FUZZY ELLIPSOIDAL TECHNIQUES TO INVESTIGATE NECKING PHENOMENA IN METALLIC SHEETS

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Abstract

Non-Destructive Testing in the field of defects identification in metallic elements plays a remarkable role with special regard to those sectors where the integrity of the material is strictly required. As a consequence, the detection of defects and discontinuous deformations in metallic plates and bars, together with the relevant shape classification, provides to the operator useful information on the actual mechanical integrity of the specimen. When solid-solid-phase transformations are being studied, the loss of uniqueness in the solution, together with the relevant fragmentation of the strain fields inside the solid can be observed, the equilibrium coming out as a solution represented from a fine mixture among phases. In this paper, firstly, a theoretical characterization of a mono-dimensional model with respect to the computational aspects are presented. Then, a comparison is done with data from an experimental non-destructive investigation technique based on the eddy current principle. Finally, a novel approach to solve the inverse problem by means Fuzzy Ellipsoidal Inference Systems is proposed. In particular, Mamdani's Inferences have been taken into account to investigate the local discontinuities of the metallic specimen subject to plastic deformations due to phase transformations when applying experimental mono-axial traction. After that, the stress field, which wouldn't be otherwise measurable, can be calculated just applying the relevant constitutive law.

Keywords: Phase transformation, interfacial energy, fuzzy approach, eddy currents.

Presenting Author's biography

Michele Buonsanti. Degree in Architecture, and specialized in Computational Mechanics of Materials and Structures, is Research with The Department of Mechanics and Materials of the Faculty of Engineering at the University of Reggio Calabria. He is author and co-author of many scientific papers in the field of mechanics of materials particularly, biomechanics, microstructure and variational elasticity.

1 Introduction to the problem

In order to improve manufacturing quality and ensure public safety, components and structures are commonly inspected for early detection of defects or faults which may reduce their structural integrity. Non Destructive Testing and Evaluation (NDT/NDE) techniques present the advantage of leaving the specimens undamaged after inspection. NDT involves treating defect detection and characterization as inverse problems. In experimental NDT/NDE, the available measurement data are exploited in order so some clues may emerge in the inspection signal that are possibly representative of structural modification of the specimen, like cracks, flaws and phase transformations that develop to discontinuous deformations. The solid-solid phase transformation theory, deduced from Van der Walls gas theory, has properly been calibrated in order to become a tool able to describe remarkable solid state physical phenomena. Ericksen [1] developed an effectual methodology to study necking-type phenomena, phase changes, hysteresis, just exploiting the elastic theory. When metallic materials are considered, a stress-strain step function is found, similarly close to Savart-Masson effect [2]. This practical-theoretical correspondence suggests that behavioral models of materials can be formulated, based on the concept of stability, as bifurcation phenomena of stress-strain curve, having as a consequence the loss of the uniqueness, as well as inhomogeneity of the deformation, or better, at intervals homogeneity. Analytical consequences entail non-monotonic relations and non-convex elastic potentials, thus making difficult the computational approach [3], [4]. The difficulty in this kind of models is that phase-rearrangements are equi-energetic, and consequently, minimizing of strain fields are in general highly non-unique [5], [6]. The local discontinuities of the metallic specimen subject to plastic deformations can be investigated by using the eddy current-based non destructive testing. If discontinuities are present in the sample, they cause local modifications both in electric and magnetic properties of the specimen itself, being consequently modified the equilibrium condition, relevant to the absence of defects. In the framework of the research, a number of test campaigns have been carried out at our NDT Lab to experimentally verify that, when strains are applied, metallic materials exhibit some interesting microstructures; in two-dimensional configurations a shear band distribution can be seen, as a consequence of deformation gradients. Similarly, in one-dimensional configurations of metallic materials a two-phase behavior can be observed, that results in two separate zones at different deformation levels, with a non sharp interface in the middle as predicted in the second order gradient theory, representing the interface between two phases. The path of the eddy currents is thus disturbed, and a variation of the total magnetic field is produced. In the exploited probe, a FLUXSET ® type one [7], this implies a phase shifting in the voltage across a pick-up coil. Automatic step-by-step scanning procedures were performed on metallic plates subject to mono-axial traction. This kind of investigation procedure, known as hard device type, consists in applying a displacement in order to determine the field strain (stretch and shear). The subsequent deformative condition of the material under investigation shows interesting alternating gradient microstructures. Having acquired this result, the stress field, which couldn't be calculated just applying the relevant constitutive law, which is not otherwise directly measurable. The proposed model turns out very useful in multi-physics approaches, i.e. a number of practical situations where mechanical integrity is compromised by non-compatible stress fields, for example thermal-mechanic, magnetic-mechanic, chemical-mechanic and so on. Finally, a novel technique based on the concept of Fuzzy Inference Systems (FIS) is here proposed for solving the inverse problem at hand. FISs are very good tools to treat uncertainty, as they hold the nonlinear universal approximation. They are suitable to handle experimental data as well as a priori knowledge on the unknown solution, which is expressed by inferential linguistic rules in the form IF...THEN whose antecedents and consequents utilize fuzzy sets instead of crisp numbers. The rest of the paper is organized as follows. In section 2 prominence is given to mechanics of microstructures aiming to point out the physics of the problem at micro-scale level, where necking stands out as coexistence of phases. The computational aspect is developed in section 3 under the hypothesis of a polyconvex, Blatz-ko- type deformation potential [8]. A second order term is found, and a problem of minimum, together with "optimal profile" problem are formulated, this last one representing the shape of the transition zone among phases. It is solved by Γ convergence method. In section 4 the employed non-destructive investigation methodology is expounded as well as the achieved experimental outcomes. The solution of the inverse problem by fuzzy techniques is developed in section 5, being a validation of the chosen approach. Remarks and conclusions can be found in section 6, which is devoted to summarize this treatment

