COMPARISON OF RESPONSES OBTAINED BY THE MAGNETICALLY NONLINEAR TRANSFORMER MODEL USING VARIABLE DYNAMIC AND STATIC INDUCTANCES

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Abstract

This work deals whit the magnetically nonlinear dynamic model of a transformer. In order to achieve the best possible agreement between measured and calculated responses the transformer model is completed by the magnetically nonlinear iron core model. The iron core model is given by the magnetically nonlinear characteristic of flux linkage versus magnetizing current, where the magnetizing current represents the sum of magnetomotive forces. The characteristic is used to define two variable and magnetizing current dependent inductances. The so called dynamic inductance is defined as a partial derivative of the flux linkage with respect to the magnetizing current while the static inductance is defined as a ratio between the flux linkage and the magnetizing current. This paper shows that responses obtained by the magnetically nonlinear dynamic transformer model substantially differ in the cases when variable dynamic and static inductances are used. The derivation presented in the paper and the comparison of measured and calculated results clearly show that only use of dynamic inductances gives acceptable results. The comparison of the measured and the dynamic model calculated results is given for the case of transformer steady state operation at rated load and for the case of switch–on of unloaded transformer.

Keywords: Transformer, dynamic model, iron core model, static inductance, dynamic inductance

Presenting Author's biography

Sebastijan Seme received B.Sc. degree in electrical engineering from University in Maribor, Faculty of Electrical Engineering and Computer Science in year 2006. He is currently involved in postgraduate study at the same institution where he is employed as a junior researcher.

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1 Introduction

Nowadays, there exist many electromagnetic devices with magnetically nonlinear iron core. For analysis of these devices and for their control design dynamic models are required. When the magnetically nonlinear properties of the device iron core are neglected, then we have to do with magnetically linear models. Such models cannot provide a good agreement between the calculated responses and those measured on the real device. In order to improve agreement between the measured and calculated responses, the magnetically nonlinear iron core behavior can be accounted for in the dynamic model [1]-[2] of the electromagnetic device in different ways. One of the numerous available approaches is based on the current dependent static inductance, while the other one is on based the current dependent dynamic inductance. Both approaches are evaluated in this work. The evaluation is based on the case study performed on a single phase transformer. The results presented show that the use of dynamic inductance in the dynamic model of a single phase transformer provides much better agreement between the measure and calculated results than the model where the static inductance is used.

2 Static and dynamic inductance

The magnetically nonlinear behavior of material is normally given by the $B(H)$ characteristics, where B denotes the flux density while H is the magnetic field strength. When this material is built in an electromagnetic device, the magnetically nonlinear behavior of the entire device can be described by the flux linkage versus magnetomotive force characteristic $\psi(\theta)$. This characteristic can be normally determined by the tests performed on the device terminals [3]-[4].

In the case of an air inductor, the ratio between the flux linkage and current is constant. It is called the inductance. However, when we have to do with an iron core inductor or any other device with an iron core, then the ratio between the flux linkage and current is no longer constant. Like in the case of magnetically nonlinear material, where the static permeability $\mu = B/H$ and the dynamic permeability $\mu_d = \partial B/\partial H$ can be defined, the static inductance $L(i)$ (1) and the dynamic inductance $L_d(i)$ (2) can be defined in the case of an iron core inductor [5].

$$L = \frac{\psi}{i} \tag{1}$$

$$L_d = \frac{\partial \psi}{\partial i} \tag{2}$$

Fig. 1 shows $\psi(\theta)$ characteristic of the tested single phase transformer. In the given case i is the magnetizing current defined as $i = \theta/N_1$, $\theta = N_1 i_1 + N_2 i_2$ where N_1 and N_2 are the number of turns of the primary and secondary winding while i_1 and i_2 are the primary and the secondary currents. Corresponding current dependent static and dynamic inductances, determined by (1) and (2) from the $\psi(i)$ characteristic shown in Fig. 1, are shown in Fig. 2.

