

# MODELING THE LASER DROPLET-FORMATION PROCESS AND OPTIMIZING WITH GENETIC ALGORITHMS

**Tadej Kokalj<sup>1</sup>, Igor Grabec<sup>2</sup>, Edvard Govekar<sup>2</sup>**

<sup>1</sup>Institute of Metals and Technology  
1001 Ljubljana, Lepi pot 11, Slovenia

<sup>2</sup>University of Ljubljana, Faculty of Mechanical Engineering  
1000 Ljubljana, Aškerčeva 6, Slovenia

*tadej.kokalj@imt.si (Tadej Kokalj)*

## Abstract

A numerical model of laser droplet formation (LDF) from a metal wire was built for the purpose of process optimization. In the LDF process three laser beams placed around the circumference of the wire are used to melt the wire tip. The basic equation of the quasi-symmetrical model is the heat equation, including nonlinear material properties and phase transitions. Numerical solutions of the model make possible the calculation of the time development of the temperature field of the wire for different sets of process parameters. We considered the LDF process in two phases: pendant-droplet formation and droplet detachment. In order to obtain the desired droplet properties the time course of the laser-pulse power was optimized for the first phase of the process. Based on the calculated temperature field and the selected objective function the optimization of the LDF process was carried out using a genetic algorithms method. In the second phase the laser-beam “key-hole” phenomenon is used to detach the pendant droplet. To determine the pulse for droplet detachment, a simplified key-hole model was used. The models of droplet formation and detachment were applied to determine the optimal laser pulses. Theoretically determined laser pulses for the case of nickel wire were verified experimentally. The experimental results of the LDF process were characterized by the variability of the droplet size and the radial scattering of the position of the deposited droplets. Experiments showed that in comparison with heuristically selected pulses, the model-based optimized pulse yields the lowest variability of the process and a small number of undesired splashes of metal during the LDF process.

**Keywords: Droplet formation, Modeling, Optimization, Experimental verification.**

## Presenting Author’s biography

Tadej Kokalj. Bachelor degree in Physics at the Faculty of Mathematics and Physics, University of Ljubljana. Ph.D. degree in “Modeling and optimization of laser droplet formation from metal wire” at the Faculty of Mechanical engineering, University of Ljubljana. Presently, a researcher at the Institute of Metals and Technology ([www.imt.si](http://www.imt.si)) in the Laboratory for Measurements in Heat Engineering, Ljubljana, Slovenia.



## 1 Introduction

The laser droplet-formation process is a novel technological process in which laser beams are used to heat the tip of a metal wire in order to produce a molten metal droplet [1,2]. This droplet can be subsequently used to fill gaps in a substrate or to form a high-temperature joint.

The potential uses for laser droplet-formation technology include micro-welding, micro-casting, and rapid prototyping.

For any industrial application of this technology a repeatable formation of droplets with the desired properties – but without any undesired side effects like splashes of the melt on the substrate and the radial scatter of deposited droplets – is needed.

Despite intensive experimental work [3,4] we still face some challenges on the road to widespread industrial use. With the aim to improve the process's performance and to get a clearer insight into the process we decided to build a numerical model of the process and to determine the process parameters on the basis of this model.

Since the different stages of the process are dominated by different physical phenomena, we divided the process into two parts: the formation of a pendant droplet at the tip of the wire, and the detachment of the droplet from the wire. In the first part of the process the heat input into the wire is of interest in order to ensure a completely melted droplet with sufficient heat content, and in the second part the influencing forces should ensure the detachment of the droplet.

We built a numerical model that predicts the development with time of the temperature field of the wire during both parts of the laser droplet-formation process. This model is more accurate for the first part than for the second part of the process, where some robust simplifications were assumed.

The numerical model makes possible the simulation of the process for different sets of process parameters, and by applying a suitable optimization criterion and method we can find the optimal set of parameters. We optimized the first part of the process where the main properties of the droplet, such as heat content and temperature field, are to a large extent determined.