2 Mechanical and computational aspects of microstructures

One of the most important problems in the field of materials strength is to determine mechanical conditions, which result in permanent deformations or fracture. In crystalline solids, the plastic phase comes out as a number of physical phenomena very well defining the behavior of the microstructure [9]. In particular, slips phenomenon stands out, that is the parallel transition of elements inside the crystalline reticule. This kind of deformation turns out in several parallel stripes well observable along the surface of the solid. The slips-line computational model is very well known in fracture mechanics. The localization of the plastic flux inside shear bands can be often seen in ductile solids. At the beginning, the deformation develops in a regular manner, but beyond a given threshold, high strain concentration zones are found. As a consequence, the deformation grows up inside the shear band without affecting the whole deformation of the solid and the occurrence of points of failure is highly probable [10]. This behavior can be easily seen when considering necking phenomenon where the solid becomes homogeneously deformed until a given value of the hard or soft device; after that, in correspondence with the next increase of the load, at the middle point two concentrations are produced, together with a sudden drop in load itself. If the external load is further on increased, the deformation spreads with a constant value of the load. Necking is the typical phenomenon to give evidence that strain conditions of the same class can coexist, but having different values (phase transformation, Fig. 3). In polymers, such as in metallic materials [11], contraction takes place together with the rising of localized shears in the region which divides big strains from small ones, clearly inclined with respect to the longitudinal axis of the solid. Analytical difficulties in formulating non-convex potential models gave rise up to today to the building of several mono-dimensional models, satisfactorily close to the actual behavior. Due to the high computational complexity, the formulation of a bi-dimensional model turns out extremely complicated. Starting from the assumption to build an Ericksen's model from tri-dimensional elastic theory, a suitable outcome was found, even in presence of simple approximations [1]. Therefore, one can assume [12], for a reference configuration Ω of a continuum, isotropic, incompressible, hyperelastic material body $\mathcal{B}$, that the deformation energy density $\mathbf{W}$ $(\nabla\mathbf{y})$: $\mathrm{Lin}^+ \to \mathbb{R}$ is a function of the eingenvalues of the gradient of the deformation $\mathbf{F}=\nabla\mathbf{y}$ supposed to comply with the material objectivity requirements, and to respect the well-known Ball poly-convexity condition [11]. In particular a deformation function $\mathbf{y}$, verifying the usual hypothesis, i.e. regularity, injectivity, det $\nabla\mathbf{y} > 0$, is considered

