MATLAB-SIMULINK NONLINEAR MODELING AND SIMULATION OF AIRCRAFT LONGITUDINAL DYNAMICS

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Abstract

A safe and cost-effective way to establish an extended flying qualities database is to conduct exploratory investigations using piloted simulations. In this technical report, a nonlinear mathematical model is obtained and some simulations techniques are investigated with Matlab-Simulink. The aircraft aerodynamics in sense of aerodynamic lift, drag and pitch moment coefficients, simple thrust of a engine and rigid body dynamics for longitudinal motion of a four turbofan engine large passenger commercial aircraft Airbus A340-300 are presented. The modeling steps are explained clearly for all sub block-sets of the model and all required equations for the mathematical model are given in the report except the function specified in the text with quotation marks. Three different flying maneuvers are performed to illustrate modeled aircraft motion for an assumed flying altitude and condition. For such case a standard atmospheric mathematical model is used. The time responses of three different simulations which examine longitudinal aircraft dynamics are added at the end of the report and results are discussed in sense of the stability of aircraft motion. The presented nonlinear aircraft model in this study is flexible for any type of fixed wing aircrafts and can be adapted by changing aerodynamic coefficients and thrust value of engines.

Keywords: Nonlinear modeling, longitudinal dynamics, aircraft, Simulink.

Presenting Author's biography

Erkan Abdulhamitbilal received a Bachelor of Science degree in 2002 and Master of Science in 2005 on Astronautics Engineering from Istanbul Technical University. The areas of interest and working are system dynamics, modeling, simulation, stability analysis and automatic control techniques of aerospace vehicles.

1 Introduction

A safe and cost-effective way to establish an extended flying qualities database is to conduct exploratory investigations using piloted simulations. So this study concerns with modeling and simulation of longitudinal dynamics of large passenger aircraft. The nonlinear modeling technique is simple and may be applied for any fixed wing aircraft. In literature so many different modeling techniques and software are used to illustrate aircraft behaviors during the flight.

In this technical report, aircraft aerodynamics and rigid body dynamics for longitudinal motion of a large passenger commercial aircraft Airbus-A340 are presented. A nonlinear mathematical model is obtained and some simulations techniques are investigated with Matlab-Simulink. The modeling steps are explained clearly and all equations are given in the report. Three different flying maneuvers are performed to illustrate modeled aircraft motion with a standard atmospheric mathematical model. The time responses of simulations are added at the end of the report and results are discussed in sense of the stability of aircraft motion.

The presented nonlinear aircraft model in this study is flexible for any type of fixed wing aircrafts and can be adapted by changing aerodynamics coefficients and thrust value of engines.

2 Aircraft model

In this topic firstly aircraft aerodynamics are studied. Then thrust effects of all engines are taken into account. And finally the rigid body aircraft dynamics are given with six degree of freedom for a completely nonlinear longitudinal model.

2.1 Aerodynamics

In this section the aerodynamic forces and moments expressions for Airbus-A320 (see Fig.1) is investigated. Therefore, the general lift and drag forces can be presented as [1]

$$L = c_L q S_w \tag{1}$$

$$D = c_D q S_w \tag{2}$$

where c_L is the lift coefficient, c_D is the drag coefficient, q is the dynamic pressure and S_w is the area of the wing. Then, dynamic pressure may be calculated from relation as

$$q = \frac{1}{2}\rho V^2 \tag{3}$$

where ρ is the air density and V is the velocity vector calculated by u, v, w as follows

$$V = \sqrt{u^2 + v^2 + w^2} \tag{4}$$

The aerodynamic pitch moment can be calculated by

$$M = c_M q S_w C_w \tag{5}$$

where C_w is the chord length of the wing at the aerodynamic center, and c_M is the pitch moment coefficient.

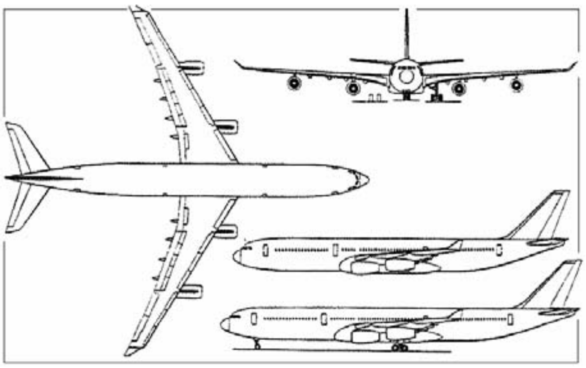

Fig.1 Airbus A340 large passenger aircraft [2]

2.1.1 Lift coefficient

Lift coefficient of an aircraft can be calculated from relation, [3]:

$$c_L = c_{L_0} + c_{L_\alpha}\alpha + c_{L_{i_h}} i_h + c_{L_{\delta_e}}\delta_e \tag{6}$$

where α is the angle of attack, i_h is the incidence of the horizontal stabilizer and δ_e is the control angle of the elevator. Also c_{L_0} is the lift coefficient at zero angle of attack, c_{L_α} is the lift coefficient at arbitrary angle of attack, $c_{L_{i_h}}$ is the lift coefficient of the horizontal stabilizer and $c_{L_{\delta_e}}$ is the lift coefficient produced by elevator deflection.