The presented model makes it possible to optimize any variable process parameter; however, in earlier research we focused on determining the time course of the laser power, i.e., the laser pulse. Other process parameters were selected in advance; of particular importance is the wire's velocity during the process, which was selected in accordance with the desired droplet size.

The laser pulses for the droplet detachment were further determined on the basis of the same numerical model, but with a different pulse-determination procedure.

Theoretically determined laser pulses were validated on the specially built experimental setup for laser droplet formation.

## 2 Numerical model and determination of the laser pulse

With the aim of theoretically determining the laser pulse we built a numerical model of the process. The basic equation of this model is the heat equation in cylindrical coordinates  $(r, \vartheta, z)$ :

$$\frac{\partial T(r, \vartheta, z, t)}{\partial t} = D(T) \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T(r, \vartheta, z, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T(r, \vartheta, z, t)}{\partial \vartheta^2} + \frac{\partial^2 T(r, \vartheta, z, t)}{\partial z^2} \right] \quad (1),$$

where  $T$  is the temperature and  $D(T) = k(T)/\rho(T)c_p(T)$  is the temperature-dependent diffusion coefficient. Temperature-dependent material parameters, like thermal conductivity  $k(T)$ , density  $\rho(T)$  and specific heat  $c_p(T)$ , were acquired from the literature in the case of nickel wire [5,6,7,8,9,10,11].

The phase transitions in our model were treated as jumps in the specific heat  $c_{pt}(T)$ :

$$c_{pt}(T) = c_p(T) + L\delta(T - T_f) \quad (2),$$

where  $c_{pt}$  is the total specific heat, combining the real specific heat  $c_p$  and the latent heat  $L$  jump at the temperature of the phase transition  $T_f$ . The Dirac delta function was, for the purposes of numerical implementation, approximated by a Gaussian function:

$$\delta(T - T_f) \approx \frac{1}{\sqrt{2\pi}\Delta T} \exp\left(-\frac{(T - T_f)^2}{2\Delta T^2}\right) \quad (3),$$

where  $T_f$  is the temperature of the phase transition and  $\Delta T$  is the selected width of the Gaussian function.

The described treatment of the phase transitions was compared with the enthalpy method found in the literature [12]. Numerical experiments show very good agreement between these two methods. The selected method was used in the model because it is easier for a numerical implementation.

When the wire is heated by three laser beams with diameters similar to the wire diameter we can assume uniform heating around the circumference of the wire, as shown in Fig. 1. Consequently, the  $\theta$  coordinate in Eq. (1) can be abandoned and the equation is reduced

to two spatial dimensions, which significantly reduces the computational time.

For the initial condition we used the initial temperature of the wire  $T_0$ :

$$(4), \quad T(r,0) = T_0$$

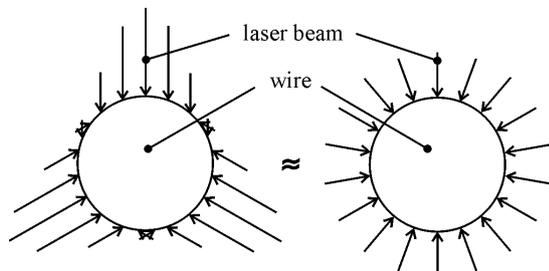


Fig. 1: Three-laser-beam heating approximates to uniform heating around the circumference.

For the boundary condition we take into account the total energy flux  $j_c$  through the surface of the wire. This incorporates the absorbed laser-light energy flux and the energy flux radiated from the surface of the wire. Finally, we get a Neumann boundary condition:

$$k(T) \frac{\partial T}{\partial n} = j_c \quad (5),$$

where  $k(T)$  is the heat conductivity,  $T$  is the temperature,  $n$  is normal to the surface and  $j_c$  is the total energy flux through the surface.

