APPLICATION OF STOCHASTIC PROGRAMMING FOR SUPPLY CHAIN INVENTORY OPTIMIZATION UNDER UNCERTAIN DEMAND AND LEAD TIME

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Abstract

Supply chain management deals with the management of information and material in a network of producers, retailers and customers. This task is rather complex since a high degree of uncertainty occurs in reality due to the fact that key factors such as product demands, stock availability, prices, facilities reliability, all have significant stochastic components. Stochastic programming is main representative approach to capture some aspects of the uncertainty due to the limited knowledge of the model input parameters. Although it has many contributions in supply chain management, they are mainly focused on the strategic decision level, where there is not enough concrete information about the features of uncertainties on which to base long-term decisions. Within a current research, it is supposed that the most of the uncertainty could be handled adequately using a stochastic modeling approach precisely at the tactical planning level, mostly employing stochastic values for customer demand, supplier lead times, production costs and/or price fluctuations. The paper illustrates the application of stochastic programming approach for inventory optimization under uncertain demand and lead time. The mathematical model is created and tested by using the algebraic modeling language AMPL, which uses the large-scale optimization solver CPLEX. The developed inventory model is tested, leading to the conclusion that stochastic programming provides superior planning decisions in comparison with deterministic equivalent. The effect of the scenario number on the model performance is evaluated. The conclusion is that the optimal number of scenario exists and it is not necessary to make the model work under all possible scenarios.

Keywords: supply chain, tactical planning, stochastic programming, uncertainty, inventory model.

Presenting Author's biography

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1 Introduction

The task of effective supply chain management is rather complex since a high degree of uncertainty occurs in reality due to the fact that key factors such as product demands, feed stock availability and their prices, technical parameters such as product yields and facilities parameters such as reliability/availability, all have significant stochastic components. Underestimating uncertainty and its impact can lead to planning decisions which will not safeguard a company against the threats.

The main representative approach to capture some aspects of the uncertainty is called stochastic programming which deals with uncertainty due to the limited knowledge of the model input parameters. The term stochastic approach is usually pointing at the goal of creating a robust model of the supply chain. To be applied to SCM problems, stochastic programming provides a near-optimal solution which nevertheless stays valid over a larger range of variable values, at a predictable but higher cost [1].

The paper illustrates the application of stochastic approach for inventory model optimization under uncertain demand and lead time. The paper is structured as follows. Section 2 provides literature review of application of stochastic programming approach in supply chain management. Section 3 deals with definition of robust supply chain. In section 4 a developed optimization model with uncertain demand for well-known beer game supply chain is described. A computational results and analysis are provided here as well. The enhanced optimization model under two uncertain parameter (demand and lead time), and computational results are shown in section 5. Section 6 presents some conclusions and some directions for further research.

2 Literature review

During the last years the number of stochastic programming application to complex supply chain management problems has grown considerably. The experience of successful application of stochastic programming for complex real life problem solving [2, 3] stimulates researchers to apply this technique to unsolved problems. Sahinidis [4] presents the detailed overview on stochastic programming applications.

Although stochastic programming has many contributions in SCM, they are mainly focused on the strategic decision level which effects the performance of the system over a relatively long time horizon ranging from 5 to 10 years. Generally, the objective function of stochastic models is a net present value of the associated investment, operating cost, and revenue streams. Most commonly, the dominant uncertain parameters are the product demands. Problems of design and planning of production plants under uncertainty have been treated in the operations research literature [5, 6, 7, 8, 9, 10].

According to [1], on the strategic level there is not enough concrete information about the features of uncertainties on which to base long-term decisions. At this level, the supply chain infrastructure is laid down (such as the location of plants and warehouses) and the physical boundaries for tactical planning are provided, which are per definition inflexible to adaptation. On the other hand, at the operational level, characterized by timeframes of 1-2 weeks, the planning horizon is too short to react to uncertainties. Hence, most of the uncertainty could be handled adequately using a stochastic modeling approach precisely at the tactical planning level, which address planning horizons of 1-2 years, mostly employing stochastic values for customer demand, supplier lead times, production costs and/or price fluctuations.