2.1.2 Drag coefficient

Drag coefficient of an aircraft can be calculated from relation [3]:

$$c_D = c_{D_0} + c_{D_\alpha}\alpha \tag{7}$$

where c_{D_0} is the drag coefficient at zero angle of attack, c_{D_α} is the drag coefficient at arbitrary angle of attack and α is the angle of attack. Therefore, c_{D_α} can be calculated from

$$c_{D_\alpha} = 2 c_L c_{L_\alpha} / \pi \mathrm{AR}\,\mathrm{e} \tag{8}$$

where AR is the aspect ratio $sapn^2 / area$ and e is the Oswald coefficient. And c_{D_0} can be obtained from

$$c_{D_0} = \overline{c}_{D_0} + \left(2 c_L^2 / \pi \mathrm{AR}\,\mathrm{e}\right) - c_{D_\alpha}\alpha \tag{9}$$

2.1.3 Pitch moment coefficient

Pitch moment coefficient of an aircraft can be calculated similarly as lift coefficient from [3]:

$$c_M = c_{M_0} + c_{M_\alpha}\alpha + c_{M_{i_h}} i_h + c_{M_{\delta_e}}\delta_e \tag{10}$$

where c_{M_0} is the pitch moment coefficient at zero angle of attack, c_{M_α} is the pitch moment coefficient at arbitrary angle of attack, $c_{M_{i_h}}$ is the pitch moment coefficient of the horizontal stabilizer and $c_{M_{\delta_e}}$ is the pitch moment coefficient produced by elevator deflection.

2.2 Thrust effect

Thrust force expression of four turbofan engines is given by

$$T = T_{\max,SL}\sigma\delta_T \tag{11}$$

where $T_{\max,SL}$ is the maximum thrust of an engine at sea level, σ is a density coefficient, and δ_T is the thrust percent given by thrust arm.

2.2.1 A standard atmosphere model

The mathematical model of the standard atmosphere is given for troposphere ($0 \le h \le 11000$ m altitude). The air temperature at an arbitrary altitude can be calculated as [4]

$$t = \lambda t_{SL} \tag{12}$$

where t_{SL} is the temperature at sea level ($288.2^\circ K$) and λ is the temperature coefficient

$$\lambda = 1 - 22.556 \times 10^{-6} h \tag{13}$$

Hence, the air pressure at an arbitrary altitude can be calculated from temperature coefficient as

$$P = \delta P_{SL} = \lambda^{5.2561} P_{SL} \tag{14}$$

where P_{SL} is the pressure at sea level ($101.300\, N/m^2$) and δ is the pressure coefficient which can be obtained from temperature coefficient as

$$\delta = \lambda^{5.2561} \tag{15}$$

Finally the air density at an arbitrary altitude can be calculated as follows:

$$\rho = \sigma\rho_{SL} = \lambda^{4.2561}\rho_{SL} \tag{16}$$

where ρ_{SL} is the air density at sea level ($1.225\, kg/m^3$) and σ is the air density coefficient which can obtained as:

$$\sigma = \lambda^{4.2561} \tag{17}$$

2.3 Aircraft dynamics

The equation of motion of the long-range passenger aircraft with six degree of freedom can be obtained with assumptions such that the mass of the aircraft is constant, aircraft is a rigid body, Earth axis is an inertial frame, the mass distribution of the aircraft is constant, and x-z plane is the plane of symmetry.

Therefore, nonlinear rigid-body dynamics for the aircraft can be written as [5, 6]

$$\dot{u} = -qw + rv - g\sin\theta + (1/m)X \tag{18}$$

$$\dot{v} = -ru + pw + g\sin\phi\cos\theta + (1/m)Y \tag{19}$$

$$\dot{w} = -pv + qu + g\cos\phi\cos\theta + (1/m)Z \tag{20}$$

$$\dot{p} = \left(R_4 + R_6\left(I_{xz}/I_{xx}\right)\right)\Big/\left(1 - \left(I_{xz}^2/I_{zz}I_{xx}\right)\right) \tag{21}$$

$$\dot{q} = \left(1/I_{yy}\right)\left[pr\left(I_{zz} - I_{xx}\right) - \left(p^2 - r^2\right)I_{xz} + M\right] \tag{22}$$

$$\dot{r} = \left(R_6 + R_4\left(I_{xz}/I_{zz}\right)\right)\Big/\left(1 - \left(I_{xz}^2/I_{xx}I_{zz}\right)\right) \tag{23}$$

$$\dot{\phi} = p + R_9\sin\theta \tag{24}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{25}$$

$$\dot{\psi} = R_9 \tag{26}$$

$$\begin{aligned}\dot{x}_e &= u\cos\theta\cos\psi \\ &\quad + v\left(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi\right) \\ &\quad + w\left(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi\right)\end{aligned} \tag{27}$$

$$\begin{aligned}\dot{y}_e &= u\cos\phi\sin\psi \\ &\quad + v\left(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi\right) \\ &\quad + w\left(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi\right)\end{aligned} \tag{28}$$

$$\dot{z}_e = -u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\cos\theta \tag{29}$$

where I_{xx}, I_{yy}, I_{zz} are the moments of inertia; I_{xz} is the product of inertia; ϕ, θ, ψ are the roll, pitch, yaw attitude angles; p, q, r are the attitude rates; u, v, w are the longitudinal, lateral and vertical body velocities; m is the mass of the aircraft; and g is the gravitational acceleration;

$$R_4 = \left(1/I_{xx}\right)\left[-qr\left(I_{zz} - I_{yy}\right) + pqI_{xz} + L\right] \tag{30}$$

$$R_6 = \left(1/I_{zz}\right)\left[-pq\left(I_{yy} - I_{xx}\right) - qrI_{xz} + N\right] \tag{31}$$

