precession and nutation rotation. Powerball® is a guided gyroscope in nutation rotation with a special feature of rolling connection in precession and spin degrees of freedom. Nutational rotation causes precession torque reaction which: the first press spinning rotors shaft on a housing to roll in a same direction and the second precession torque acts on the rotor. So the first mode of operation is start-up to the minimal necessary spin. It can be done with a starting rope or some other starting device. The gyroscope after that only spin-ff because there is friction in the bearings. The same is valid for Powerball® when the initial spin is not to low and independent from applied or not applied nutation rotation.

When the spin is high enough two additional modes occur. Energy supply to the rotor is possible due to pat. pended precession ring which enables direct connection between the rotor shaft and the housing of the Powerball®.

2.1 Rolling connection

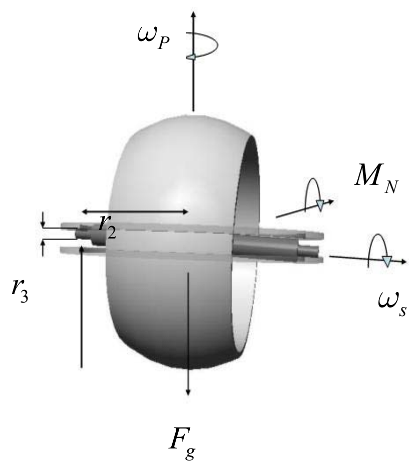

Fig. 2 Rotor reaction on torque M_N

With this rotor and housing connection the additional friction effect is observed . When the nutation reaction torque M_N is to low, only dissipation of energy is observed. But when the reaction torque causes normal forces on a connection in a degree that the friction is high enough, the rotor shaft begins to roll. The connection with the housing takes place in a gap so two connecting pairs are possible. One is up-down for left precession and another is down-up for opposite precession rotation. The situation for the force pair acting on the shaft is illustrated on Fig. 2.

When the rolling is without sliding the precession speed ω_p is proportional to the spin speed ω_s given with the equation (1). Direction of the precession depends on amount and direction of torque M_N, which acts against the torque from gravitation on the rotor μ. In the worst case the rotor axes is horizontal and the equation (2) is valid. The ratio in diameters p of the shaft r_3 and the ring r_2 is given with equation (3) and reflects rolling velocities ratio.

$$\omega_P = \begin{cases} -1/p \cdot \omega_S;\ M_N \geq \mu \\ 1/p \cdot \omega_S;\ M_N \leq -\mu \end{cases} \tag{1}$$

$$\mu \leq \mu_{\max} = \left| F_g \cdot r_2 \right| \tag{2}$$

$$p = r_2 / r_3 \tag{3}$$

Due to two possible rolling modes transition between them is not trivial. It takes place thru the free rotor mode. This transitions are called reversion and the rotor spin direction is not influenced. In both precession directions the same spin-up is possible.

2.2 Rotor spin-up

After rolling takes place the wrist exerciser stays in this mode because the precession reacts on the nutation support. On this way the connection with the rotor take place. When no work is delivered to the device this state will vanish due to the unavoidable dissipation. Supplying the energy with nutation M_N thus transmits to the rotor kinetic energy and the rotor spin-up. Precessing rotor direction changes against nutation rotation, so the nutation should follow the rotor.

This can be done with the two DOF of device housing in a mans hand or with the robot wrist or only with the one DOF. Using only the one DOF simplifies the analysis as the experimenting setup so this is done in our work. This simplified case incorporates singularity which must be avoided at the start-up and the transmitted energy is only one half of the full activated case.

The described principle is quit simplified. The detail discussion is possible only with the modeling and the appropriate simulation. This will be done in next paragraphs

3 Model of the wrist exerciser

The masses of housing and the ring are neglected as are the friction in bearings. Only the friction due to normal force acting on the ends of rotor shaft will bi explicitly discussed. It conditioned left and right precession rolling and the free rotor spin.

The wrist exerciser has 2 DOF when it is free and one additional DOF belongs to the robot wrist.

The torque $\mathbf{M}$ or impulse applied to the rotating body changes its rotating quantity $\mathbf{\Gamma}$:

$$\mathbf{M} = \frac{d}{dt}\mathbf{\Gamma} = \frac{d}{dt}\left(\mathbf{J} \cdot \boldsymbol{\omega}\right) \tag{4}$$

Where is $\mathbf{J}$ the inertia tensor and the $\boldsymbol{\omega}$ the rotation velocity vector. The equation (4) holds only in the inertial coordinate system (CS). The entries of the $\mathbf{J}$ changes with the selection of coordinate system. The usual way to start modeling is in the coordinate system of the rotor. In our case the coordinate system of the ring is chosen. Thus the vectors and tensor components are most simplified.

