ROBUST CONTROL DESIGN METHODS FOR REDUCED ORDER MODELS

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Abstract

The quality of a controller is not only dependent on the design method. Rather the correspondence between a model and a plant is an important criteria as well. In practice models are often adequate or the controller is a priori robust to model errors. Usually no additional considerations concerning robustness are necessary. But especially mechanical system tend to have a very high system order. For example by replacing the distributed parameters structure by point masses especially high order modes are missing in a model. Also models generated by finite element methods (FEM) have a high order that makes the controller design more demanding. In this case an order reduction is used to get a lower order than the original system. In general the correlation between model and plant is better in the lower frequency range. But also here the exact position of the poles is not known. Additional poles can be found in the upper frequency range. Especially pole placement is difficult for such systems. The unconsidered poles of the plant and the not exact position of the considered poles in the model can move to the right s-domain using a model based state space controller. This article presents different methods to deal with such reduced order and not exact models using state space theory. All techniques excel as *easy* compared to modern concepts like H_∞ or μ-synthesis [1]. In addition the complete design process of the controller is highly automated and could be a method to introduce state space theory in an industrial environment. The design methods will be compared and tested by simulations at a power train of a CNC-machine tool.

Keywords: Pole placement, Pole region assignment, Robustness, Model errors, Output feedback control.

Presenting Author's Biography

Ulrich Ahrholdt received his Master's degree in mechanical engineering from the Georgia Institute of Technology, Atlanta, GA, United States in 2002, the Diploma in mechanical engineering from the Technical University of Clausthal, Germany, in 2003. Currently he is working toward his Ph.D. degree in model based control theory of machine tools using state space methods at the electrical engineering department of the Technical University of Darmstadt, Germany. The project is funded by *Siemens, Automation & Drives (A&D)*.

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1 Introduction

In many cases already the design method of creating a model does not consider the behavior of the plant in the higher frequency region. This results in models that have correlation to the plant only in the lower frequency domain. Additional poles of the plant can be found in the higher frequency range. Errors compared to the model are mainly in the lower frequency region.

In the following some examples of reduced order models in practice are presented.

- Today computer based engineering and design (CAD) is widely spread [2]. Already in early development stages one can get easily exact FEM-models of systems, e.g. mechanical. Therefore the existence of a prototype is not necessary and the future behavior in case of controllability of the plant is known. The subsequent creation of a high order state-space model, based on a FEM-model, is state of the art, but to handle such models an order reduction with loss of information is necessary.

- Another method of modeling mechanical systems is to fit a measured Bode-diagram to a Bode-diagram of a multi body system (MBS). Because auf measurement accuracy the Bode-Diagram will be more exact in the lower frequency region. Above a certain frequency noise is dominating and higher modes of the plant will be invisible. Also here the model is not as exact as the real system.

- To model mechanical systems often point masses are used instead of a distributed parameter model. By this some of the eigenmodes are neglected. For example the beam of an inverse pendulum can be such a system. It may oscillate with a high frequency that is not considered in the model based on point masses.

This paper describes different methods to design a pole placement controller based on a faulty model. The goal is to transfer the controller, based on the model, to the plant. Robustness concerning the described modeling errors is demanded.

The paper results from a cooperation of the TU-Darmstadt and *Siemens, Automation and Drives (A&D)*. Therefore the desired plant is a computer numerical controlled (CNC) machine tool. In section 2 such a machine is shortly presented in detail.

To compare the different model based design techniques the results are tested at a plant of higher order. The basic characteristics of model and plant are also presented in section 2.

The control structure and the controller design will be explained in section 3. A description of the different robustness methods can be found in section 4. Here one can also find simulation results. Finally a conclusion is presented (section 5).

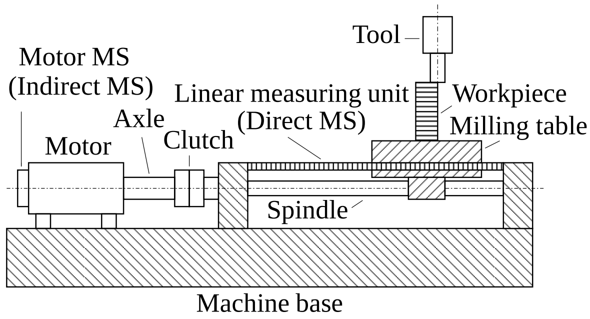

Fig. 1 A typical construction of an CNC-machine tool axis

2 Plant and Model

In the following sections first a CNC-Machine tool will be described in detail. Afterwards the creation of the model is presented. Finally plant and model are compared with respect to dynamic behaviour and the requirements for the closed loop system are introduced.