$$R_9 = \left(r\cos\phi + q\sin\phi\right)/\cos\theta \tag{32}$$

The aerodynamic, thrust, and other forces and moments can be obtained as

$$X = -D\cos\alpha + L\sin\alpha + T\cos\phi_T \tag{33}$$

$$Y = F_{A_Y} + F_{T_Y} \tag{34}$$

$$Z = -D\sin\alpha - L\cos\alpha + T\sin\phi_T \tag{35}$$

$$L = L_A + L_T \tag{36}$$

$$M = M_A + M_T \quad (37)$$

$$N = N_A + N_T \quad (38)$$

3 Matlab-Simulink nonlinear modeling and simulation

The aircraft model is prepared by commercial software Matlab-Simulink. The top view of the block diagram is shown in Fig.2. The model consists of five sub block-sets as follows. Aircraft block-set calculates aerodynamic forces and moments according to aerodynamic coefficients of the aircraft. Thrust block-set calculates total thrust effect of four turbofan engines. Gross weight block-set transforms the weight in body axes system with the direction cosine matrix obtained by attitude angles. Six-degree-of-freedom (6DoF) block-set includes a set of equations of motion. Control block-set produces the pilot inputs for desired maneuvers. Before run the simulation in Simulink the codes for aircraft data given in Table 1 and codes for initial conditions given in Table 2 should be executed in Matlab command window.

Some steady state initial conditions data given in Table 2 are obtained via trim algorithm at 10.000 m altitude written for this model. The trim variables for piloted flight for the modeled aircraft are calculated at $\alpha = 3.7355\,\text{deg}$, $i_h = -4.6426\,\text{deg}$, $\delta_T = 0.6912$.

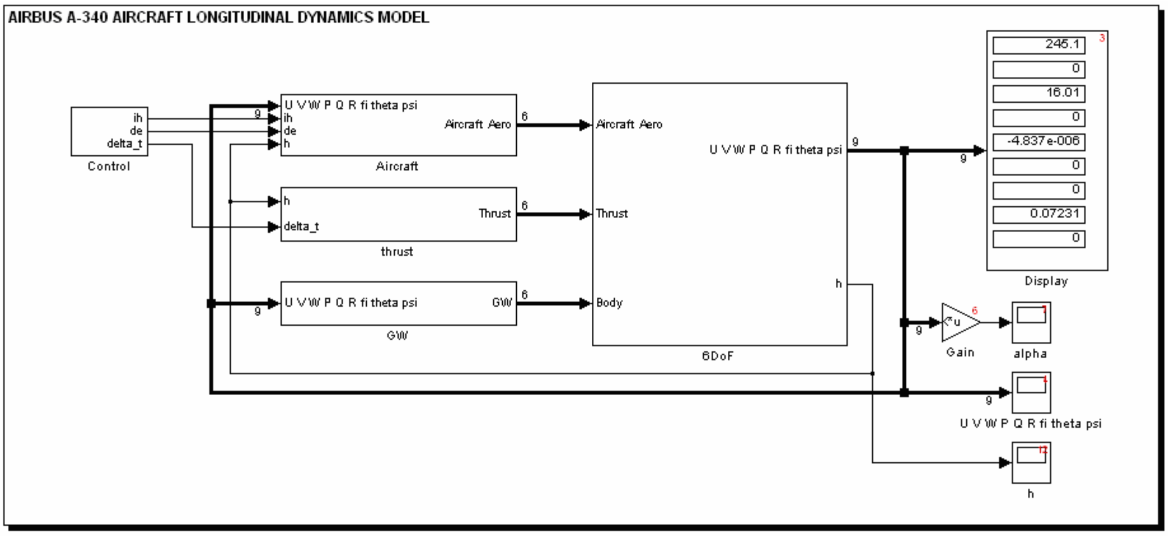

Fig.2 Top view of the Matlab-Simulink block diagram of AIRBUS-A340-300 passenger aircraft

Table 1 Airbus-A340 data

`sig  = 1.4;`	`u    = a*M;`	`Cla    = 5.9598;`
`Tsl  = 288.2;  % oK`	`q    = 0.5*rho*u^2;`	`Cl0    = 0.2301;`
`rhosl= 1.225;`	`sigma= act^4.2561;`	`Clih   = 0.8299;`
`h    = 10000;  % m`		`Clde   = 0.2391;`
`M    = 0.82;`	`dt      = -2.0;     % m`	`e      = 0.85;`
`R    = 287;    % J/kg oK`	`Sw      = 363.12;`	`Cm0    = -0.0812;`
`act  = (1-22.556e-6*h);`	`ARw     = 10.03;`	`Cma    = -3.1069;`
`T    = act*Tsl;`	`Cw      = 7.49;`	`Cmih   = -3.4077;`
`rhosl= 1.225;`	`bw      = 60.35;`	`Cmde   = -0.9816;`
`rho  = rhosl*act^4.2561;`	`Tmaxsl = 4*138800;`	`Cd_0   = 0.0172;`
`a    = sqrt(sig*R*T);`	`fi_t    = 0.0;`	