$$\mathbf{y} : \Omega \to \mathbf{R}^{\mathbf{n}} \tag{1}$$

with a multi-convex associated Blatz-Ko potential, where a = cost, φ a convex function.

$$\mathbf{W}\,(\mathbf{F}) = \mathrm{a}\,(\mathbf{F}\cdot\mathbf{F} + \varphi(\det \mathbf{F})) \tag{2}$$

If the transversal contraction (necking) is considered, a particular kinematics for $\mathcal{B}$, defined trough a class of deformations where a particle at $\mathbf{x} = x\mathbf{i} + y\mathbf{j} + k\mathbf{z}$ is mapped to

$$\mathbf{y} = [(1+\nu)x - \nu x\partial_z\, f(x,y,z)]\mathbf{i} + [(1+\nu)y - \nu y\, \partial_z f(x,y,z)]\mathbf{j} + f(x,y,z)\mathbf{k} \tag{3}$$

being the relevant deformation gradient:

$$[\mathbf{F}] = \begin{bmatrix} 1+\nu-\nu f_{,z} & -\nu x f_{,zy} & -\nu x f_{,zz} \\ -\nu y f_{,zx} & 1+\nu-\nu y f_{,zy} & -\nu y f_{,z} z \\ f_{,x} & f_{,y} & f_{,z} \end{bmatrix} \tag{4}$$

The aim is therefore to find for the energy functional $\mathbf{E}=\int_\Omega \mathbf{W}(\mathbf{F})$ the

$$\min_{\mathbf{f}\in A} \mathbf{E}(\mathbf{W}) \text{ with } \mathbf{A}= \{\, \mathbf{f}: \Omega\to\mathbf{R}^3, \quad \mathbf{v}(0) = 0,\ \mathbf{v}(\mathrm{L}) = \beta\mathrm{L} \,\} \tag{5}$$

The problem is simplified by introducing internal cinematic restrictions, i.e. the motion preserves the flatness of transverse planes normal to the solid axis. This conditions is equivalent to assume that $f\,(x,y,z)$ does not depend upon x and y; for convenience, lets keeping the same notation, being simply $f = f(z)$ the new corresponding function and $f'(z)$ its derivative. A similar restricted kinematics was earlier considered by Coleman and Newman [13]. The total energy can be integrated in the cross section, to give:

$$\mathbf{E}\,[\mathrm{f}] = a\mathrm{A}\int_0^{\mathbf{L}} (\Lambda(\mathbf{f}') + \frac{1}{2}\mathbf{k}\mathbf{f}''^2)\, dz \tag{6}$$

Here, $\Lambda = \Lambda(t)$ is an opportune not-convex function, and $\mathbf{k}$ is a constant depending from the second order polar moment of inertia of the solid cross-sectional area. In this way it has been shown that the classical non-linear elasticity theory, under kinematical assumptions, may directly provide models where the stored energy is a non-convex function of the axial strain. This implies the natural appearance of a second-order term in the resulting energy (6) which plays the important role of penalizing the interfaces between heterogeneous material phases.
Setting $u = f'$ the problem under consideration becomes the following [12]