2.1 CNC-Maschine Tool

Modern CNC-machine tools have up to five axis to handle free formed surfaces. So they are multi-input-multi-output (MIMO) system with 5 inputs and 5 outputs. Because of the translational arrangement of the axis the system is mechanically decoupled. So one can design the controller of each axis separately. Thats why this article is focused on designing the controller for a single axis instead of all.

In Fig. 1 the mechanical structure of one axis is displayed. Basically it is an electrical motor that powers an axle. A spindle connected by a clutch transforms the rotational movement to a translative movement. Besides of this most common construction one can also think of a linear motor. Here the motor transforms the force directly in a translative movement.

The input of the plant is the torque of the motor M_{m}. The outputs depends on the used measurement systems (MS). Fig. 1 includes two different MS

- Indirect MS
- Direct MS.

The main differences of the two different MS are illustrated in Tab. 1. The indirect MS is integral part of every power train and is placed directly at the motor. However the control variable is not the position φ_m of the motor but the position of the workpiece respectively the position x_{mt} of the milling table. High sophisticated machine tools have an additional direct MS not for the workpiece but the milling table. In this case the measurement position is much closer to the control variable than in the case of an indirect MS. Low end machines have only an indirect MS. Here you have no feedback of the desired control variable x_{mt} as only the position

Tab. 1 Comparison of indirect MS and direct MS ($M-$ torque, $\varphi-$ angle position, $\omega-$ angular velocity, $x-$ position, $v-$ velocity).

	Indirect MS	Additional direct MS
Input	M_m	M_m
Output	$\varphi_\mathrm{m}, \omega_\mathrm{m}$	$\varphi_\mathrm{m}, \omega_\mathrm{m}, x_\mathrm{mt}, v_\mathrm{mt}$
Control variable	φ_m	x_mt
Desired control variable	x_mt	x_mt

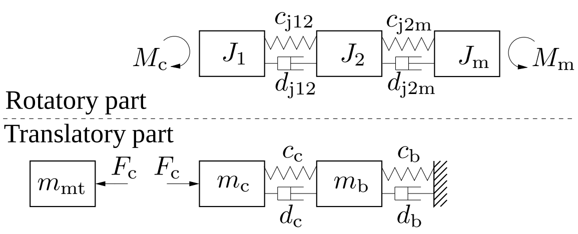

Fig. 2 Example of a 10th order MBS-model representing a CNC-maschine tool axis ($m-$ mass, $c-$ stiffness, $d-$ damping, $F-$ force, $M-$ torque, $J-$ inerta).

of the motor φ_m is known. So different measurements can be used for the controller.

2.2 Models

Fig. 2 shows an example for a 10th order MBS-system ($m_c = 0$ kg). The motor torque is feed in the system at the rotatory mass J_m. The milling table can be thought at m_mt. As an axis can consist of a rotatory and a translatory part the model is divided adequately. The two parts has to be connected by a kinematic constraint.

The MBS-model can be described as a state space model. Here the plant will be represented by a state space description of order $n = 10$

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\,\mathbf{x} + \mathbf{b}\,u \\ \mathbf{y} &= \mathbf{C}\,\mathbf{x}\,. \end{aligned} \tag{1}$$

The nominal model of the plant has only one instead of three inertia (Fig. 2). Then the order will reduce by four with a resulting state space model of order $n_M = 6$

$$\begin{aligned} \dot{\mathbf{x}}_\mathrm{M} &= \mathbf{A}_\mathrm{M}\,\mathbf{x}_\mathrm{M} + \mathbf{b}_\mathrm{M}\,u \\ \mathbf{y}_\mathrm{M} &= \mathbf{C}_\mathrm{M}\,\mathbf{x}_\mathrm{M}\,. \end{aligned} \tag{2}$$

In practise the model will be calibrated by comparing the frequency response of the model with the measured one of the real system. As this procedure is always kind of faulty and in addition parameters like the mass of the workpiece, damping values and stiffness can vary, alternatively a model family can be defined. If one put all changing parameters in a vector ξ_i the resulting model

Tab. 2 Poles of model and *plant I*

Model	Plant I
0	0
0	0
$-5.53 \pm 203.68i$	$-5.76 \pm 209.14i$
$-23.83 \pm 443.55i$	$-25.63 \pm 453.40i$
	$-87.21 \pm 5090.14i$
	$-1032.58 \pm 23846.67i$