Table 2 Initial Conditions

`M      = 0.82;`	`P      = X0(3);`	`sR0    = 0;`
`h      = 10000;      % m`	`q      = X0(4);`	`sfi0   = 0;`
`g      = 9.81;`	`a      = X0(5);`	`stheta0= 0+alpha0;`
`GW     = 2500000;    % N`	`athet  = X0(6);`	`spsi0  = 0;`
`Ixx    = 22906211.6; % kg-m2`	`asigma = X0(7);`	`sxe0   = 0;`
`Iyy    = 30513547.0; % kg-m2`	`adelt  = X0(8);`	`sye0   = 0;`
`Izz    = 52921985.1; % kg-m2`		`sze0   = 0;`
`Ixz    = 0.0;        % kg-m2`	`% TRIM CONDITIONS`	
`m      = GW/g;`	`alpha0 = 3.635493494*pi/180;`	`% INITIAL CONTROL INPUTS`
`Inertia= [Ixx 0 0;`	`a3     = 3*pi/180;`	`ih     = -4.642563651*pi/180;`
`          0 Iyy 0;`	`sU0    = a*M*cos(alpha0);  % m/s`	`delta_t= 0.6911766218;`
`          0 0 Izz];`	`sV0    = 0;`	
`X0     = atmos(h);`	`sW0    = a*M*sin(alpha0);  % m/s`	`de     = 0*pi/180;`
`T      = X0(1);`	`sP0    = 0;`	
`rho    = X0(2);`	`sQ0    = 0;`	

3.1 Aircraft block-set

Aircraft block-set calculates lift force L, drag force D and pitch moment M which are the elements of output of aerodynamic forces and moments acting on the aircraft. The inputs of this block-set are the body velocities u, v, w; the attitude angles ϕ, θ, ψ; the attitude rates p, q, r; altitude, h; the incidence of horizontal tail i_h; and the elevator deflection δ_e. Generally this block-set is described with Equations (1)-(10) and (33)-(38). The block diagram is shown Fig.3. The user defined function "aero_fm" calculates L, D, M, α and "aerox1" transform the forces and the moment into body fixed coordinates at the center of gravity of the aircraft.

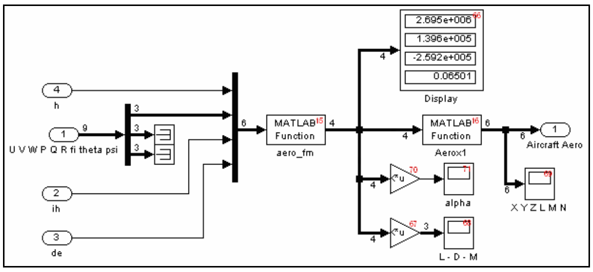

Fig.3 Block diagram of aircraft block-set

3.2 Thrust block-set

The thrust block-set calculates the total thrust T of four turbofan engines. The inputs of this block-set are the altitude h and as a control input the thrust percentage δ_T. The block-set is described with Equations (11)-(17) and (33)-(38). The block diagram is shown in Fig.4. The user defined function "mthrust" calculates the total thrust force of four engines and "thrx3" transform the force into body fixed coordinates at the center of gravity of the aircraft.

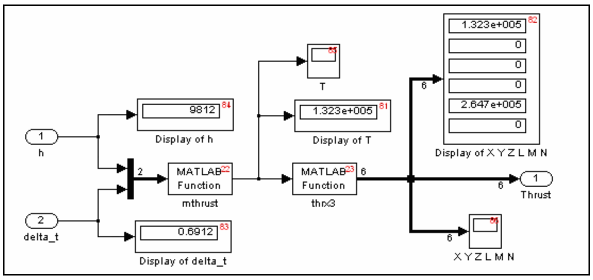

Fig.4 Block diagram of thrust block-set

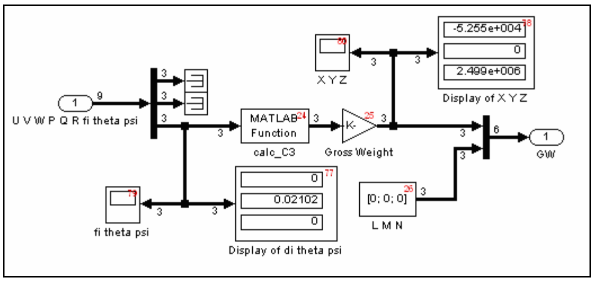

Fig.5 Block diagram of GW block-set

3.3 GW (Gross Weight) block-set

Gross weight block-set calculates the forces cause by the weight of the aircraft in body fixed coordinates at the center of gravity. The inputs are the attitude angles and the gross weight of the aircraft. The block diagram is shown Fig.5. The user defined function "calc_c3" transform the weight with proper direction cosine matrix obtained by Euler angles ϕ, θ, ψ in body fixed coordinate system.

3.4 Six-degree of freedom (6DoF) block-set

This block-set calculates the body velocities u, v, w, the attitude angles ϕ, θ, ψ, the attitude rates p, q, r and the displacements in Earth axis system x_e, y_e, z_e. The inputs are the forces X, Y, Z, the moments L, M, N, the aircraft mass m and the inertia matrix I of the aircraft. This block set is described with a set of Equations (18)-(32) to calculate aircraft dynamics. The block diagram is shown Fig.6

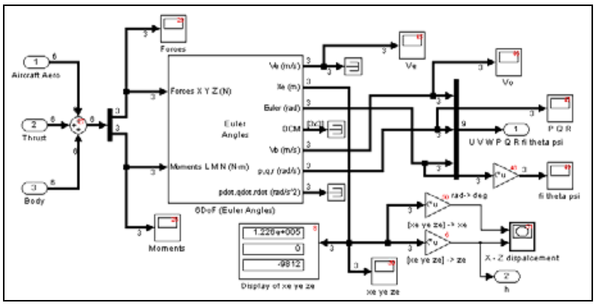

Fig.6 Block diagram of 6DoF block-set.