By using these initial and boundary conditions we can solve Eq. (1) numerically; we used an explicit numerical scheme [13]. The numerical solution gives a prediction of the time development of the temperature field of the wire, as shown in Fig. 2. The figure shows an example of the temperature field for the pre-selected duration of a pendant-droplet formation pulse,  $t=12$ ms.

In the pendant-droplet-formation part of the process the droplet's properties are strongly influenced and therefore the laser pulse should be determined with respect to the desired properties of the droplets.

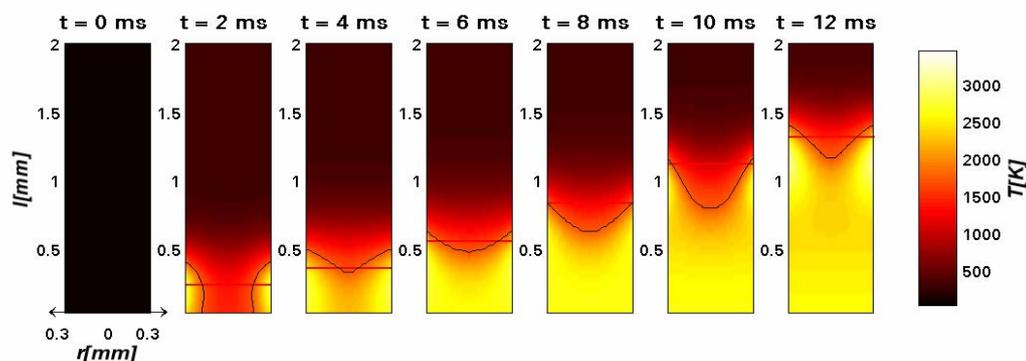


Fig. 2: Temperature field of the wire cross-section at different times. The red horizontal line denotes the position of laser beams and the black line divides the material into liquid and solid phases.

Generally, we want to achieve a completely melted metal droplet with the required uniform temperature. Furthermore, intense vaporization is not desired during the first part of the process. An example of an optimization criterion, written in the form of a mathematical expression, is:

$$J = \frac{\sqrt{\sum_{j=1}^N (T_j(t_p) - T_D)^2 \cdot \frac{m_j}{m_T}}}{T_D} + \frac{1}{M} \sum_{k=1}^M \frac{m_{vk}(t_k)}{m_T} \quad (2),$$

where the first term describes the difference in the droplet temperature compared to the desired droplet temperature  $T_D$  at the end of the pulse  $t_p$ .  $T_j$  is the temperature of  $j$ -th volume element of the wire and  $m_j$  is its mass.  $N$  is the total number of volume elements and  $m_T$  is the total mass of the part of the wire under consideration. The second term describes the vaporization rate during the process. The laser pulse is divided into  $M$  equal time intervals and at the end of each time interval  $t_k$  the ratio of the vaporized wire material  $m_v$  to the total mass of the part of wire under consideration  $m_T$  is calculated.

The numerical value of the function  $J$  can be calculated from a numerical simulation of the process. It is obvious from the construction of the function  $J$  that a smaller value of this function means a more suitable process. Finding the set of process parameters that minimizes the value of the function  $J$  is the aim of the process optimization.

Since the wire-heating process itself is inherently nonlinear and the fitness function  $J$  shows non-convex behavior we selected a genetic algorithm method to perform the optimization of the process parameters.

For the purposes of optimization we set all the process parameters, such as wire geometry and material parameters, laser beam parameters and wire velocity,

to selected values. We then focused on determining just the time course of the laser power. We parameterized the laser pulse with seven values of laser power in equal time intervals of 2ms, as shown in Fig. 3. These seven parameters were the final subject of our optimization.