Tab. 3 Pole of the modified plants *plant II* and *plant III*

Plant II	Plant III
0	0
0	0
$-6.47 \pm 231.68i$	$-6.71 \pm 206.05i$
$-21.94 \pm 388.99i$	$-15.44 \pm 296.71i$
$-80.97 \pm 5081.59i$	$-71.83 \pm 5071.76i$
$-1032.06 \pm 23847.53i$	$-1031.29 \pm 23846.37i$

family includes ϱ different single models

$$\begin{aligned} \dot{\mathbf{x}}_{\mathrm{M}i} &= \mathbf{A}_\mathrm{M}(\xi_i)\,\mathbf{x}_{\mathrm{M}i} + \mathbf{b}_\mathrm{M}(\xi_i)\,u_{\mathrm{M}i} \\ \mathbf{y}_{\mathrm{M}i} &= \mathbf{C}_\mathrm{M}(\xi_i)\,\mathbf{x}_{\mathrm{M}i} \\ i &= 1, \ldots, \varrho\,. \end{aligned} \tag{3}$$

2.3 Analysis of Plant and Model

Plant and model have two unstable poles ($s_{1,2} = 0$). The other poles are all complex conjugate. The poles of the model and the plant can be found in Tab. 2. The complex conjugate poles are poorly damped. The pole correlation of model and plant in the lower frequency region is high but not exact.

To analyze the robustness of the controller with respect to errors between model and plant two additional plants of 10th order are defined. The new *plant II* has greater pole location errors compared to the model than *plant I*. *Plant III* has greater variations than *plant II*. The pole location of the model and the three plants is displayed in Fig. 3 for the lower frequency region. *Plant III* has significant errors at the second resonance frequency.

The model is always of lower order than the real system. Because of that it is necessary to find a strategy to avoid unstable or poorly damped poles in the real system closed loop, here represented by the three plant models.

The general requirements of an industrial used machine tool are the following

- Stability and high dynamics
- Enough damping
- Robustness concerning model errors
- Precision in the range of some 10^{-6} m.

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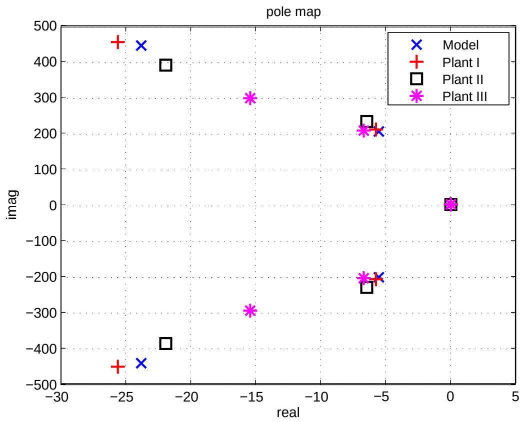


Fig. 3 Poles of model and plants in the lower frequency region

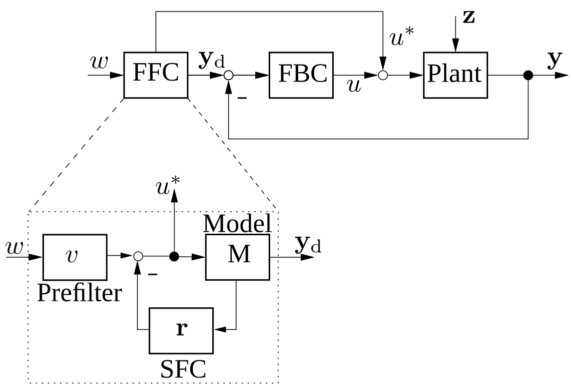


Fig. 4 New control structure with feed forward control (FFC) (FBC-Feedback controller, SFC-State feedback controller)

The controller should meet all these specifications.

3 Controller Design

First the structure of the controller will be described. Afterwards a design method is presented, that already addresses robustness concerning model errors.

3.1 Two-Degrees-of-Freedom structure

The control structure is presented in Fig. 4. It is based on the two-degrees-of-freedom control which consists of two parts [3]. The feed forward control is responsible for the command action and the controller is responsible for the disturbance response.

As simulations show, the feed forward control works already very well if designed by a low order model [4]. As all states of the model are known the controller $\mathbf{r}$ can be design by pole placement. The feed forward control can be also interpreted as a low pass. The system will only be activate in the bandwidth, defined by the poles of closed loop feed forward system.