3.5 Control block-set

This block-set produces pilot inputs or control system inputs such as incidence of the horizontal tail, elevator deflection and thrust percentage. Three different simulations specified by (39), (40) and (41) are performed to illustrate the flying qualities in longitudinal motion of the aircraft for elevator input by pilot, for thrust change, and for a change of angle of attack in this study.

$$\text{Case A: } \delta_e = \begin{cases} 0s \le t \le 100s & \delta_e = \delta_{e,trimmed} = 0^\circ \\ 100s < t \le 110s & \delta_e = 5^\circ \\ t > 110s & \delta_e = \delta_{e,trimmed} = 0^\circ \end{cases} \quad (39)$$

$$\text{Case B: } \delta_T = \begin{cases} 0s \le t \le 100s & \delta_T = \delta_{T,trimmed} \\ 100s < t \le 110s & \delta_T = 1.1\delta_{T,trimmed} \\ t > 100s & \delta_T = \delta_{T,trimmed} \end{cases} \quad (40)$$

$$\text{Case C: } \alpha = \alpha_{trimmed} + 3^\circ \text{ for } t > 0s. \quad (41)$$

4 Simulation results

Three different simulations techniques are performed to illustrate the behavior of the modeled aircraft. The first maneuver "Case A" described with (39) examples the aircraft according to elevation deflection caused by longitudinal stick input. The second maneuver "Case B" described with (40) examples the aircraft according to thrust of engines by thrust arm input. The third maneuver "Case C" described with (41) examples the aircraft for a change of angle of attack.

The elevation effect for the input of "Case A" is simulated and the time responses of the body velocities, angular rates, attitude angles, aerodynamic forces and moments, total thrust, angle of attack, flight altitude and deflection of elevator of the modeled aircraft are shown in Fig.7-14, respectively. As seen from the figures the aircraft is stable. Short-period dynamics reach steady state approximately in 15 seconds. However, damping ratios of long-period dynamics are too small and system reaches steady state more than 30 minutes. For high elevator deflection of horizontal tail/stabilizer, the aircraft oscillates totally up and down at first side with maximum gain of 75 meters of altitude, 30 m/s of forward speed and 10 degrees of pitch angle near trim conditions.

The thrust change for the input of "Case B" is simulated and the time responses of the body velocities, angular rates, attitude angles, aerodynamic forces and moments, total thrust, angle of attack, flight altitude and thrust input of the modeled aircraft are shown in Fig.15-22, respectively. For the given thrust at trimmed flight in (40) the aircraft oscillates with maximum gain of 18 meters of altitude, 2-3m/s of forward speed and 0.7-0.8 degrees of pitch angle. After setting the thrust back to the trimmed value the aircraft is stabilized approximately in 25 minutes.

The change of angle of attack considered as disturbance effect shown in "Case C" is simulated and the time responses of the body velocities, angular rates, attitude angles, aerodynamic forces and moments, total thrust, angle of attack and flight altitude of the modeled aircraft are shown in Fig.23-29, respectively. For the disturbed angle of attack at trimmed flight, the aircraft oscillates with maximum gain of 40 meters of altitude, 24 m/s of forward speed and 6.5 degrees of pitch angle.

5 Conclusions

In this technical report, aircraft aerodynamics and rigid body dynamics for longitudinal motion of a large passenger commercial aircraft Airbus-A340 are presented. A nonlinear mathematical model is obtained and some simulations techniques are investigated with Matlab-Simulink. The modeling steps are explained clearly and all equations are given in the report. Three different flying maneuvers are performed to illustrate modeled aircraft motion with a standard atmospheric mathematical model.

The aircraft model can be advanced by selection of aerodynamic coefficient from look-up table which may illustrate different flying conditions for different Reynolds numbers. Then the fuel flow can be associated with the mass and inertias changes for the aircraft dynamics to simulate the aircraft range. Moreover, a precise atmospheric model can be replaced instead of considered one and optionally the landing assembly can be added for simulation of take-off or landing maneuvers. Finally, lateral dynamics can be investigated in similar manner for a full passenger aircraft model.

6 References

[1] John David Anderson, 2001. Fundamentals of Aerodynamics, McGraw-Hill, Boston.

[2] http://www.airbus.com/en/aircraftfamilies/a330a340/a340-300/specifications.html

[3] Thomas R. Yechout, Steven L. Morris, David E. Bossert, Wayne F. Hallgren, 2003. Introduction to Aircraft Flight Mechnaics, AIAA Education Series, Virginia.

[4] http://www.grc.nasa.gov/WWW/K-12/airplane/atmos.html

[5] Donald McLean, 1990, Automatic Flight Control Systems, Prentice Hall, London.

[6] Warren F. Philips, 2004, Mechanics of Flight, Wiley, New Jersey.