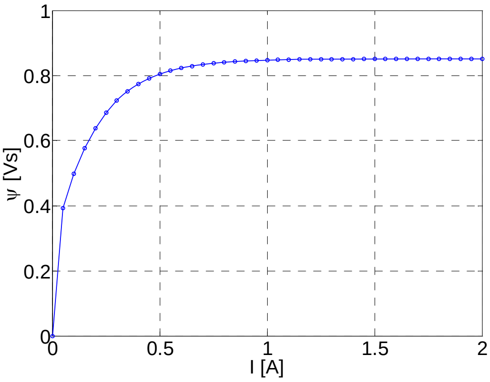

Fig 1: Magnetically nonlinear characteristic $\psi(i)$ of the tested single phase transformer

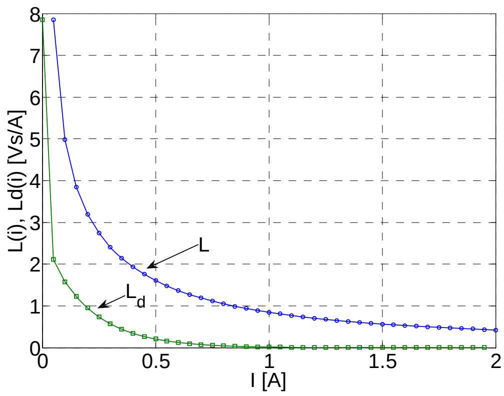

Fig 2: Static inductance $L(i)$ and dynamic inductance $L_d(i)$ determined from $\psi(i)$ characteristic in Fig.1

3 Transformer dynamic model

This section deals with the magnetically nonlinear dynamic model of a single phase transformer. It is schematically presented in Fig. 3:

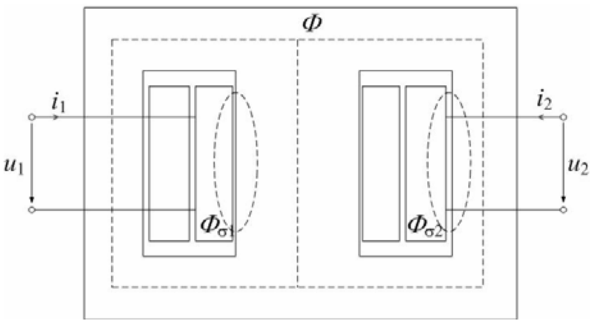

Fig. 3: Schematic presentation of a single phase transformer

where u_1, u_2 and i_1, i_2 denote the primary and the secondary voltages and currents, $\phi_{\sigma 1}$ and $\phi_{\sigma 2}$ are the primary and the secondary leakage fluxes, ϕ is the flux while the magneto motive force. When the eddy current losses and the hysteresis losses are neglected, the voltage balances in the primary and the secondary winding of the single phase transformer can be described by (3) and (4):

$$u_1 = i_1 R_1 + \frac{d}{dt}\left[\psi_{\sigma 1} + \psi_1\right] \tag{3}$$

$$u_2 = i_2 R_2 + \frac{d}{dt}\left[\psi_{\sigma 2} + \psi_2\right] \tag{4}$$

where R_1 and R_2 are the primary and secondary resistances, $\psi_{\sigma 1}$ and $\psi_{\sigma 2}$ are the primary and secondary leakage flux linkages, while ψ_1 and ψ_2 are the primary and secondary current-dependent flux linkages.

The leakage flux linkages can be expressed by the constant primary and secondary leakage inductances $L_{\sigma 1}$ (5) and $L_{\sigma 2}$ (6):

$$\psi_{\sigma 1} = L_{\sigma 1} i_1 \tag{5}$$

$$\psi_{\sigma 2} = L_{\sigma 2} i_2 \tag{6}$$

which leads to (7) and (8).

$$\frac{d\psi_{\sigma 1}}{dt} = L_{\sigma 1}\frac{di_1}{dt} \tag{7}$$

$$\frac{d\psi_{\sigma 2}}{dt} = L_{\sigma 2}\frac{di_2}{dt} \tag{8}$$

The primary flux linkage ψ_1 and the secondary flux linkage ψ_2 are caused by the main flux ϕ. They can be expressed by (9) and (10):

$$\psi_1 = N_1 \phi \tag{9}$$

$$\psi_2 = N_2 \phi \tag{10}$$

The main flux ϕ depends on the magnetomotive force θ (11), which can be expressed by the magnetizing current i_m:

$$\theta = N_1 i_1 + N_2 i_2, \quad i_m = \frac{\theta}{N_1} = i_1 + \frac{N_2}{N_1} i_2 \tag{11}$$