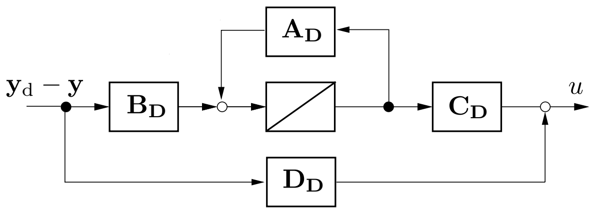


Fig. 5 Structure of a dynamic state space controller

Errors because of differences between model and plant are kind of disturbances. So the controller does not have to care for the command action but only for the disturbance response. This structure was proved of value to handle the control requirements and all following methods are based on it.

The sophisticated part is not the design of the feed forward control but the design of the feedback controller. It must be robust concerning model errors.

3.2 Dynamic output controller

As a new approach state space theory is used for the complete design structure of the machine tool. For the controller FBC in Fig. 4 an output feedback controller will be used

$$\mathbf{u} = \tilde{\mathbf{K}}(\mathbf{y}_\mathrm{d} - \mathbf{y}) \; . \tag{4}$$

As the number of outputs q of a system is usually smaller than the number of states n_M a dynamic controller has to be used to place all poles. The structure of a dynamic state space controller of order r is shown in Fig. 5.

To design the controller the system has to be extended by the dynamic controller. Afterwards you get a similar equation like (4) with [5]

$$\mathbf{K} = \begin{bmatrix} \mathbf{D}_D & \mathbf{C}_D \\ \mathbf{B}_D & \mathbf{A}_D \end{bmatrix} \; . \tag{5}$$

The degrees of freedom f_c of the controller is defined by the number of independent parameters of the controller matrix $\mathbf{K}$ and can be calculated as

$$f_\mathrm{c} = (p+q)\, r + p \cdot q \; . \tag{6}$$

p is the number of inputs of the used model. To place all poles of the closed loop system exactly, the number of degrees of freedom should be at least the number of poles one wants to place ($f_\mathrm{c} \geq n_M + r$).

To assure steady-state behavior it is essential to have an integrator in the controller. As a dynamic state-space controller is used this can be provided by a pole of the controller in zero ($s_{\mathrm{c}1} = 0$). So one eigenvalue of the $\mathbf{A}_D$ matrix has to be zero.

To design $\mathbf{K}$ by pole placement only a numerical solution is possible as an analytical one is not possible [5]. So an optimization problem has to be formulated.

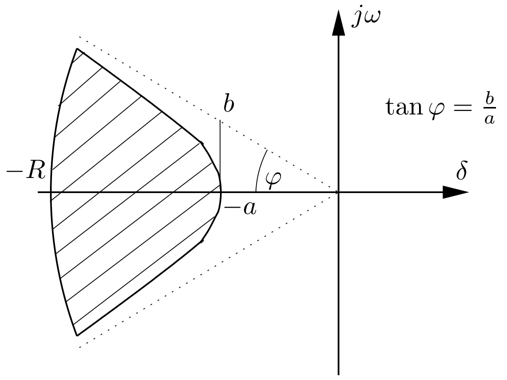


Fig. 6 Closed loop pole region

3.3 Pole region assignment

In order to increase robustness instead of pole placement, pole region assignment is used to design the controller FBC in Fig. 4. The poles are not placed at exact predefined positions, but one defines an area in which all closed loop poles have to be located. As this requirement is less restrictive it can be interpreted as a kind of robustness. The method is described in detail in [6, 7]. The desired pole area can be defined by a hyperbola at the right border to guarantee a minimum speed (a) and damping (φ). At the left side a circle with radius R minimizes the maximum eigenfrequency of the closed loop modes (Fig. 6). Thus the user can design a complex output feedback controller by choosing only three parameters.

The optimization problem is to place all poles of the closed loop systems

$$s_{ik} = \delta_{ik} + \mathrm{j}\omega_{ik}, \quad i = 1, \ldots, \varrho, \quad k = 1, \ldots, n_{\mathrm{M}} + r, \tag{7}$$

inside the desired pole region by the control law (4). A penalty function evaluates each pole by its location inside or outside the desired pole region

$$J = \sum_{i=1}^{\varrho} \sum_{k=1}^{n_{\mathrm{M}}+r} \left(e^{p_i \cdot f_{ik}} + e^{q_i \cdot g_{ik}} \right) \tag{8}$$

with

$$f_{ik} = \delta_{ik} + \frac{a_i}{b_i} \sqrt{b_i^2 + \omega_{ik}^2} \tag{9}$$

and

$$g_{ik} = \sqrt{\delta_{ik}^2 + \omega_{ik}^2} - R \, . \tag{10}$$

p_i and q_i are factors to weight the left and right border of the pole region assignment to each over. For the presented system class p_i should be greater than q_i ($p_i > q_i$) as the open loop poles of the plant are mainly on the right side of the pole region.