The classical illustration of tactical planning is related to the optimization of inventory models which are used to determine the optimal trade-off between inventory costs and customer demand satisfaction. In general, the model output consists of optimal order quantities or the optimal target stock level, in order to minimize the sum of backlog and inventory costs over the time horizon, dealing with stochastic customer demand, stochastic supplier lead times, price fluctuations etc. [11, 12, 13].

The random data representing the uncertainty of the future are expressed in stochastic programming by a scenario tree. A major focus of scenario generation is to create a tree structure of scenarios that "best" approximates a given underlying distribution of the random parameters of the model [14].

A scenario is the particular succession of possible random parameter's values (samples) over the periods in the time horizon.

Domenica et.al [14] describe the procedure of scenario generation which consist of four steps and point out the main methods which may be adopted for the different steps. One computational bottleneck in solving stochastic programs is the amount of possible scenarios. In many problems there are combinatorial many scenarios that require prohibitive amounts of computational time. Authors note that in many cases researchers try to apply some reduction techniques to resulting scenario tree to provide model instances, which can be realistically optimized by the available computational recourses.

One recently developed technique called Sample Average Approximation (SAA) proposed in [15] allows one to reduce the number of scenarios. The authors suggest using only a subset of the scenarios, randomly sampled according to the scenario distribution, to represent the full scenario space. Illustratively, Benisch in [16] address an issue of scheduling component of the trading agent competition in supply chain management problem. This combinatorial optimization problem with inherent uncertainty is formulated as a stochastic program, and is solved using the SAA approach.

Growe Kuska et.al [17] proposed reduction algorithms which determine a subset of the initial scenario set and assign new probabilities to the preserved scenarios. The scenario tree construction algorithms successively reduce the number of nodes of a fan of individual scenarios by modifying the tree structure and by bundling similar scenarios. Numerical experience is reported for constructing scenario trees for the load and spot market prices entering a stochastic portfolio management model of a German utility.

Stochastic programming is often mentioned as a tool for creating a robust solution for a problem with uncertain input parameters. The next part focuses on the definition of robustness.

3 Robust supply chain defined

Developed mainly from around 1960, robustness provides methods that are resistant to errors or outliers in the data, which can be arbitrarily large. The need of robustness has been recognized in a number of application areas. Even the interest in this topic increases significantly during last few years no unique definition of robustness has been accepted.

Bundschuh et.al define robustness as the extent to which a system is able to perform its intended function relatively well in the presence of failures of components or subsystems [18]. Kleijnen et al. [19] define robustness as the capability to maintain short-term service in a variety of environments. Kutanoglu et al. [20] research scheduling robustness under processing time variation. The scheduling procedure is called to be robust than an alternative if the schedules it generates achieve better performance (as defined by the objective function) under the same set of random disturbances and changes. Rosenhead [21] discuss robustness analysis as a way of supporting decision making when there is radical uncertainty. Author argues for the wide prevalence of uncertainty in strategic decision making and the potential relevance of robustness at this planning level.

Vincke [22] distinguish four concepts of robustness: the concept of robust decision in a dynamic context, the concept of robust solution in optimization problems, the concepts of robust conclusion, and the concept of robust methods. In the case where the decision problem is modeled as an optimization problem, the robust solution is that which is good in most versions (i.e. sets of values for the data and the parameters of the model) and not too bad in the other. Although author states that a robustness of decision depends on the mire or less great margin the decision-maker is ready to concede in the information. The novelty adopted by the author within his research is the term “version” which is used instead of “scenario” in order to avoid any reference to an unknown future and to the traditional probabilistic approaches.