Considering the dependence $\phi(\theta)$ and expressions (9) to (11) the time derivatives of ψ_1 and ψ_2 can be expressed by (12) and (13).

$$\frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt} = N_1 \frac{\partial \phi}{\partial \theta}\left\{N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt}\right\} \tag{12}$$

$$\frac{d\psi_2}{dt} = N_2 \frac{d\phi}{dt} = N_2 \frac{\partial \phi}{\partial \theta}\left\{N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt}\right\} \tag{13}$$

When the magnetomotive force is expressed by the magnetizing current $\theta = N_1 i_m$ and a new symbol for the primary flux linkage is introduced as $\psi_0 = N_1 \phi$, expressions (12) and (13) can be rewritten in the forms (14) and (15).

$$\frac{d\psi_1}{dt} = \frac{1}{N_1}\frac{\partial \psi_0}{\partial i_m}\left\{N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt}\right\} \tag{14}$$

$$\frac{d\psi_2}{dt} = \frac{N_2}{N_1^2}\frac{\partial \psi_0}{\partial i_m}\left\{N_1 \frac{di_1}{dt} + N_2 \frac{di_2}{dt}\right\} \tag{15}$$

By inserting (7), (8), (14) and (15) into (3) and (4), voltage balances in the primary and secondary windings can be expressed by (16) and (17).

$$u_1 = i_1 R_1 + L_{\sigma 1}\frac{di_1}{dt} + \frac{\partial \psi_0}{\partial i_m}\left[\frac{di_1}{dt} + \frac{N_2}{N_1}\frac{di_2}{dt}\right] \tag{16}$$

$$u_2 = i_2 R_2 + L_{\sigma 2}\frac{di_2}{dt} + \frac{\partial \psi_0}{\partial i_m}\left[\frac{N_2}{N_1}\frac{di_1}{dt} + \frac{N_2^2}{N_1^2}\frac{di_2}{dt}\right] \tag{17}$$

Let us now introduce the dynamic inductance $L_d(i_m)$ by (18) and the static inductance $L(i_m)$ by (19).

$$L_d(i_m) = \frac{\partial \psi_0}{\partial i_m} \tag{18}$$

$$L(i_m) = \frac{\psi_0}{i_m} \tag{19}$$

In order to simplify (16) and (17) expressions (20) to (22) can be introduced:

$$L_{11} = L_{\sigma 1} + \frac{\partial \psi_0}{\partial i_m} = L_{\sigma 1} + L_d \tag{20}$$

$$L_{12} = \frac{N_2}{N_1}\frac{\partial \psi_0}{\partial i_m} = L_d \tag{21}$$

$$L_{22} = L_{\sigma 2} + \left(\frac{N_2}{N_1}\right)^2 \frac{\partial \psi_0}{\partial i_m} = L_{\sigma 2} + \left(\frac{N_2}{N_1}\right)^2 L_d \tag{22}$$

where L_{11} is the self inductance of the primary winding, L_{22} is the self inductance of the secondary winding, while L_{12} is the mutual (magnetizing) inductance. If (20) to (22) are considered in (16) and (17) it yields (23) and (24).

$$u_1 = i_1 R_1 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} \tag{23}$$

$$u_2 = i_2 R_2 + L_{22}\frac{di_2}{dt} + L_{12}\frac{di_1}{dt} \tag{24}$$

Expressions (23) and (24) can be written in the modified form (25), (26) from whose the time derivatives of both currents can be expresses in the explicit form (27), (28).

$$u_1 - \frac{L_{12}}{L_{22}}u_2 = i_1 R_1 - \frac{L_{12}}{L_{22}}i_2 R_2 + \left(L_{11} - \frac{L_{12}^2}{L_{22}}\right)\frac{di_1}{dt} \tag{25}$$

$$u_2 - \frac{L_{12}}{L_{11}}u_1 = i_2 R_2 - \frac{L_{12}}{L_{11}}i_1 R_1 + \left(L_{22} - \frac{L_{12}^2}{L_{11}}\right)\frac{di_2}{dt} \tag{26}$$

The obtained expressions (27) and (28) are appropriate to be solved by the explicit integration methods.

$$\frac{di_1}{dt} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2}\left(u_1 - \frac{L_{12}}{L_{22}}u_2 - i_1 R_1 + \frac{L_{12}}{L_{22}}i_2 R_2\right) \tag{27}$$

$$\frac{di_2}{dt} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}\left(u_2 - \frac{L_{12}}{L_{11}}u_1 - i_2 R_2 + \frac{L_{12}}{L_{11}}i_1 R_1\right) \tag{28}$$

It is evident form (16) and (17) that the dynamic inductance must be used in the transformer dynamic model. However, some authors try to extend properties of the magnetically liner systems to the magnetically nonlinear systems without to be aware of consequences. They use the current dependent static inductance $L(i_m)$ (18) instead of the dynamic inductance $L_d(i_m)$ (19) in expressions (20) to (22).