The functions (9) is defined, that poles on the right side of the hyperbola get very high values, poles on the left side very low ones. Function (10) delivers great values for poles on the left side of the circle and lower ones for poles on the right side.

To minimize the penalty function the use of a gradient method is useful. Therefore an analytical expression for the gradient, the partial differentiation of the penalty function with respect to the controller elements, has to be known.

If the elements of the controller matrix $\mathbf{K}$ are denoted by k_{lm} with $l = 1, \ldots, p$, $m = 1, \ldots, q$ the partial differentiation of (8) yields

$$\frac{\partial J}{\partial k_{lm}} = \sum_{i=1}^{\varrho} \sum_{k=1}^{n_{\mathrm{M}}+r} \left(p_i \frac{\partial f_{ik}}{\partial k_{lm}} e^{p_i \cdot f_{ik}} + q_i \frac{\partial g_{ik}}{\partial k_{lm}} e^{q_i \cdot g_{ik}} \right) \tag{11}$$

with

$$\frac{\partial f_{ik}}{\partial k_{lm}} = \frac{\partial \delta_{ik}}{\partial k_{lm}} + \frac{a_i \cdot \frac{\partial \omega_{ik}}{\partial k_{lm}} \cdot \omega_{ik}}{b_i \cdot \sqrt{b_i^2 + \omega_{ik}^2}} \tag{12}$$

and

$$\frac{\partial g_{ik}}{\partial k_{lm}} = \frac{\frac{\partial \delta_{ik}}{\partial k_{lm}} \cdot \delta_{ik} + \frac{\partial \omega_{ik}}{\partial k_{lm}} \cdot \omega_{ik}}{\sqrt{\delta_{ik}^2 + \omega_{ik}^2}} \, . \tag{13}$$

The partial differential equations in (12) and (13) are the real and imaginary part of the pole sensitivity [8]

$$\frac{\partial s_{ik}}{\partial k_{lm}} = \frac{\partial \delta_{ik}}{\partial k_{lm}} + \frac{\partial \omega_{ik}}{\partial k_{lm}} \, .$$

The pole sensitivity function indicates how much a change of the controller element k_{lm} affects the location of the closed loop poles. This is the main advantage of the pole region assignment compared to a traditional pole placement. The method will find automatically the poles that fulfil the requirements (defined by the pole region) as the inner dynamic is considered. The user has not to choose exact pole locations of the closed loop system that are perhaps difficult to reach.

The pole sensitivity function can be calculated by the equation [8]

$$\frac{\partial s_{ik}}{\partial k_{lm}} = - \frac{\mathbf{w}_{ik}^T \mathbf{b}_i \mathbf{c}_m^T \mathbf{v}_{ik}}{\mathbf{w}_{ik}^T \mathbf{v}_{ik}} \, . \tag{14}$$

Here $\mathbf{b}_i$ and $\mathbf{c}_m^T$ are the collum vectors of $\mathbf{B}$ and the row vectors of $\mathbf{C}$ respectively. $\mathbf{v}_{ik}$ and $\mathbf{w}_{ik}^T$ are the right eigenvectors and and left eigenvectors of the poles p_{ik}.

To realize steady state behavior predefined elements of the controller matrix k_{lm} must be zero (section 3.2). To guarantee this the gradient of some elements must be defined as

$$\frac{\partial J}{\partial k_{lm}} = 0 \, .$$

3.4 Problem definition

Designing a controller by pole region assignment using just one model (2) results in good results concerning the model. But applying the model based controller to the plant (1) of higher order and non exact correlation in the lower frequency region results in poles of the closed loop system outside the desired pole region or even unstable behavior.

In the following section different methods to get a better robustness concerning model errors will be presented.

4 Methods to handle model errors and simulation results

In this section different methods of dealing with modeling errors are presented. The first two techniques change the model the controller design is based on. The third approach analysis the influence of the number of output variables that feed the controller. An overview of the different methods is presented in the following list:

- Extending the model
- Integrating a model family
- Increasing the number of outputs
- Combination of methods.

Each technique will be tested by a simulation. Next to robustness concerning model errors also the dynamic will be evaluated.