In system dynamics robustness is mostly refers to the extent to which the real system can deviate from the assumptions of the model without invalidation policy recommendation based upon it [23].

The discussion about reliability and robustness of supply chain networks has gained momentum mainly after September 11, which has led to an increased perception of risk and vulnerability in general as well as in today’s production-distribution systems. In the context of supply chain, robustness describes how much the output of a supply chain is affected by its participant (i.e. suppliers, customers) failures. Authors point that there are two aspects of supply chain robustness that can be quantified. The first one is a number of supplier failures before a supply chain is completely disrupted. The second is the standard deviation of the output, which is a common measure for risk-induced variability of performance indicators. The standard deviation however is strongly influenced by reliability as well [18].

Mulvey et al. [24] separate solution robustness and model robustness. The robust solution is robust with respect to optimality if it is “close” to the optimal solution for any set of realizations of the scenarios s which describe possible statements of environments. Authors call the model is robust when a solution is robust with respect to feasibility, i.e. it is “almost” feasible for any realization of the scenario s. They modify two-stage stochastic model by adding a weight of variability of the second-stage cost to the objective function. Varying the weight forces the optimization process to produce solutions that may present higher expected total cost with lower second-stage cost-deviations. This paradigm is then called as robust optimization. The application of this paradigm in power capacity expansion problem showed that the solutions gained within robust optimization has higher expected cost then the stochastic linear programming solution, but it has substantially lower standard deviation.

Aghezzaf [25] discovers robust planning in production systems with uncertain demand. He presents two alternative models to generate plans that are robust to the variability resulting from uncertainty of demand. The robustness is measured through the average extra cost resulting from adaptation of the plan when extreme scenarios occur.

4 Beer-game inventory model under uncertain demand

The model that is used within current research is the Beer Game supply chain, which represents a single-product linear supply chain consisting of 4 echelons – factory, warehouse, wholesaler and retailer. A fifth echelon is constituted by the market, passing on the customer's demand to the retailer echelon. Every period each echelon places its order to the lower echelon, trying to meet the optimal trade-off between customer demand satisfaction and inventory costs. There are two costs involved in the game: inventory carrying costs and backlog costs. To determine the optimal trade-off (i.e. the order strategy with the lowest total supply cost), an inventory planning model can be derived, optimizing the inventory position on the efficiency frontier.

A deterministic beer game model, not taking into account the real life fluctuation of customer demand, supplier lead times, etc., provides an inadequate representation of reality and will result in inferior planning decisions. One can always assume the average occurrence of demand and establish a production plan, but in practice the resulting plans are weak. To reduce the total costs and the variability inherent to planning in a multi-echelon supply chain as simulated by the beer game, a scenario approach should be utilized.

Since demand variability can be considered as the key source of uncertainty in planning and controlling supply chains, a beer game model under uncertainty of demand is developed initially, which results already in a far more robust planning model, attempting to define the optimal trade-off between customer demand satisfaction and inventory costs.

4.1 Mathematical model

The beer game can be considered to represent an inventory problem in a single-product linear supply chain consisting of 4 echelons – factory, warehouse, wholesaler and retailer. The factory has an infinite inventory without inventory costs or backlog costs. The products ordered by the warehouse are immediately shipped by the factory without any delay. The objective of the beer game is to minimize the total inventory cost, i.e. the sum over all periods and levels of the backlog costs and the inventory costs. The objective function of the basic beer game model under uncertainty of demand is given by (1).

The decisions to be made in the beer game are beer quantities to order at each echelon at each period. Thus, decision variables in the model are order quantities O_{lt} per level per period. However, in reality for companies it is no doubt more useful to work with a stock target level, i.e. a level of inventory which has to be maintained, instead of revising the optimal order quantities again every period for every level. With this Periodic Order Review (POR) policy, the order quantities are defined as the difference between the desired target level and the stock on hand, in order to keep this target level fixed. In this case, the order quantities are calculated for every level, every period and, in contrast to the regular model, for every scenario, and the output of this stochastic model consists of the optimal target stock level, which minimizes the total inventory costs.