The rest of this paper shows the consequences of the incorrect use of magnetically nonlinear iron core characteristics in the dynamic model of a single phase transformer. It compares the measured results with those calculated by the transformer dynamic model where static and dynamic inductances are applied in order to account for the magnetically nonlinear behaviour of the transformer iron core.

4 Results

The testing object is a small single phase laboratory transformer. Its data are show in table I.

Table 1: Testing transformer data

N_1	The number of primary turns	425
N_2	The number of secondary turns	1722
R_1	The primary resistance	11 Ω
R_2	The secondary resistance	141.8 Ω
$L_{\sigma 1}$	the primary leakage inductance	33 mH
$L_{\sigma 2}$	the secondary leakage inductance	33 mH

All simulations are performed in the program package Matlab/Simulink using the dynamic model of a single-phase transformer given by equations (27) and (28) and the magnetically nonlinear iron core characteristics shown in Fig. 1. The magnetically nonlinear behavior of the transformer iron core is accounted fore by the static and dynamic inductances (1) and (2) shown in Fig. 2.

Figs 4 and 7 shows the primary voltage measured during the no-load test and the primary voltage measured during the test preformed at loaded transformer.

Figs. 5, 6 and 8, 9 show the comparison of measured and calculated transformer currents in different operating conditions. In all figures presented, the measured currents are marked with i, the dynamic model calculated ones using the dynamic inductance are marked with i_{Ld}, while the dynamic model calculated ones using the static inductance are marked with i_L.

Fig. 4 shows the mesured primary voltage u_1 applied during the no-load test. The same voltage is used in the dynamic model. Its amplitude is 136.7 V at the frequency of 50 Hz.

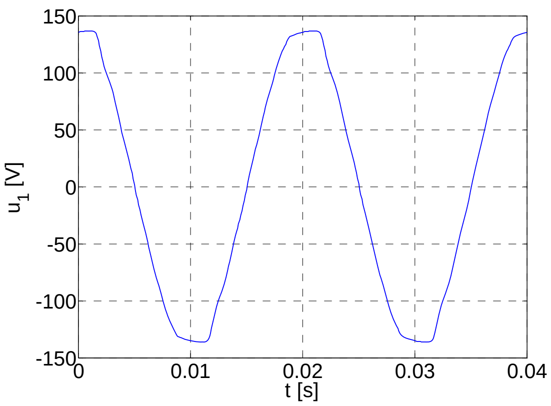

Fig 4: Primary voltage u_1 measured during the no-load test

Fig. 5 shows the comparison of measured and calculated current for the steady state operation at no load. The agreement between the measured and the calculated results is very good when dynamic inductance is used in the model.

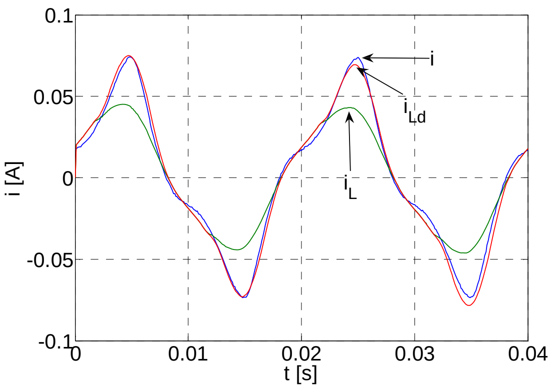

Fig 5: Steady state at no load: measured current i, current calculated with static inductance i_L and current calculated with dynamic inductance i_{Ld}

The comparison of measured and calculated currents during switch-on of the unloaded testing transformer is shown in Fig. 6. The agreement between measured and calculated results is very good if the dynamic inductance is used. However, in the case when static inductance is used in the model the agreement between measured and calculated results is not good.