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TIME RESPONSES OF SIMULATION FOR CASE A

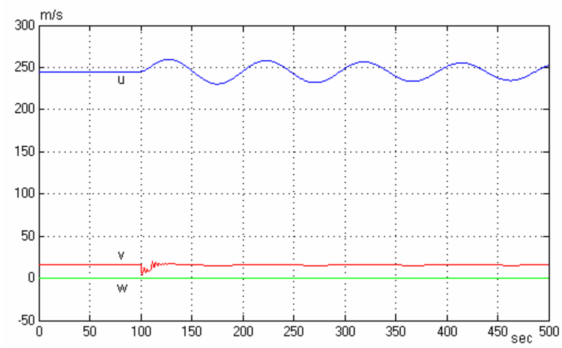

Fig.7 Body velocities: u, v, w

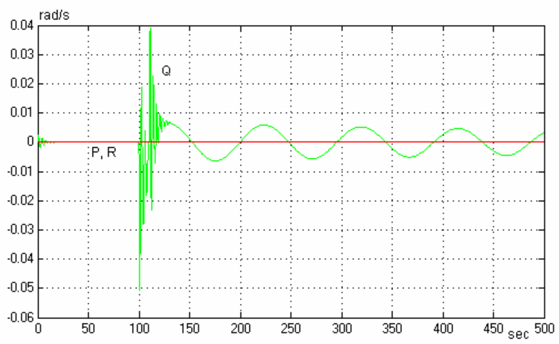

Fig.8 Attitude rates: p-roll rate q-pitch rate r-yaw rate

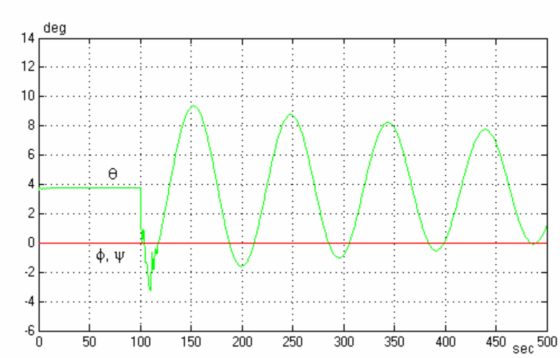

Fig.9 Attitude angles: ϕ-roll θ-pitch ψ-yaw

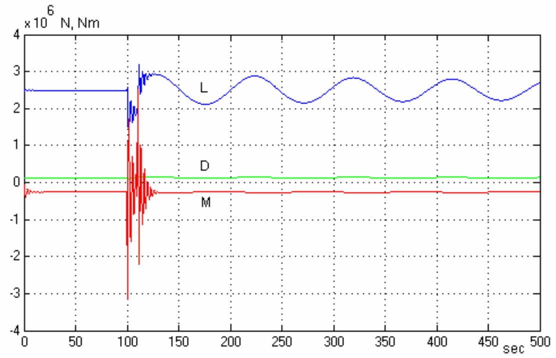

Fig.10 Aerodynamic forces and moments (L, D, M)

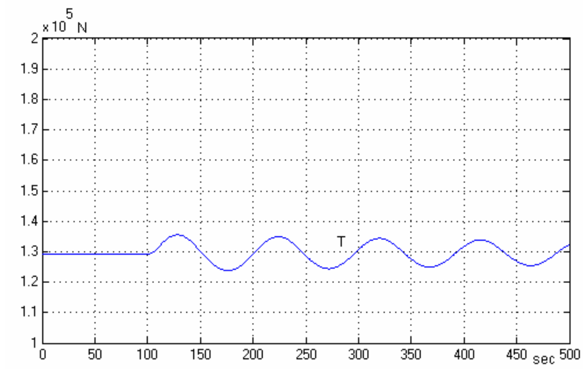

Fig.11 Total thrust produced by 4 turbofan engines

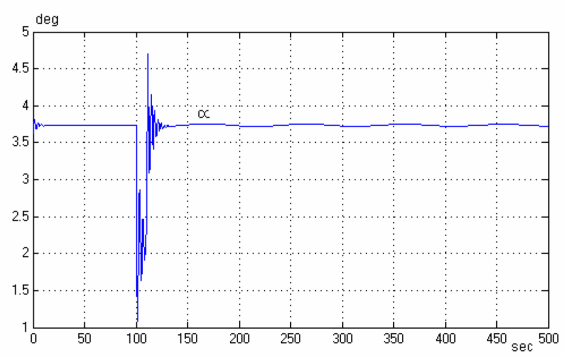

Fig 12 Angle of attack: α

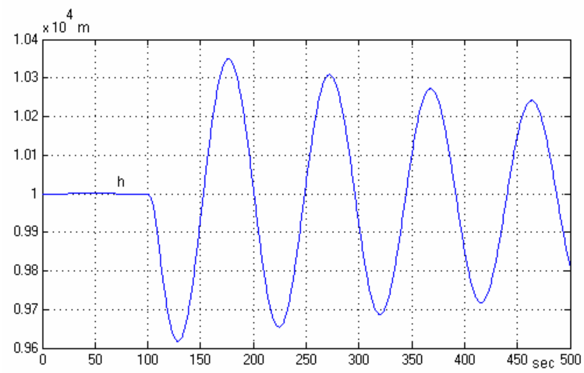

Fig.13 Flight Altitude: h

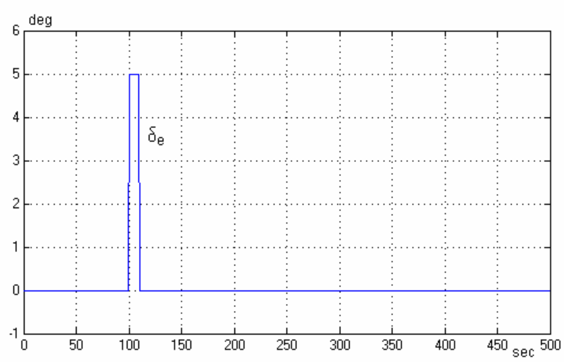

Fig.14 Deflection of elevator

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TIME RESPONSES OF SIMULATION FOR CASE B