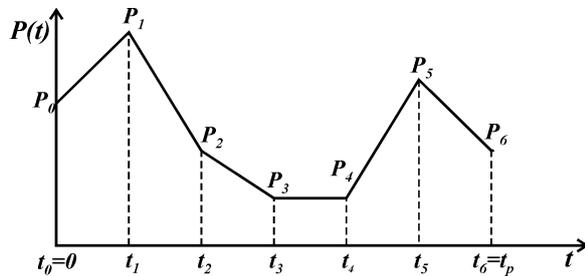


Fig. 3: Parameterization of the laser pulse for pendant-droplet formation with seven parameters.

To ensure a rapid simulation of the process, the numerical model was implemented in the C++ programming language. In addition, the optimization was performed with the “Genetic Algorithm Tool” toolbox of the Matlab software package. Most of the preset optimization parameters were used in the optimization, except for the population size and the number of generations, which were set to 70 and 150, respectively, on the basis of preliminary numerical experiments.

The laser pulse for the pendant-droplet formation was found for several different fitness functions and different optimization objectives. The laser pulse for the droplet detachment was determined on the basis of the same numerical model, but with a different pulse-determination method.

Droplet detachment from the wire is the consequence of the so-called “key-hole” effect. The basis for the onset of a key-hole is the self-focusing of the laser beam, induced by dynamic changes to the surface and vaporization. For a key-hole to appear, however, intensive laser radiation is needed [14]. When the self-

focusing appears in the process a deep, thin hole is rapidly formed at the spot with the highest laser-light intensity. Rapid vaporization of the material creates an overpressure in the hole, and the force acting on the droplet in the downward direction is sufficient to detach the droplet, at almost the same moment as the key-hole reaches the centre of the wire, as shown in [15].

Because of the complexity of the development of the key-hole effect we treated it as the vaporization of material. In order to determine the laser-detachment pulse we selected a laser power and calculated the time required for the key-hole to reach the center of the wire. The procedure is shown schematically in Fig. 4. A simulation of the key-hole’s development on the basis of a simplified model is shown in Fig. 5.

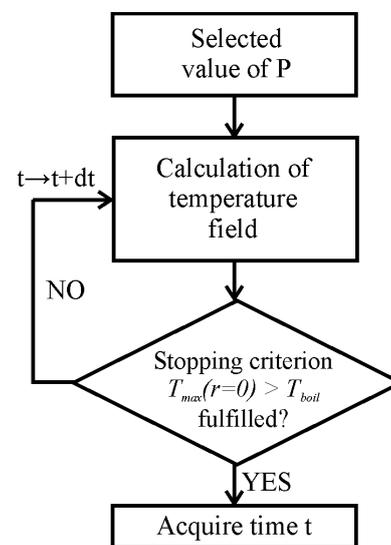


Fig. 4: Schematic representation of determining the droplet-detachment pulse. We select the laser power  $P$ , calculate the temperature field and check if the maximum temperature on the axis of the wire  $T_{max}(r=0)$  has reached the boiling temperature  $T_{boil}$ . If yes, we acquire the time, otherwise we continue with the heating.

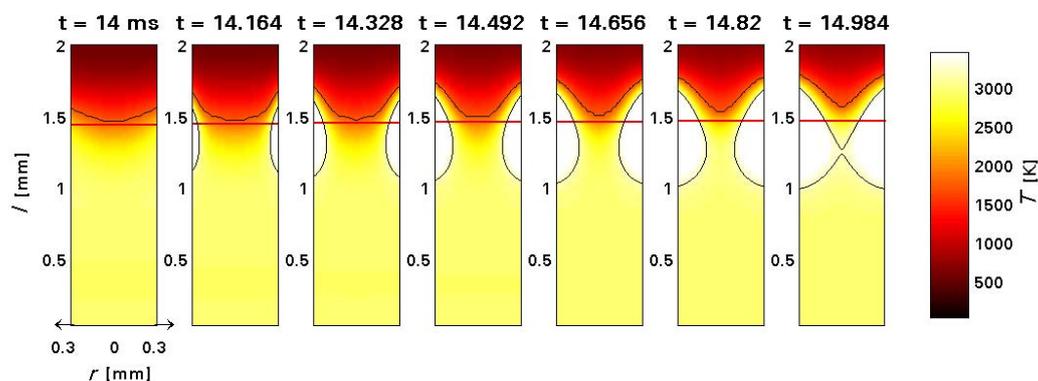


Fig. 5: Temperature field of wire cross-section at key-hole development. Red horizontal line denotes the position of laser beams and black lines are dividing material in solid, liquid and gas phase. Vaporized material is represented by white color.