- **Problem $\Pi°$:** minimize

$$\Pi[u] = a\mathrm{A}\int_0^{\mathbf{l}} \Lambda\,(u(z)) + 1/2k(u'(z))^2\, dz \tag{7}$$

over all $u\in \mathrm{W}^{1,2}(0,\mathrm{L})$, $u>0$. Several formulations of this model can be found in literature. It was firstly proposed by Coleman [14] and by Carr, Gurtin & Slemrod [15] as a development of the original idea by Ericksen. In [15], [16] a well known theorem states that for k infinitesimal, the higher order term causes the two-phase solutions with least energy to become the single interface solution. As this very important case is concerned, we refer to an elegant approach by Alberti [17], providing an elementary characterization of the transition zone among heterogeneous phases. The two phases are connected by a transition zone and consequently the following optimal profile problem may be considered to determinate the shape of the transition zone.

- **Problem Π^*** : (optimal profile) find u: R→R such that

$$aA \int_{-\infty}^{+\infty} [\Lambda^*(u(z))+1/2k(u'(z))^2]\, dz = \min \qquad (8)$$

$$\lim_{z\to-\infty} u(z) = \alpha_1 \quad \text{and} \quad \lim_{z\to+\infty} u(z) = \alpha_2$$

and by means of a heuristic approach, applying Γ-convergence, the non-sharp profile is found, relevant to the interface among phases, minimizing in addition the energy. The jumps of the displacements in relationship with the consumption of the energy are practically taken into account. If a minimum is concerned, only one interface is found and the jump between the two phases of the displacement function just represents necking. This model is valid both in mono-dimensional and bi-dimensional solids. Its configuration in 1D formulation respects the restrain of undeformability of the transversal sections and consequently it allows to model the typical restraining among elastic and plastic phases. In 2D case, one can neglect the transversal section when considering thin specimens. The modeled degree-2 materials allow a soft type approach of the interface among phases. Experimental comparisons find similar interfaces in case of necking formation in plane thin-solids, where the concentrated deformation is highly evident, as well as in mono-axial traction with high-intensity loads . Again, phase interface is a optimal profile either in mono and bi-dimensional specimens after the hard device experimental test is applied.

3 The exploited database

The microscopic structure of metallic material has a correlation with its local magnetic properties [18]; in particular, local discontinuities of a some amount, such are those in the structure of a specimen subject to plastic deformations, can be investigated by using an electromagnetic non destructive testing method. Eddy current non destructive investigation technique (ECT), which is based, under the point of view of physics, in exciting the specimen by electromagnetic waves at frequencies usually ranging from few hundreds of Hz up to 300 kHz turns out very efficient to investigate cracks and flaws in metallic parts, for instance in industrial equipments, both at on-line production phase and during their operative life. Application fields of non-destructive electromagnetic diagnostic cover many areas. The analysis by eddy currents, as well as other non destructive investigation techniques, allows both the selection of faulty materials, this way saving the cost of following machining, and verifying the integrity of the material itself at the end of the manufacturing or in service. As for technology is concerned, main features on which ECT devices are based on are: 1) easiness in using, thanks to hardware and software devices; 2) versatility (large assortment of probes); 3) quickness in getting expected results mainly for on-line applications; 4) PC-linking capability for data post-processing; 6) small influence on the measurements due to external disturbances. Eddy current based investigation techniques exploits induction phenomena, by means of which, inside the specimen excited by an external electromagnetic source (primary filed), eddy currents arise that act as sources of a new field (reaction field), contributing to define the actual distribution of the overall field. Of course the distribution of the induced field depends on the geometry of the specimen and consequently on the presence of defects if any; that's way from the analysis of magnetic field variations or other related quantities, information can be achieved both on the presence and the characteristics of the defect. By exploiting this principle, the control of the specimen can be done without neither electric nor mechanic contact. Let consider a main coil (exciting coil) driven by an alternate current with a FLUXSET probe located along its longitudinal axis. Inside the cylinder an alternative magnetic field H_0 will appear, which in his turn will induce circular pattern eddy currents. These latter will give rise to a reaction magnetic filed such to oppose the main field, producing a variation in the exciting signal. By the analysis of its phase variations in the voltage of FLUXSET probe due to the presence of defects, one can diagnose the presence of it as well as evaluate other relevant parameters, such as the position, the depth and the width. ECT methodology nevertheless presents some limitations, mainly due to the decrease of both eddy currents intensity and density with the increase of the investigation depth. At this regard care must be taken in choosing the proper exciting frequency, f, considering that the penetration depth δ of eddy currents varies according to:

$$\delta = (1/2\pi f\mu\sigma)^{1/2} \qquad (9)$$

(μ and σ magnetic permeability and electrical conductivity of the specimen).

In this paper the effects caused on a steel specimen plate, 110 x 220 width, 1 mm thick, subject to 27 kN monoaxial traction, which produced a plastic deformation, were considered. The exploited experimental methodology is based on the well-known principle of eddy currents. An exciting **B** magnetic field was generated by a sinusoidal current flowing in a primary coil. This field induces an eddy current flow in the metallic plate, which follow concentric paths along planes perpendicular to the direction of **B**, according to the equation rot($\mathbf{B}/\mu$) = $\mathbf{J}$, where $\mathbf{J}$ is the current density (from a circuital point of view the coil and the specimen are the primary and the secondary of a transformer respectively). If discontinuities are present in the sample, they cause local modifications both in electric and magnetic properties of the specimen itself, being consequently modified the equilibrium condition, relevant to the absence of defects; the path of the eddy currents is thus disturbed, and a local variation of the primary magnetic field density is produced. In the exploited probe, a FLUXSET type one [7], this variation implies the rising of phase shifting in the voltage across the pick-up coil; these shifts turn out proportional to the component of the magnetic field parallel to the longitudinal axis of the pick-up coil. The FLUXSET sensor has an excellent vertical sensitivity and a very good signal to noise ratio. These features, together with a marked miniaturization, makes FLUXSET especially suitable to investigate with great accuracy plane metallic structures. The experimental investigation concerned an automatic step-by-step scanning procedure (1 mm scanning step) along both x and y axes, over a 140 x 90 surface (center of mass to the area where defect is waited to be located). The exciting current was set to 107 mA RMS, 1 kHz, sinusoidal, the driving signal (to saturate the core material of the FLUXSET) was equal to 125 kHz, triangular, 9 Vpp in amplitude. Experimental tests have been performed at the Non Destructive Testing Lab, DIMET Department, University of Reggio Calabria. Four matrixes, i.e. our database, come out from the scanning procedure, respectively representing the real part, the imaginary part (Fig. 2), the magnitude (Fig. 1) and the phase of the picked-up voltage in each point of the above-mentioned scanning. The maps we got represent the stress condition[1] of the specimen in terms of eddy currents. The evidence of necking phenomenon inside the specimen clearly turns out as a “concentration” of magnetic field lines (red dark coloring on the left part of Fig. 1 and 2) where traction took place.

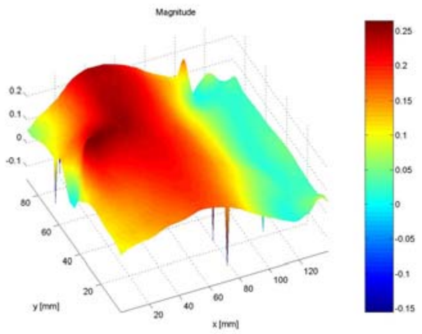

Fig. 1 Map of the magnitude ([V]) of the measured signal, applying 1kHz, 107 mA RMS sinusoidal exciting current.