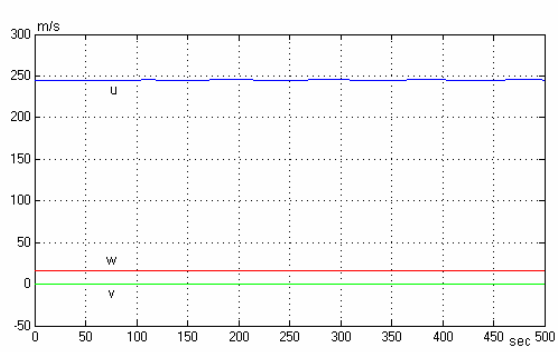

Fig.15 Body velocities: u, v, w

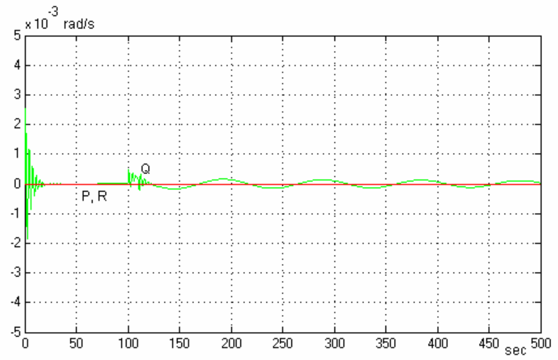

Fig.16 Attitude rates: *p*-roll rate *q*-pitch rate *r*-yaw rate

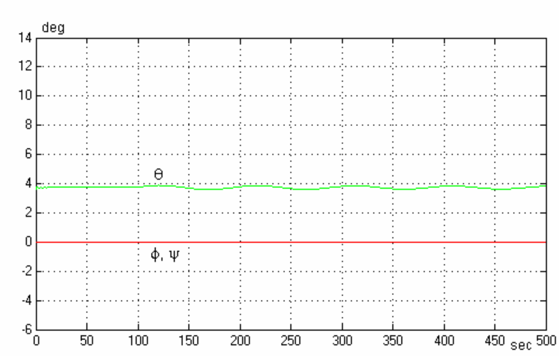

Fig.17 Attitude angles: ϕ-roll θ-pitch ψ-yaw

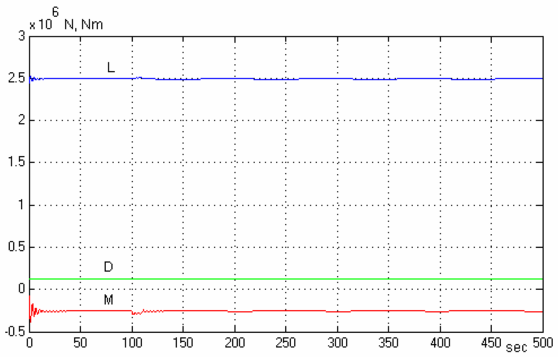

Fig.18 Aerodynamic forces and moments (*L*, *D*, *M*)