The position of the coordinate system is selected centered for the wrist exerciser which is also the TCP of the robot. The situation is represented on Fig. 3.

.

For the cos and the sin the abbreviations c and s are used. Transformations are given with:

$${}^{2}\mathbf{T}_3(q_3) = Rot(z, q_3) = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$${}^{1}\mathbf{T}_2(q_2) = Rot(y, q_2) = \begin{bmatrix} cq_2 & 0 & sq_2 \\ 0 & 1 & 0 \\ -sq_2 & 0 & cq_2 \end{bmatrix}, \tag{6}$$

$${}^{0}\mathbf{T}_1(q_1) = Rot(z,\ q_1) = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

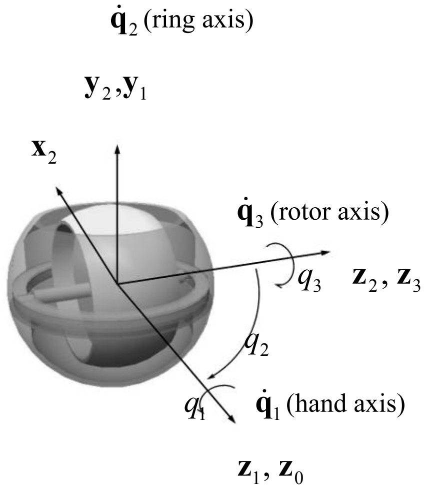

Fig. 3 coordinate systems, axis, and rotation velocity vectors.

Let the coordinate system CS 0 be chosen as inertial and the transformation (4) results in

$$ {}^0\mathbf{M} = \frac{d}{dt}\,{}^0\mathbf{\Gamma} = \frac{d}{dt}\left({}^0\mathbf{T}_2 \cdot {}^2\mathbf{\Gamma}\right) \quad , \tag{8} $$

$$ {}^2\mathbf{M} = {}^2\mathbf{T}_0\,{}^0\mathbf{M} = {}^2\mathbf{T}_0 \frac{d}{dt}\left({}^0\mathbf{T}_2\right){}^2\mathbf{\Gamma} + \frac{d}{dt}\left({}^2\mathbf{\Gamma}\right), \tag{9} $$

Where the ${}^0\mathbf{T}_2 = {}^0\mathbf{T}_1\,{}^1\mathbf{T}_2$ and ${}^2\mathbf{T}_0 = \left({}^0\mathbf{T}_2\right)^{-1}$.

Express the transformation derivative with the tensor calculus:

$$ \frac{d}{dt}\,{}^0\mathbf{T}_2 = \mathbf{R}\left({}^0\mathbf{\Omega}\right)\cdot {}^0\mathbf{T}_2 \quad , \tag{10} $$

where is ${}^0\mathbf{\Omega}$ a rotation velocity vector of CS 2 in the CS 0.

Then from equation (9):

$$ {}^2\mathbf{M} = {}^2\mathbf{T}_0\mathbf{R}\left({}^0\mathbf{\Omega}\right){}^0\mathbf{T}_2\,{}^2\mathbf{\Gamma} + {}^2\dot{\mathbf{\Gamma}} \,, \tag{11} $$

and

$$ {}^2\mathbf{T}_0 \cdot \mathbf{R}\left({}^0\mathbf{\Omega}\right){}^0\mathbf{T}_2 = \mathbf{R}\left({}^2\mathbf{\Omega}\right), \tag{12} $$

torque in the CS of the ring is expressed with:

$$ {}^2\mathbf{M} = \mathbf{R}\left({}^2\mathbf{\Omega}\right){}^2\mathbf{\Gamma} + {}^2\dot{\mathbf{\Gamma}} \,\cdot \tag{13} $$

The expression (13) is known as transport theorem:

$$ {}^2\mathbf{M} = {}^2\mathbf{\Omega} \times {}^2\mathbf{\Gamma} + {}^2\dot{\mathbf{\Gamma}} \,, \tag{14} $$

where are quantities valid for CS 2 and are defined as:

$$ {}^2\mathbf{M} = \left[{}^2M_x, {}^2M_y, {}^2M_z\right]^T, \tag{15} $$

$$ \mathbf{R}\left({}^2\mathbf{\Omega}\right) = \begin{bmatrix} 0 & -{}^2\Omega_z & {}^2\Omega_y \\ {}^2\Omega_z & 0 & -{}^2\Omega_x \\ -{}^2\Omega_y & {}^2\Omega_x & 0 \end{bmatrix}, \tag{16} $$