4.1 Extending the model

The nominal model of the plant is defined by the state space model (2). As the order of the model n_M is lower than the order of the real system n ($n_M < n$) the closed loop can have a poor damping or can even be unstable if the higher modes of the the real system are not considered during the development of the controller.

To increase the accuracy of the model the plant can be extended. The easiest way to do this is to to increase the number of inertia of the MBS-system (Fig. 2). This results in a new model with the order $n_{\mathrm{ex}} = n_{\mathrm{M}} + 2$

$$\begin{aligned} \dot{\mathbf{x}}_{\mathrm{ex}} &= \mathbf{A}_M \, \mathbf{x}_{\mathrm{ex}} + \mathbf{b}_{\mathrm{ex}} \, u \\ \mathbf{y}_{\mathrm{ex}} &= \mathbf{C}_{\mathrm{ex}} \, \mathbf{x}_{\mathrm{ex}} \,. \end{aligned} \qquad (15)$$

The complex conjugate poles of the new model part should be all faster than the known poles of the model, as the low frequency poles are assumed to be relatively correct. In addition it is known that the damping of the unconsidered poles of the plant will be low, as only such poles tend to be unstable or low damped in the closed loop. Therefore the additional poles should be outside a certain circle around the origin (velocity) and with a maximum damping $d_{\max}$ (Fig. 7). A controller designed with the extended model will be more robust concerning model errors in the higher frequency region, because of additional poles approximately account for the unknown plant dynamics in the higher frequency range.

As output variables for the simulation position x_{mt} and velocity v_{mt} of the machine table are assumed. For the controller design a model like (15) is used. To get enough degrees of freedoms for the optimization a controller of 4th order has to be used. The details of the defined pole area are presented in Tab. 4.

In Fig. 8 the pole locations of the extended model and the plants (*plant I*, *plant II*, *plant III*) are presented for the critical area around the origin of complex s-plane.

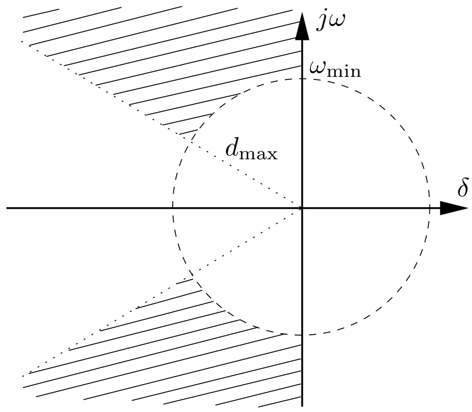


Fig. 7 Area of the additional poles for the model extension (hatched)

Tab. 4 Control design details of the method *extension*

Number of outputs	$q = 2$
Order of controller	$r = 4$
Pole region	$a = 150$ $b = 200$ $R = 600$

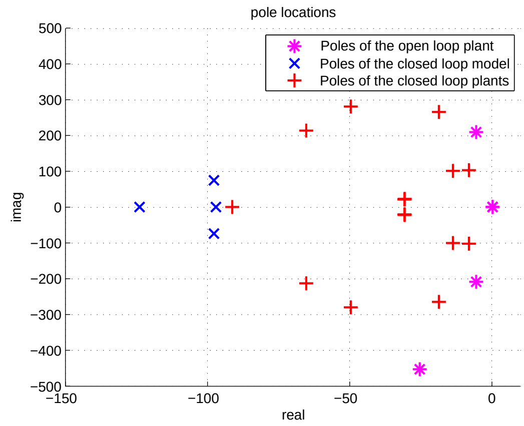


Fig. 8 Poles of model and plants (Method *extension*)

Tab. 5 Control design details of the method *family*

Number of outputs	$q = 2$
Order of controller	$r = 4$
Pole region	$a = 25$ $b = 70$ $R = 600$

Tab. 6 Control design details of the method *outputs*

Number of outputs	$q = 4$
Order of controller	$r = 2$
Pole region	$a = 90$ $b = 155$ $R = 600$

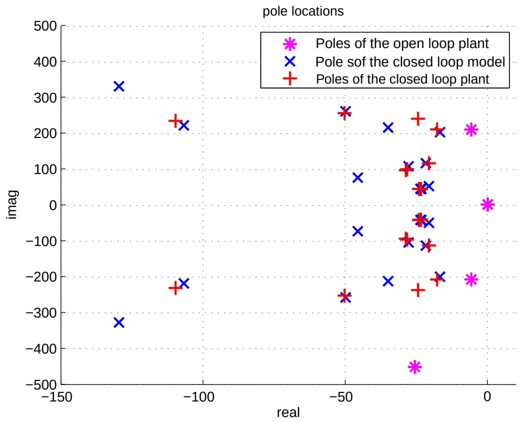


Fig. 9 Poles of model and plants (Method *family*)