This study provides three models, for three different order policies. In the first case, the regular model, the optimal order quantities minimizing the total inventory costs is returned, per level per period. For the POR model, the optimal stock target level is returned as model output. Finally, the POR version is extended to the model, returning an optimal stock target level for each echelon separately, as in reality different branches of the supply chain network have different operating logistics (called SCR, Supply Chain Reorder point).

For testing this basic beer game model, the deterministic lead time is set to 2 periods, the initial inventory is 100 for all echelons and the initial backlog is 0. Consequently, the concerned model is subject to following constraints:

The initial inventory for each echelon l at period 0 is set to 100 units for each scenario (3);

The initial backlog for each echelon l is set to 0 units for each scenario. The supply chain has no history at period 0 (4);

No former shipments p are coming for the first lead time interval for each echelon l for each scenario. The supply chain has no history at period 0 (5);

The order quantity for the customer (l=5) for each period is set to the demand of that period (6);

Assuming that the plant inventory is unconstrained, the order quantity for each period at the warehouse factory is equal to the shipped quantity at the plant for that period. This means that the plant always can satisfy the orders from the warehouse factory (7);

The increase of total backlog for one period for each echelon is equal to the difference of the ordered quantity at the next echelon for that period and the shipped quantity to that echelon (10);

The increase of total inventory for one period for each echelon is equal to the difference of the shipped quantity at the echelon for that period and the shipped quantity at a higher echelon, respecting a time interval that is the lead time N (11).

The restrictions above are independent of the used order policy. For the POR model only three extra constraints are needed.

The on order quantity *Ond* has to be defined, i.e. the quantity ordered in previous periods which has not arrived yet (in the regular model, there's no need for such an on order quantity variable (9);

Assuming the supply chain has to start from scratch, the on order quantity is 0 for the first period in each echelon l for each scenario (2);

An extra order constraint using the desired stock target level ζ is needed to define the order quantity per level per period and per scenario, being the difference between the target level and the stock on hand (i.e. inventory and on order quantity minus the backlog (8).

Simply adding an extra level index *l* to the stock target level ζ upgrades the POR model to a SCR model, returning the optimal stock target level for each echelon separately as model output (12):

$$O_{lts} = \zeta_l - \left(I_{(l,t-1,s)} - B_{(l,t-1,s)} + Ond_{(l,t-1,s)} \right).$$

The only extra difference between the two described models is the missing of a scenario index *s* in the decision variable $O_{l,t}$ in the regular model, where in the POR model an extra scenario index *s* indicates that the order quantity is calculated for each level, each period, and each scenario separately (the order quantities are calculated for each scenario separately in order to determine the optimal stock target level as model output). Thus, in the POR and SCR model the decision variable $O_{l,t}$ is provided with an extra scenario index *s*. The entire POR model becomes then:

$$\min \sum_{s}^{S} V_s \left(\sum_{l=2}^{4} \sum_{t=1}^{T} ic_{lt}.I_{lts} + \sum_{l=2}^{4} \sum_{t=1}^{T} bc_{lt}.B_{lts} \right) \quad (1)$$

$s.t.$

$Ond_{0ls} = 0; \quad (2)$

$I_{0ls} = 100; \quad (3)$

$B_{0ls} = 0; \quad (4)$

$P_{(l-1,-2,s)} = P_{(l-1,-1,s)} = 0; \quad (5)$

$O_{5,t,s} = D_{ts}; \quad (6)$

$O_{2,t,s} = P_{(1,t)}; \quad (7)$

$O_{lts} = \zeta - I_{(l,t-1,s)} + B_{(l,t-1,s)} - Ond_{(l,t-1,s)}; \quad (8)$

$Ond_{lts} = Ond_{(l,t-1,s)} + O_{lts} - P_{(l-1,t-N,s)}; \quad (9)$

$B_{lts} = B_{(l,t-1,s)} + O_{(l+1,t,s)} - P_{lts}; \quad (10)$

$I_{lts} = I_{(l,t-1,s)} + P_{(l-1,t-N,s)} - P_{lts}; \quad (11)$

$T, L, V_S > 0;$

$\zeta, N, D_{ts}, ic_{lt}, bc_{lt}, O_{lts}, I_{lts}, B_{lts}, P_{lts}, Ond_{lts} \geq 0;$

with:

S: The number of demand scenarios in the demand tree.

L: The number of level.

T: The user defined number of periods in the time horizon.

V_s: The probability that a demand scenario s occurs.

ic_{lt}: Inventory cost for level l at period t.

I_{lts}: Inventory for level l at period t for each scenario.

bc_{lt}: Backlog cost at period t for level l.

B_{lts}: Backlog for level l at period t for each scenario.

To obtain concrete results, the mathematical models described above can be incorporated in AMPL, a computer language for solving large-scale optimization and mathematical programming problems. The CPLEX solver implements a modified primal and dual simplex algorithm to find an optimal solution.

4.2 Scenario generation

However, with P the number of periods in the time horizon and N the number of possible demand values, the number of demand scenarios S (i.e. the number of all possible combinations of the demand values over all periods) increases as $S = N^P$. In this study three scenario development methods are employed:

(1) scenario tree (see Fig.1): the developed model employs 3 possible demand values, each having the same probability of occurrence. Assuming a normal demand distribution (mathematical expectation μ_D = 50 units per period and standard deviation σ_D = 20 units), 3 samples can be derived as: S1=27.2, S2=50, and S3=72.8 (the average demand values are used to generate the demand data tree).

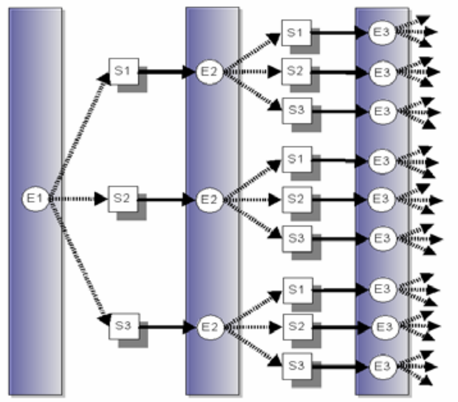

Fig.1 Demand data tree generation

(2) a set of random generated demand sequences (see Fig.2): instead of employing user defined demand values, a data structure can also be generated using totally random demand samples.

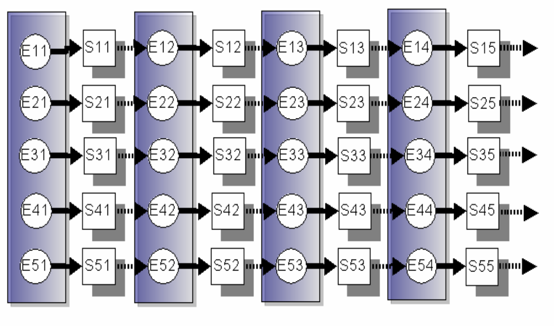

Fig. 2 Random data generation

(3) a hybrid data tree (see Fig.3): non-random periods employing user defined demand values in a demand data tree are alternated with totally random periods.