[1] not otherwise directly measurable

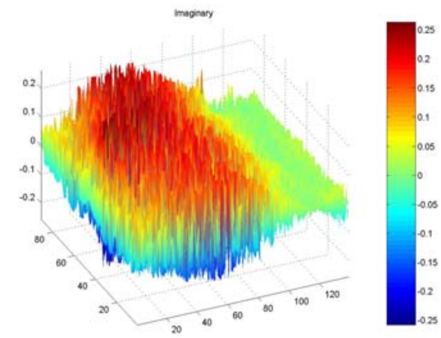

Fig. 2 Map of the imaginary part ([V]) of the measured signal, applying 1kHz, 107 mA RMS sinusoidal exciting current

4 Ellipsoidal Fuzzy Inference Systems to solve inverse problems

The main goal of this part of the work is to show how the information extracted from the database can drive the construction of simple yet efficient models for investigating the local discontinuities of the specimen. The solution implies the analysis of large databases in order to extract some relevant regularities (i.e. features). The exploited algorithms must discern important features from noise, where noise can be conceptualized as anything that is not a feature. A feature is a particular phenomenon that exists in the volume of data. From the application viewpoint, a feature is a high-level multi-dimensional object that has a number of important distinguishing characteristics. The extraction of features is performed by identifying the essential phenomena in the data according to the feature specification. The problem here is to associate a pair of x-y coordinates on the specimen to the magnitude of the stress in that point. Quite often, the emergence of the decision about the magnitude of stress derives from the organization of recognized features into compact abstractions possibly amenable to visualization and storage in compressed form.

The process of merging features is explicitly carried out in Artificial Neural Network (ANN) [19] approaches by means of a learning process. In ANNs the output of hidden layers of neurons build an internal representation of the problem. To be useful such intermediate representation of the data must preserve the distance among patterns. This means that “similar” patterns must be represented by “similar” feature vectors. The data are thus clustered around specific classes of patterns (Figures 5 and 6).This is precisely what we are asked for in our problem. One of the most relevant problems in real time applications is indeed the possibility for the deformation of passing from a configuration to another one. This implies a first derivative discontinuity of the mapping geometric coordinates – magnetic measurements. Global regressions typically show an error level about twice the least accurate individual category regression

[20]. This inability is mainly related to the handling of the first order discontinuities in the behavior of the parameter across transitions (Figures 5 and 6). Fuzzy Inference Systems (FISs) are very good tools to treat uncertainty (transition zones), as they hold the nonlinear universal approximation. They are suitable to handle experimental data as well as a priori knowledge on the unknown solution, which is expressed by inferential linguistic rules in the form IF-THEN whose antecedents and consequents utilize fuzzy sets instead of crisp numbers. The problem to represent the magnetic field distribution in a given specimen can be formulated as the search of a suitable matching inside the set of available measurements (pick-up voltage values, each one considered with their own relevant geometric x and y coordinates). In this work, the above inverse problem is solved by means of a system that extracts information on the specimen under test and implements a priori constraints to facilitate the detection and characterization of the defect. The method utilizes the concepts of fuzzy inference in order to estimate the magnitude of the pick-up voltage from geometric x and y coordinates. The numeric input and output are fuzzified in the first block of the FIS. This implies that the output is associated to more than one linguistic label with a finite degree of confidence. In other words, the measurement is treated as a fuzzy variable which admits a number of possible fuzzy values (Very Small, Small, Medium, Large, …). Each fuzzy value of the fuzzy variable is a subset of the entire domain (inverse of discourse) and it is characterized by a Fuzzy Membership Function (FMF) that maps the universe of discourse to the real interval [0, 1]. The degree to which a measured value belongs to a particular class is its grade of membership. Although it could seem inappropriate to fuzzify a numeric variable, this step can be interpreted in the framework of ill-posdness to be mapped in the corresponding geometric point. Each FMF expresses a membership measure to each of the linguistic properties. FMF are usually scaled between zero and unity, and they overlap. Gaussian FMFs have been used to improve the flexibility of the applied model. In order to design a FIS, one needs to fuzzify the variables (inputs/outputs), to extract a bank of fuzzy rules (linguistic frameworks), and finally to defuzzify the output variables.