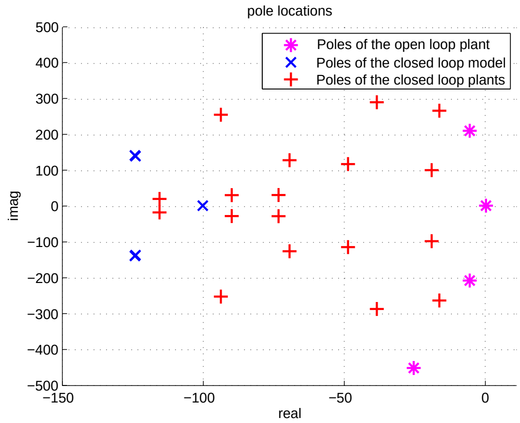


Fig. 10 Poles of model and plants (Method *outputs*)

Compared with the single low order model (2) this time a stable controller design of the closed loop plants is possible. The not considered high frequency poles of the open loop plant do not move away from the original position in the closed loop system. The extended model has a positive effect to the robustness concerning model errors. Unfortunately the poles of the closed loop plants are much closer to the imaginary axis than the designed closed loop model poles. In practise this can result in an unstable system.

The method of an extended model leads to a better controller design. But the effect is to small to use it in practise.

4.2 Model family

The errors of the model with regard to the plants that result in not identical pole locations (Fig. 3) can be considered by the model-family (3). It can be generated from the MBS-model (Fig. 2). The masses are usually known quite exact and damping values can be approximated by experience values very accurately. The main uncertainty is the stiffness so that the used model family is based on a variation of the stiffness parameters c_{c} and c_{b} in Fig. 2. The model family approach can be implemented by pole region assignment optimization (section 3.3).

For the simulation a model family of the form (3) with three models is used. The outputs are the same as in the example *extension*. The design details can be find in Tab. 5. The resulting pole locations are presented in Fig. 9.

It is not possible to place the closed loop model poles as fast as with the method *extension*. This can be explained by the optimization process. It is more difficult to find a solution if more models are included in the optimization.

This method is much better to predict the location of the closed loop plant poles as all plant poles are surrounded by closed loop model poles. So the area of the known closed loop model poles are in the same area as the in practise not known closed loop plant poles. The high frequency poles of the plant stay at the same pole area.

The model family approach seems to be a useful way to integrate robustness in the controller design process.

4.3 Increasing the number of outputs

For the design of the controller a dynamic feedback controller is used (section 3.2). This structure offers the possibility to integrate as many outputs as possible. More outputs result in more degrees of freedom to design the controller. This increases the optimization parameters.

Instead of only two all four output variables φ_{m}, ω_{m}, x_{mt} and v_{mt} are used. The nominal model of (2) is used for the controller design. Because of this the order of the controller can be reduced to $r = 2$ (Tab. 6). The pole locations are plotted in Fig. 10.

The poles of model and plants can be located more left (faster) than in the previous designs. Increasing the number of outputs q has a more positive effect than increasing the order of the controller r. But as only

Tab. 7 Control design details of the method *combination*

Number of outputs	$r = 4$
Order of controller	$r = 2$
Pole region	$a = 50$ $b = 100$ $R = 600$

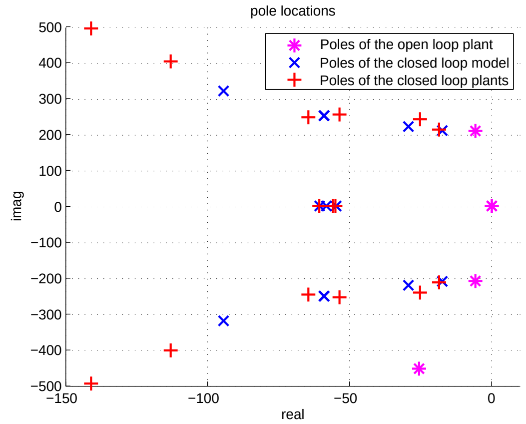


Fig. 11 Poles of model and plants (Method *combination*)

one low order model is used the robustness concerning model errors ist not high. The closed loop plant poles are much closer to the imaginary axis than the closed loop model poles.

4.4 Combination of methods

Especially the *family* and *outputs* approaches seem to be useful to design a controller for the presented plant. So the two methods will be combined. The same model family is used like before (Tab. 7). The pole location are shown in Fig. 11.