$$ {}^2\mathbf{\Omega} = \left[\Omega_x, \Omega_y, \Omega_z,\right]^T = \dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_2 = \dot{q}_1\,{}^2\mathbf{z}_1 + \dot{q}_2\,{}^2\mathbf{y}_2 \tag{17} $$

$$ {}^2\mathbf{\Gamma} = {}^2\mathbf{J}\cdot{}^2\boldsymbol{\omega} \,, \tag{18} $$

$$ {}^2\mathbf{J} = {}^3\mathbf{J} = \begin{bmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J_3 \end{bmatrix}. \tag{19} $$

The entry J_3 is an principal angular momentum of the rotor spin axes and both J are momenta of the perpendicular principal axes to J_3. Equation (19) is thus valid for the rotor type of the body. Vector of the rotor rotation velocity is given in the component form:

$$ {}^2\boldsymbol{\omega} = \dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_2 + \dot{\mathbf{q}}_3 = \dot{q}_1\,{}^2\mathbf{z}_1 + \dot{q}_2\,{}^2\mathbf{y}_2 + \dot{q}_3\,{}^2\mathbf{z}_2 \tag{20} $$

When the matrix components are:

$$ {}^2\mathbf{T}_1 = \left({}^1\mathbf{T}_2\right)^T = \left[{}^2\mathbf{x}_1, {}^2\mathbf{y}_1, {}^2\mathbf{z}_1\right] = \begin{bmatrix} cq_2 & 0 & -sq_2 \\ 0 & 1 & 0 \\ sq_2 & 0 & cq_2 \end{bmatrix} \tag{21} $$

$$ {}^2\mathbf{z}_1 = \begin{bmatrix} -sq_2 \\ 0 \\ cq_2 \end{bmatrix}, \quad {}^2\mathbf{z}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad {}^2\mathbf{y}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \tag{22} $$

then the velocities are given with:

$$ {}^2\mathbf{\Omega} = \begin{bmatrix} -sq_2 \cdot \dot{q}_1 \\ \dot{q}_2 \\ cq_2 \cdot \dot{q}_1 \end{bmatrix} \quad \text{and} \quad {}^2\boldsymbol{\omega} = \begin{bmatrix} -sq_2 \cdot \dot{q}_1 \\ \dot{q}_2 \\ cq_2 \cdot \dot{q}_1 + \dot{q}_3 \end{bmatrix}. \tag{23} $$

When (15-23) are inserted in to the equation (13) the dynamical model of the free spinning rotor expressed in the ring CS is:

$$ \begin{bmatrix} {}^2M_x \\ {}^2M_y \\ {}^2M_z \end{bmatrix} = \begin{bmatrix} -Jsq_2 & 0 & 0 \\ 0 & J & 0 \\ J_3cq_2 & 0 & J_3 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix}; \\ \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} -(2J - J_3)cq_2 \cdot \dot{q}_1\dot{q}_2 + J_3 \cdot \dot{q}_2\dot{q}_3 \\ (J_3 - J)sq_2cq_2 \cdot \dot{q}_1^2 + J_3sq_2 \cdot \dot{q}_1\dot{q}_3 \\ -J_3sq_2 \cdot \dot{q}_1\dot{q}_2 \end{bmatrix} \tag{24} $$

The model (24) is important for expression of the nutation torque in equation (6) because the axis is parallel to the $\mathbf{x}_2$:

$$\begin{aligned} M_N &= -{}^2M_x \\ &= Jsq_2 \cdot \ddot{q}_1 + (2J - J_3)cq_2 \cdot \dot{q}_1\dot{q}_2 - J_3 \cdot \dot{q}_2\dot{q}_3 \end{aligned} \tag{25}$$

The axis torques are the projections of the vector (15) on the axes of the system. The projection on $\mathbf{z}_3$ is trivial due to $\mathbf{z}_3 = \mathbf{z}_2$ and $M_3 = {}^2M_z$. The same is valid for the ring $M_2 = {}^2M_y$. In the wrist axis the transformation $M_1 = {}^1M_z = \left({}^1\mathbf{T}_2\,{}^2\mathbf{M} \bullet {}^1\mathbf{z}_1\right) = -sq_2\,{}^2M_x + cq_2\,{}^2M_z$ is used, where the (.) is a dot product.