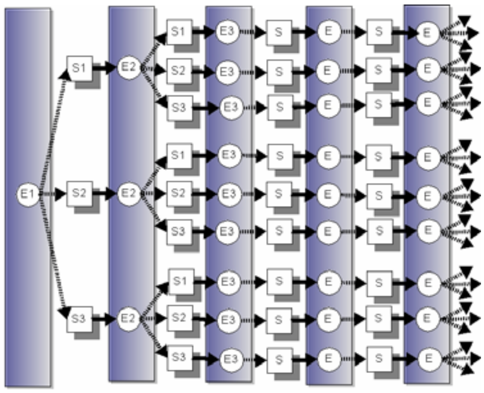

Fig. 3 Hybrid data generation

In order to create the necessary AMPL data file in a fast and easy way, a Visual Basic Application for Excel was created which generates user-defined data trees and exports the generated data to an AMPL '.dat' file. The program consists of three main sections: the Stochastic Demand Section, responsible for generating stochastic demand data trees whether they are random, hybrid, or derived from user-defined demand values; the Stochastic Lead Time Section, responsible for generating random, hybrid or user defined lead time data trees; and the Export Section responsible for creating an appropriate AMPL data file.

4.3 Results

The AMPL beer game model under uncertainty of demand was tested on a Pentium® 4 CPU 3.00GHz with 1024 MB of RAM memory. The number of periods for the data tree method was therefore limited to 6, leading to $3^6 = 729$ scenarios. The initial inventory was set to 100 units for all levels, initial backlog 0, inventory cost equaled 1 and backlog cost equaled 2. The AMPL model was tested employing all presented data generation methods. The hybrid approach consisted also of 6 periods, 4 non-random and 2 random periods, leading to 81 scenarios. For both the hybrid as well as the random method, the demand sample boundaries were set to 10 – 90 units. For the random approach, 729 scenarios were generated. Using the POR basic beer game model, these settings led to the results in Tab. 1.

Tab. 1 Quantitative results

	Tree	Hybrid	Random
Total Cost	678,1	709,8	807,2

The hybrid method introduces more randomness than the rather well-structured data tree approach, and therefore results in a higher cost. The random method provides the most robust solution against the demand uncertainty, but returns the highest cost, due to the total randomness of the demand samples, varying between 10 and 90 demand units. When the random demand sample interval is shortened (for instance to an interval of 30-70 demand units), the total cost decreases, as there will be less randomness to deal with. The returned optimal target stock level is quite similar for all experiments, as the number of periods in the time horizon is quite small. A larger number of periods results in more variety.

For the data tree and the hybrid method, the number of scenarios is defined by the number of periods in the time horizon. However for the random generation mode, the desired number of scenarios is user-defined. Therefore, the influence of the number of scenarios on the total cost should be analyzed in order to find an optimal number of scenarios to apply. To analyze if the total cost continuously decreases when the number of scenarios increases, or if there is a limit, the AMPL POR model was tested twice with random data, over 5 periods and 10 periods respectively, starting at 5 scenarios and steadily increasing this number to 729 scenarios.

In the first experiment, the total cost decreases with a growing number of random demand scenarios, to stabilize around a total cost of 520 at the number of 500 scenarios. In this case, using more than 500 scenarios will not result in a lower total cost and is therefore not recommendable, see Fig.4.