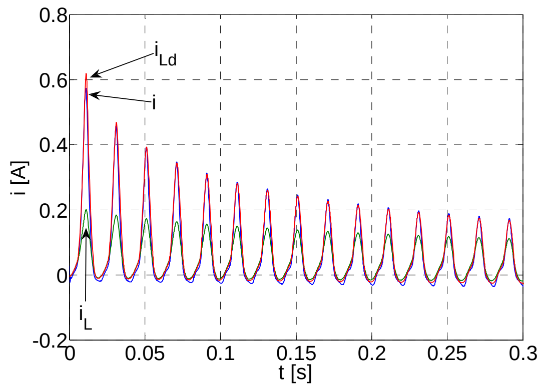

Fig 6: Inrush: measured current i, current calculated with static inductance i_L and current calculated with dynamic inductance i_{Ld}

Fig. 7 shows the measured primary voltage u_1 applied during the test performed at the loaded transformer. The same voltage is used in the dynamic model. Its amplitude is 137.8 V at the frequency of 50 Hz.

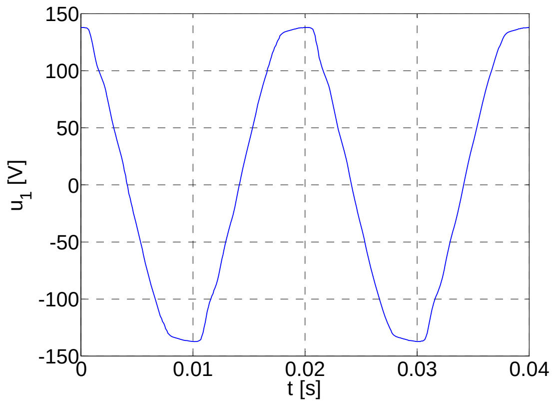

Fig 7: Primary voltage u_1 measured during the test at loaded transformer.

Figs. 8 and 9 show the comparison of measured and calculated for the case of transformer loaded with the nominal load. Fig. 8 shows steady state operation, while Fig. 9 shows the loaded transformer switch-on. In the case of loaded transformer, there is only a small difference between the currents calculated with static and dynamic inductance. This could be explained by the relatively small values of the magnetizing current in the case of loaded transformer. The agreement with the measured results is relatively good.

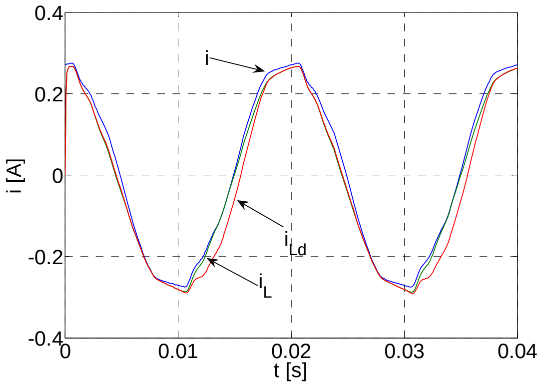

Fig 8: Steady state at nominal load: measured current i, current calculated with static inductance i_L and current calculated with dynamic inductance i_{Ld}

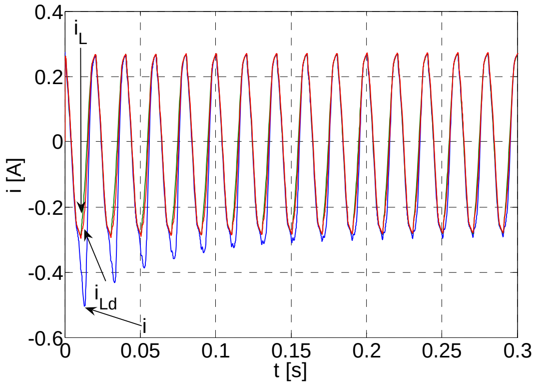

Fig 9: Loaded transformer switch-on: measured current i, current calculated with static inductance i_L and current calculated with dynamic inductance i_{Ld}

5 Conclusion

This work discusses the use of dynamic and static inductances in the single phase transformer dynamic model. The model derivation and the presented results clearly show that the dynamic inductances should be used in the model. However, the results of simulations given for the case of loaded transformer clearly show that even the use of static inductance is acceptable in such operating conditions.

6 References

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