The torques in the colocated rotations are given with:

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} H_{11} & 0 & H_{13} \\ 0 & H_{22} & 0 \\ H_{13} & 0 & H_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, \tag{26}$$

where are

$$H_{11} = Js^2q_2 + J_3c^2q_2,$$

$$H_{13} = J_3cq_2,$$

$$H_{22} = J,$$

$$H_{33} = J_3,$$

$$h_1 = -2(J_3 - J)sq_2cq_2 \cdot \dot{q}_1\dot{q}_2 - J_3sq_2 \cdot \dot{q}_2\dot{q}_3,$$

$$h_2 = h_y,$$

$$h_3 = h_z.$$

The rolling depends on relative velocity between the shaft and the precession in the housing:

$$v = |\zeta|\dot{q}_2r_2 + \zeta\dot{q}_3r_3\ ; \tag{27}$$

$$\zeta = \begin{cases} 1 & ;\ M_N > \mu \\ 0 & ;\ -\mu \le M_N \le \mu \\ -1 & ;\ M_N < -\mu \end{cases}$$

Where the ζ indicates the mode of the system. At 1 the negative part of the rotors shaft is elevated and the positive side is down and at -1 is the situation opposite, and at 0 the rotor float. The driving torques in axes 2 and 3 due to the friction are:

$$\begin{aligned} M_2 &= -f_T\left(v, M_N\right) \\ M_3 &= -\zeta \cdot \frac{1}{p} \cdot f_T\left(v, M_N\right) \end{aligned} \tag{28}$$

Where is $f_T\left(v, M_N\right)$ the friction model which depends on sign, relative amount of speed, and the normal force from the nutation torque. When only the dry friction is modeled then with $f_T\left(v, M_N\right) = k_T sig\left(v\right)$.

4 Simulation of exerciser operation

Model (26) and (28) are implemented in Matlab-Simulink. For animation the model in the Nastran is developed [7]. Only the static friction type effect is considered in this case. On the Fig. 4 experiment on real device is documented.

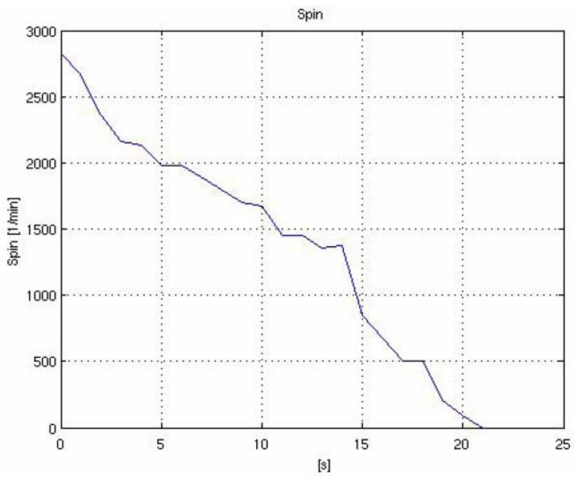

Fig. 4 The spin-off experiment

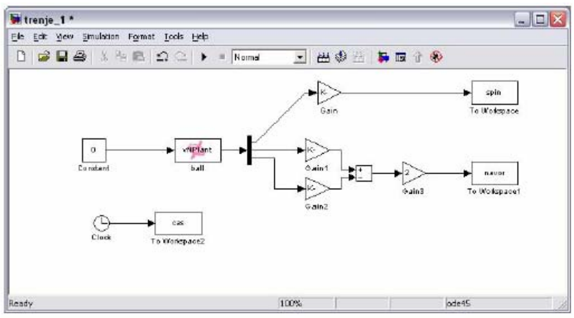

Fig. 5 Simulation model: spin-off

The spin for stopping from 2800 rpm gives model parameters for friction. Until 14 sec the precession is observable and after that only the spin was performed.

For the simulation the model on Fig. 5 was used. Obtained results have a good match to the experiment and are shown on Fig. 6.

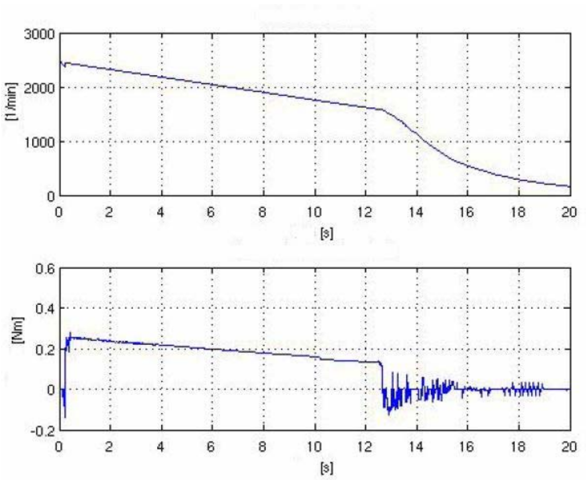

Fig. 6 Spin and the torque of the spin-off

Next the spin-on without friction and with the friction was performed. The first model is represented on Fig 7 and the results on Fig. 8.