Mamdani style inference is based on a work by Ebrahim Mamdani [21] which was proposed as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic rules obtained from experienced human operators. Mamdani style inference expects the output variables to be fuzzy sets. After the inference is complete, the resulting output variables must be defuzzified. Usually a Mamdani FIS consists of five main steps, briefly described in the following.

Step 1. Fuzzification of the inputs.

First step of Mamdani inference consists in determining the degree of membership of inputs to the different pertaining fuzzy sets. Input is always a crisp value bounded to the universe of speech of the input variable, output is a membership fuzzy degree between zero and unity.

Step 2. Application of the fuzzy operators (AND – OR).

Once inputs have been fuzzified, we know the degree to which the antecedent was satisfied for each rule. If the antecedent of a given rule shows more than a part, the fuzzy operator is applied to obtain a number representing the outcome of the antecedent for that rule. This number will then applied to the output functions. The fuzzy input operator is represented by two or more membership values arising from input variables fuzzification. The output of the operator is represented by a single truth value. As for AND operation, most used methods are:

$min\,(a,\ b)$ $\qquad prod\,(a, b)$

while for OR operator most utilized methods are:

$max\,(a,\ b)$ $\qquad probor\,(a, b)$=(a+b-ab)

where a e b are the membership functions values.

Step 3. Application of the Implication.

The implication method is defined by the fuzzy set consequent construction based on the antecedent. The implicative process is applied to each rule. The methods utilized are the same as for AND operator.

Step 4. Aggregation of the outputs.

The operation of output aggregation lies in unifying the outputs coming from each rule by graphical superposition. All fuzzy output sets for each rule are practically taken into consideration to be put together in a single fuzzy set ready to be defuzzified. The input of the aggregation process is the membership functions set (stopped or scaled) obtained by the implication process on each rule. The output is represented by a fuzzy set for each output variable. To get the final output three methods are being used: max (maximum); probor (probabilistic); sum (just the addition of each output set)

Step 5. Defuzzification of the output.

The input of the process is represented by the aggregate fuzzy output set, while the output is a crisp value obtained by the calculation of the barycentre of the figure (centroid) as it rises from Step 4.

Fuzzy rules cover the system's state space with patches of simple geometry. In order to improve the fitness of the input-output map, the fuzzy rule can have a different shape, for example, as suggested by B. Kosko [24], its shape can be an ellipsoid in the input-output state space of the system. Then an additive FIS approximates the map by covering it with ellipsoidal rule patches. According to the proposed scheme, each fuzzy rule is represented by an ellipsoidal patch on the input-output space. Fuzzy ellipsoidal systems consider each rule as an ellipsoid . The ellipsoid is described as the locus of all z that satisfy:

$$\alpha^2 = (z-c)^T A(z-c) = (z-c)^T P \Lambda P^T (z-c)$$

where:

A is the inverse of the covariance matrix,

Λ is a diagonal matrix of eingenvalues $\lambda_1, \ldots, \lambda_q$ of A,

P is an orthogonal matrix whose columns are the unit eigenvectors $e_1, \ldots, e_q$ of A.

The matrix P basically plays the role of rotating the coordinate system to the eigenvectors to orient the ellipsoid. The Euclidean half-lengths of the ellipsoid of the axes are equal to

$$\alpha/\sqrt{\lambda_1}, \ldots, \alpha/\sqrt{\lambda_q}$$

The *k*th hyper-rectangle has 2^q vertices at

$$\left(\pm \alpha_k/\sqrt{\lambda_{k1}}, \ldots, \pm \alpha_k/\sqrt{\lambda_{kq}}\right)$$

in the rotated coordinate plane. The unit eigenvectors define direction cosines for each axis of the ellipse. The projection of the *k*th hyper-rectangle onto the *i*th axis is centered at c_{ki} on the *i*th axis and has length:

$$\rho_{ki} = 2\alpha_k \sum_{j=1}^{q} \frac{|\cos \gamma_{kij}|}{\sqrt{\lambda_{kj}}}$$