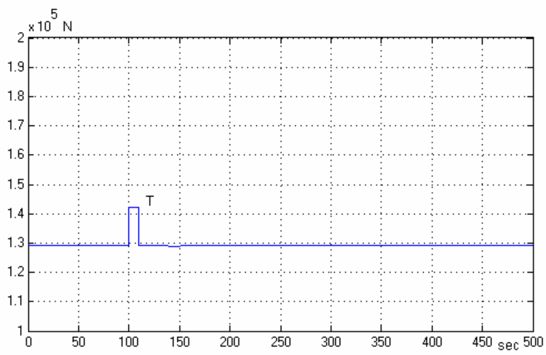

Fig.19 Total thrust produced by 4 turbofan engines

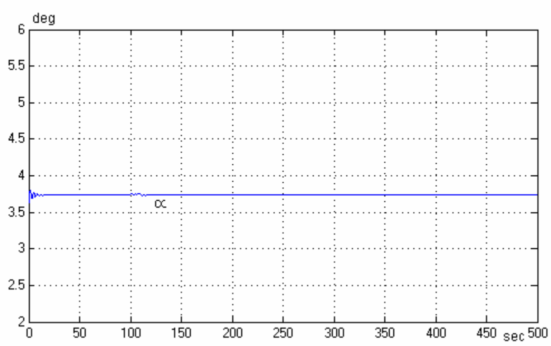

Fig.20 Angle of attack: α

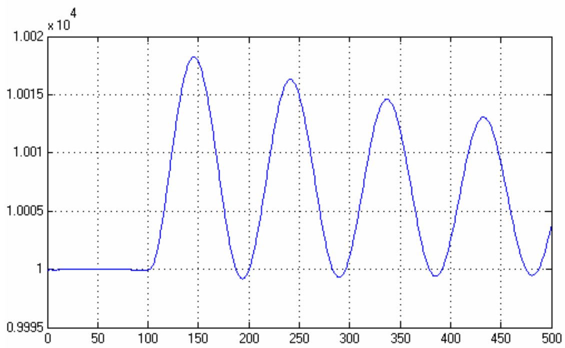

Fig.21 Flight Altitude: h

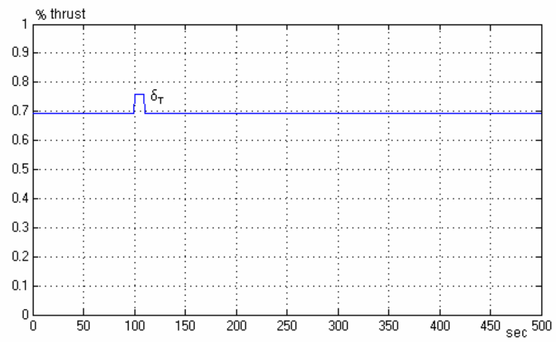

Fig.22 Thrust %

TIME RESPONSES OF SIMULATION FOR <u>CASE C</u>

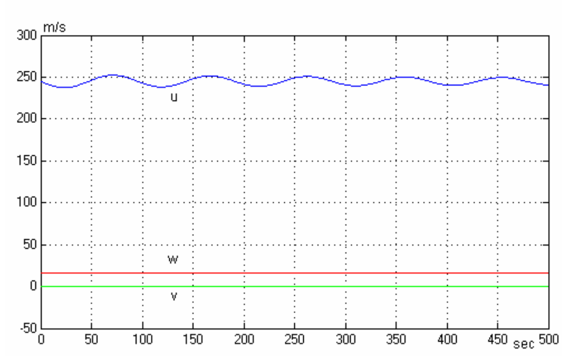

Fig.23 Body velocities: u, v, w

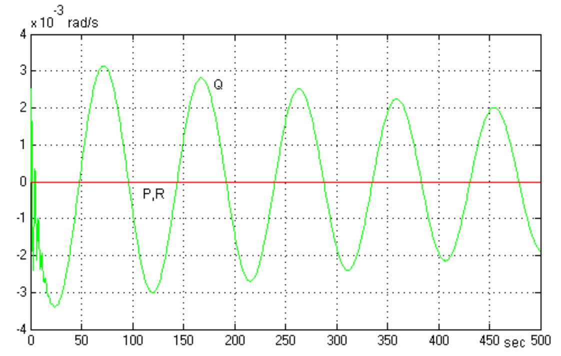

Fig.24 Attitude rates: *p*-roll rate *q*-pitch rate *r*-yaw rate

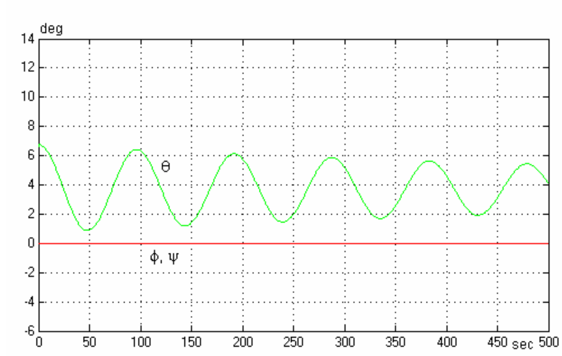

Fig.25 Attitude angles: ϕ-roll θ-pitch ψ-yaw

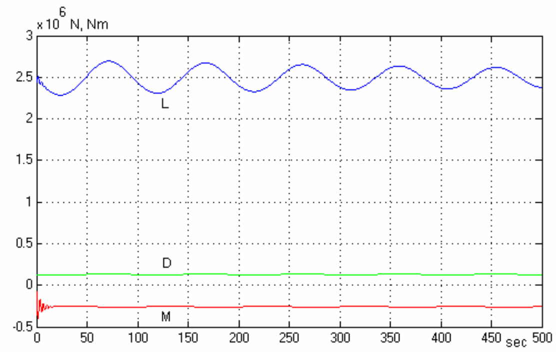

Fig.26 Aerodynamic forces and moments (*L*, *D*, *M*)

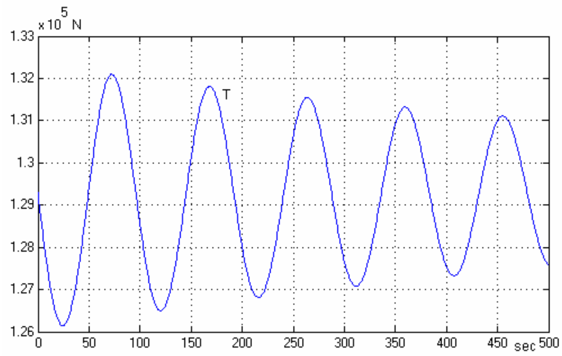

Fig.27 Total thrust produced by 4 turbofan engines

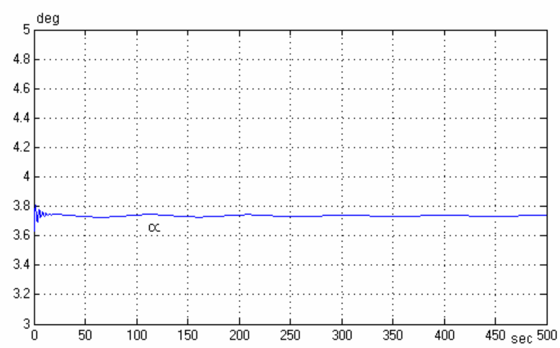

Fig.28 Angle of attack: α

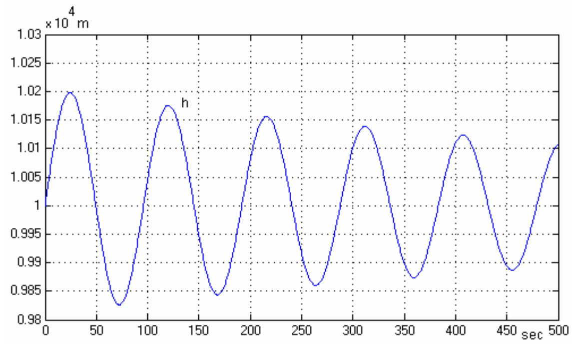

Fig.29 Flight Altitude: h

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