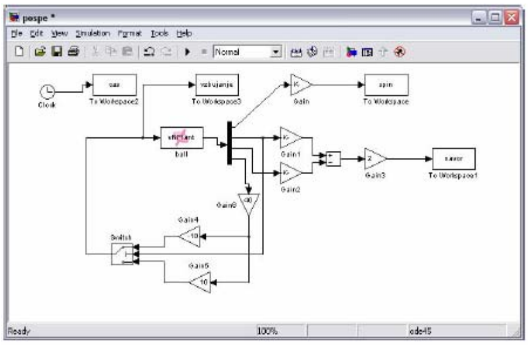

Fig.7 Simulation model without friction

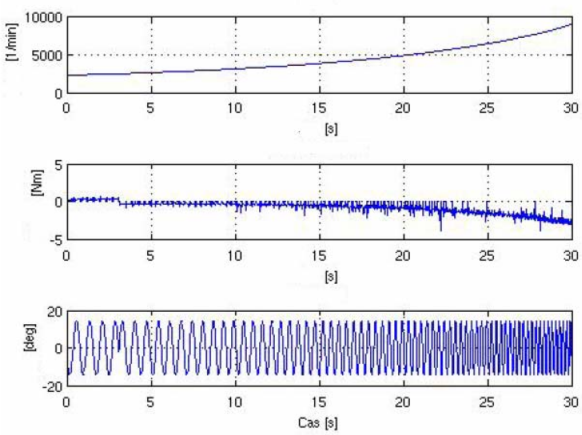

Fig. 8 Spin, torque and the excitation angle for the spin-on.

The exerciser was excited with the periodic angle at the starting spin of 2500 rpm. After 3 sec the reversion is made in nutation and the precession torque changes sign. Unsurprised the end spin is very high at 9000 rpm. When the friction is taken into account the simulated results are showed in the Fig.9

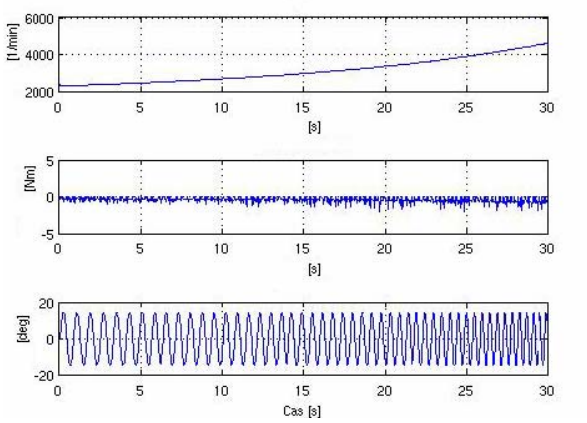

Fig. 9 Spin, torque and the excitation angle for the spin up

5 Experimenting with robot

Fig. 10 Experimental systems et up (Powerball®, sensor griper, and the robot wrist)

In the project [8] the experiment preparation was taking place. The simple torque sensor griper was build and attached to the Motoman HP6 robot (Fig. 10). The operation range was significantly restricted by the torque sensor because the torque amount can be very large and the achievable speed are therefore relatively low. Due to the fact that simulations match very good with the experimental results, the development of the control algorithms will be in the future tested first in simulator domain.

6 References

[1] A.L. Mishler, Gyroscopic device, U.S. patent 376146, 1973.

[2] L. Žlajpah, Robotic yo-yo: Modeling and control strategies, Robotica Vol. 24, Issue 2, 2006

[3] M. W. Spong: Underactuated Mechanical Systems, Control Problems in Robotics and

Automation, LNCIS, vol. 230, Springer Verlag, London, 1998

[4] D. W Gulick, O.M. Reilly: On the Dynamics of the Dynabee, ASME J. of App. Mechanics,V.67, 2000.

[5] G. Schweitzer: Antrieb eines Spielkreis, Festschrift TU Munchen, 1982.

[6] K. Ucke, H. J. Schlichtig: Fascinirendes Dynabee, Physicinunseren Zeit, 2002.

[7] T. Petric, M. Franc And B. Curk: Distributed Simulation Matlab-Nastran, Project FERI. Maribor, 2007

[8] B. Curk, Robotiziran Powerball. Preecidings *ERK 2006, Portorož, Slovenija*, pp. 167-170, 2006