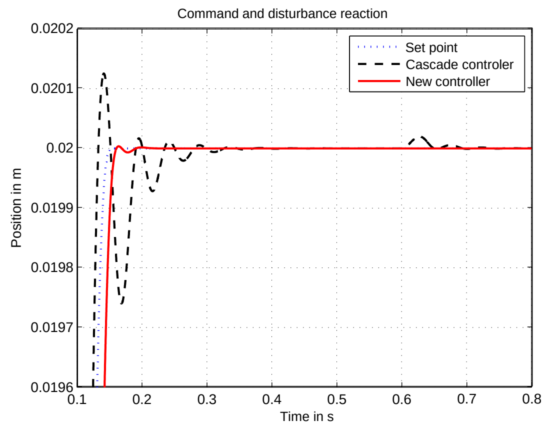
The poles of the three models surround the poles of the three plants. So a good prediction of the closed loop plant system can be done by the model family. The high frequency poles of the model stay again in the same pole area. Additionally the locations of the poles is more left than before. This will result in a good dynamic of the closed loop system.

4.5 Simulation results

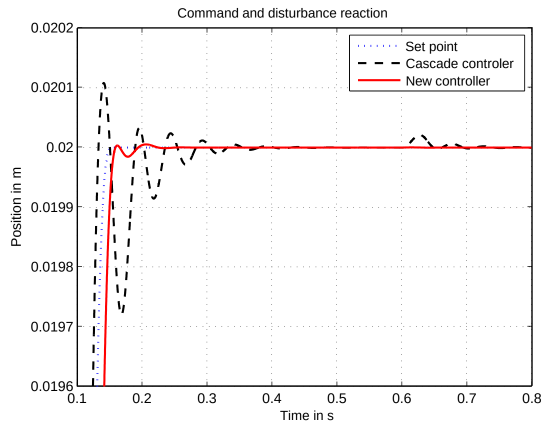
The command action and disturbance reaction of the final controller design will be tested by simulation. The benchmark system is a traditional cascade control structure, which is very common in the field of power trains [9].

In Fig. 12 the step response of a $20\ \mathrm{mm}$ positioning and the reaction to $5\ \mathrm{Nm}$ torque step is displayed. This is done for *plant I*, *plant II* and plant *plant III*.

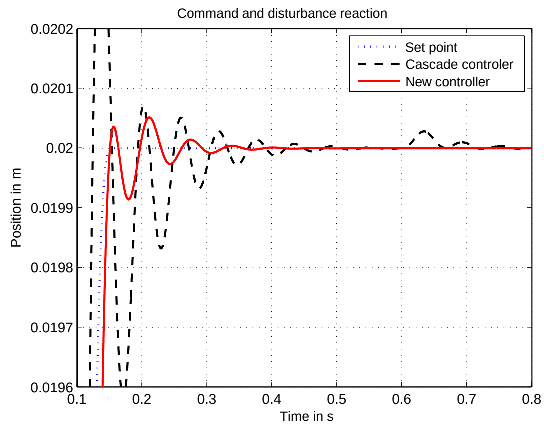
In all cases the new controller structure has advantages



(a)



(b)



(c)

Fig. 12 Simulation results of command action and disturbance reaction *combination* (a) *Plant I* (b) *Plant II* (c) *Plant III* compared to the cascade structure. The command action has less overshooting and more damping. Especially the disturbance reaction shows significant better behavior compared to the traditional structure. For *plant III* the control parameters of the cascade structure had to be adapted. So the new structure has a better robustness concerning model errors than the traditional structure.

5 Conclusion

Mathematical models of real systems are an essential part of modern engineering. Especially in the area of control engineering often the quality of the model influences the results more than the design method of a controller. Unfortunately every model is a simplification of the plant. Two different types of errors can be distinguished: A reduced order and general (parameter-) uncertainties of the model. This article highlights the problems that result from model errors with special regard to the pole placement technique.

Based on the defined problem of controlling a CNC-machine tool a method is presented to design a state space controller using pole placement. The method is highly automated and can be done without detailed knowledge of state space theory.

Problems designing the controller occur as the model has significant deviations to the plant. To prevent unstable or poor closed loop behavior different methods are presented. A model extension tries to consider the higher order of the system. With a model family parameter variations can be described. Also the number of system-outputs that are used for feedback have an influence on the controller design. It is shown by simulations that especially the combination of the model family and the increasing number of outputs results in closed loops that have more dynamic and more robustness concerning model errors than traditional cascade controllers.

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