We have separately theoretically determined the laser pulse for pendant-droplet formation and the laser pulse for droplet detachment. When we combine these two pulses one after another we obtain one pulse for laser droplet formation. An example of the laser pulse is shown in Fig. 6.

Since the pendant droplet oscillates at the tip of the wire after its formation, we inserted a short period of time ( $t=2\text{ms}$ ) after the pendant-droplet formation phase at time 12ms to allow the droplet to stabilize and to reach the proper position before the detachment pulse. During this period the laser power is reduced to the minimum power of the operating laser. This additional stabilizing time interval was determined on the basis of preliminary experiments.

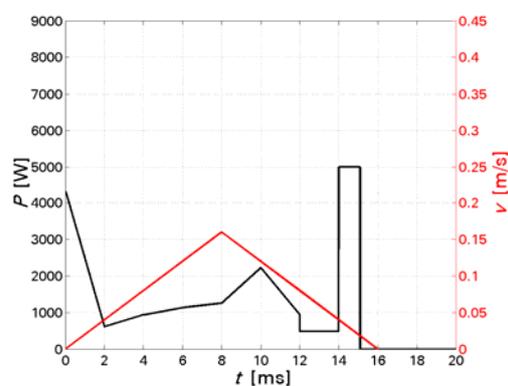


Fig. 6: Example of a theoretically determined laser pulse for laser droplet formation (black color) for a selected wire-velocity profile (red color).

### 3 Experimental verification

Experiments were performed to evaluate the theoretically determined laser pulses. Like with the theoretical determination of the laser pulses, we treated separately the pendant-droplet formation and the droplet-detachment phase.

The experiments were performed on a custom-built, laser droplet-formation system, which consisted of a Nd:YAG pulsed infrared laser system, a wire-feed system and a visual system for the process characterization and control. Nickel wire of 0.6 mm diameter was used in all the experiments.

First, the pendant-droplet formation was investigated. We performed several sets of pendant-droplet formation experiments with various theoretically optimized laser pulses. Depending on the selected optimization criterion, we obtained different experimental outcomes. The laser pulses that were determined in accordance with the low desired temperature of the wire, near the melting temperature, did not usually yield pendant droplets. The best pendant droplets were produced with the laser pulses

determined in accordance with the desired droplet temperatures near to the boiling temperature.

After the evaluation of the experimental results of the pendant-droplet formation experiments, we selected the best pendant-droplet formation laser pulse and appended different pulses for the droplet detachment. Again we produced sets of droplets, which were deposited on the substrate, as shown in Fig. 7.

The experimental results show that the high laser power of the detachment pulse causes more intensive splashes of material on the substrate and scatter of the droplets on the substrate. On the other hand, if a lower laser power is used for the detachment pulse, droplets sometimes stay attached to the wire.

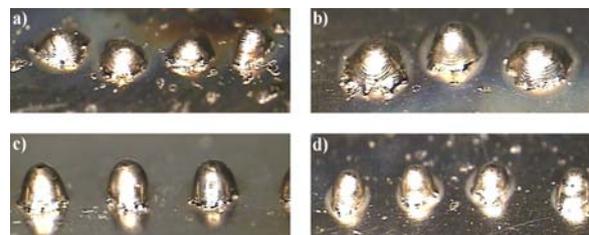


Fig. 7: Examples of nickel droplets produced with different laser pulses. The powers of the detachment pulses were (a)  $P=8000\text{W}$ , (b)  $P=6000\text{W}$ , (c)  $P=4000\text{W}$  and (d)  $P=5000\text{W}$ .