The rotation matrix for the 2-D ellipsoid is:

$$P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Thus, the hyper-rectangle projections are:

$$\rho_{k1} = 2\alpha_k \left(\frac{|\cos\theta|}{\sqrt{\lambda_{k1}}} + \frac{|\sin\theta|}{\sqrt{\lambda_{k2}}} \right)$$

$$\rho_{k2} = 2\alpha_k \left(\frac{|\sin\theta|}{\sqrt{\lambda_{k1}}} + \frac{|\cos\theta|}{\sqrt{\lambda_{k2}}} \right)$$

The projection of each ellipsoid onto the axes of the input-output state space defines a fuzzy set. The ellipsoid defines a fuzzy patch or rule between fuzzy subsets of inputs and outputs. The main advantage of using the ellipsoidal fuzzy systems regards the possibility to cover the input-output space by means of ellipsoids differently oriented in the space, in such a way reducing the number of uncovered samples and, finally, best fitting the sample distribution. The representation is also parsimonious in terms of number of rules. We shall use the concept of fuzzy ellipsoidal systems only to derive an optimal representation in terms of centroids and orientation of the rules: we do not use learning algorithms to tune the systems. The achieved results are thus improvable by introducing supervised learning in the procedure.

5 The building of strain maps

A first approach consists to use the input-output pairs to design a "*traditional*" FIS in which no concept of learning was exploited: in this case, the estimation accuracy was found to be invariably not good enough. However, the design of such a "naïve" FIS can turn out to be useful as a first guess model and when real time systems are concerned. Results were improved by using an algorithm of automatic extraction of FIS from numerical data; in particular, by means of Ellipsoidal Systems, a suitable FIS was carried out for our purpose: given a set of input and output data, this procedure extracts a set of rules that models the data behavior. The problem under study is, clearly, a problem of identification, that we try to treat by means of Fuzzy Logic. Generally,

- Fuzzy Logic identification approach exploits a set of linguistic variables either in place or in addition to numerical variables; then, the characterization of links among variables by using fuzzy conditional statements is done, and finally the implementation of complex relations by using fuzzy algorithms and calculus is carried out.

As a validation of the proposed methodology, we reconstructed the map of the magnitude of the measured signal by using Ellipsoidal approach , starting, as database, from the x and y pick-up probe scanning geometric coordinates. The map in Fig. 3 has been reconstructed by means of 45 rules; when compared to the number of processed patterns (testing database), more than 12000, the achieved outcome can be considered satisfactory. The map obtained by fuzzy inference, even not reflecting the same color distribution, is anyway qualitatively comparable to that in Fig. 1. Necking phenomenon, again, is clearly visible. The number of rules in the fuzzy set we built represents less than 9% of the samples, this way one doesn't run the risk of the explosion of the rules. On the other side, considering the reduced number of inputs (x and y coordinates), to obtain qualitatively good maps we need a number of rules having an order of magnitude of some hundreds of units.

The achieved result gives us a magnetic map similar to mechanical strain one, which last one is not otherwise directly measurable.

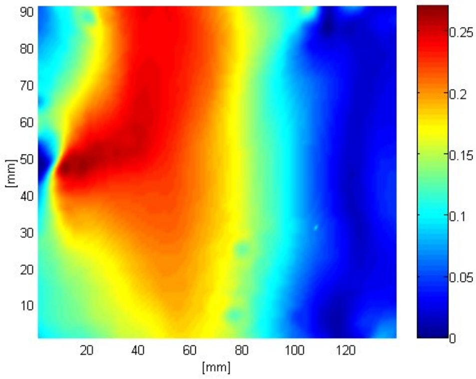

Fig. 3 Map of the magnitude of the signal (pick-up voltage, [V]) reconstructed by using fuzzy inference; inputs are the spatial coordinates, output is represented by the amplitude of the signal.

6 References

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