The experimental results were evaluated on the basis of the variability of the droplet diameter, the variability of the radial scatter of the droplets on the substrate and the presence of material splashes on the substrate. The experimental results show very little process variability and a small number of splashes for some theoretically determined laser pulses.

### 4 Conclusions

In this paper we have presented a physical and numerical model for the laser droplet-formation process. This model enables a simulation of the evaluation of the temperature field of a wire for different sets of process parameters.

With respect to the desired application or outcome of the process, a proper set of process parameters can be determined by means of process optimization.

The most important process parameter – the temporally dependent laser power – was determined for the case of nickel wire. The theoretically determined laser pulses were successfully verified experimentally and they yielded suitable process outcomes.

The results of our experimental investigations show that the presented numerical model can be successfully applied to determine the process parameters.

A numerical model of the process was developed as an additional, theoretical tool, with the aim to decrease the experimental time needed to determine the proper process parameters and to get a better insight into the process.

We believe that the construction of the numerical model brought the laser droplet-formation process closer to its real application in industry.

## 5 References

- [1] E. Govekar, B. Jahrsdörfer, J. Klemenčič, P. Mužič and I. Grabec. Characterization of laser droplet formation process by IR camera images and AE. Dynamic days, Dresden, 2001.
- [2] W. Hovig and B. Jahrsdörfer. Laser droplet weld – ein innovatives fügeverfahren. Laser in Elektronikproduktion & Feinwerktechnik, LEF 2001, 2001.
- [3] E. Govekar, J. Klemenčič, T. Kokalj, B. Jahrsdörfer, P. Mužič and I. Grabec. Characterization of laser droplet formation process by acoustic emission. *Ultrasonics*, 42:99-103, 2004.
- [4] J. Klemenčič. Characterization of laser metal droplet formation. Ph.D. thesis, Faculty of Mechanical Engineering, University of Ljubljana, 2005.
- [5] F.C. Nix and D. MacNair. The thermal expansion of pure metals: Copper, gold, aluminum, nickel and iron. *Physical Review*, 60(8):597-605, 1941.
- [6] P. Hidnert. *Journal of Research National Bureau Standards*, 58:89, 1957.
- [7] W.D. Drotzig. Thermal expansion of iron, cobalt, nickel and copper at temperatures to 600 K above melting. *High Temperatures-High Pressures*, 13:441-458, 1981.
- [8] B.J. McBride, S.Gordon and M.A. Reno. Technical paper 3287. *Technical report*, NASA, 1993.
- [9] N. Wang, X.Y. Han and B.Wei. Specific heat and thermodynamics properties of undercooled liquid cobalt. *Applied Physics Letters*, 80(1):28-30, 2002.
- [10] R.W. Powell, P.E. Liley and C.Y. Ho. Thermal conductivity of the elements. *Journal of Physical and Chemical Reference Data*, 1:279, 1972.
- [11] T. Nishi, H. Shibata, H. Ohta and Y. Waseda. Thermal conductivities of molten iron, cobalt and nickel by laser flush method. *Metallurgical and Materials Transactions A*, 34(12): 2801-2807, 2003.
- [12] D. M. Christopher. Comparison of interface-following techniques for numerical analysis of phase-change problems. *Numerical Heat Transfer, Part B*, 39: 189-206, 2001.
- [13] J.W. Thomas. Numerical Partial Differential Equations: Finite Difference Methods. Springer-Verlag, New York, 1995.
- [14] D. Bäuerle. Laser Processing and Chemistry. Springer-Verlag, Berlin, 2000.
- [15] T. Kokalj, J. Klemenčič, P. Mužič, I. Grabec and E. Govekar. Analysis of the laser droplet formation process. *Journal of Manufacturing Science and Engineering*, 128(1):307-314, 2006.