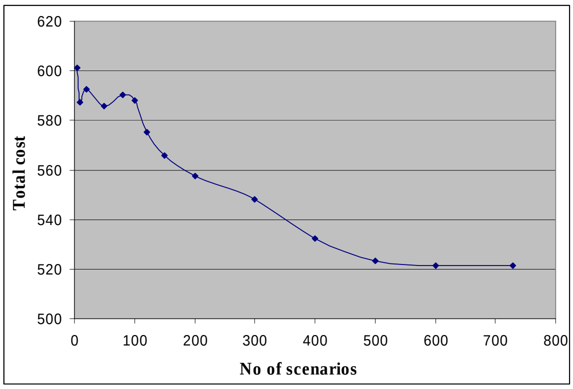

Fig. 4 Influence of the number of scenarios over 5 periods

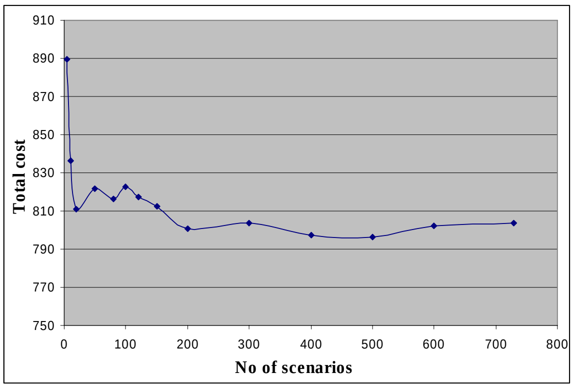

Fig. 5 Influence of the number of scenarios over 10 periods

In the second experiment, which uses random data over 10 periods, the total cost also decreases when the number of employed scenarios is increased, but the cost stabilizes more or less already at a number of 150 scenarios. In this case, no more than 150 scenarios should thus be generated, see Fig.5.

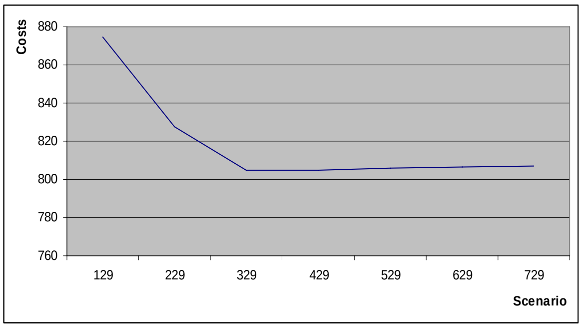

Fig. 6 Scenario reduction

Finally, the scenario reduction algorithm [17] was applied for the reduction of random scenario tree of 729 (T=6). The results are presented in Fig.6.

The experiments lead to the conclusion that the total cost can indeed be reduced by increasing the number of random demand scenarios, and this up to an optimal number after which the total cost stabilizes. This optimal number of scenarios is however dependent of the concerned number of periods in the time horizon, and can therefore not be predefined. This leads to the overall conclusion that for the random approach of the beer game model, the more periods are concerned over the time horizon, the less scenarios need to be generated to obtain a stabilized lowest total cost.

5 Enhanced model under uncertain demand and lead time

After having developed and tested a basic stochastic beer game model under uncertainty of customer demand, the next task consists of enhancing this model by varying the supplier lead times over time. In the stochastic lead time model, the lead time value N as used in the basic beer game model, is not fixed anymore, but is dependent of parameters s (lead time scenario), l (level) and t (period). A separate lead time scenario tree (with a number of periods equal to the number of periods in the demand tree) is generated. Each lead time scenario in the data tree will indeed consist of a specific lead time for each period as well for each level in the supply chain, as in reality there are different operating logistics in different branches of the network. Thus, in the objective function, inventory and backlog costs are not anymore only summated over all demand scenarios, but for each specific demand scenario all lead time scenarios are summated as well. The total number of summated scenarios is then given by DS*LS (with DS the number of demand scenarios and LS the number of generated lead time scenarios) and after adding an extra lead time scenario index ls to all variables, the new objective function becomes (for both regular and POR version) (13):

$$\min \sum_{ds}^{DS} \sum_{ls}^{LS} \left(\sum_{l=2}^{4} \sum_{t=1}^{T} ic_{l,t}.I_{l,t,ds,ls} + \sum_{l=2}^{4} \sum_{t=1}^{T} bc_{l,t}.B_{l,t,ds,ls} \right) \bullet \frac{1}{DS \cdot LS}; \quad (13)$$

The extra lead time scenario index ls is added to all variables Orders O, Shipments P, Inventory I, Backlog B and On Order Ond, in order to indicate that these variables now also depend of the actual lead time scenario, besides concerned level, period and demand scenario.

5.1 Results

The enhanced AMPL model was tested twice with random data, using the POR order policy over 4 and 8 periods respectively, starting at 3 scenarios for both customer demand and supplier lead time, and steadily increasing this number to 35 demand and 35 lead time scenarios. This total number of periods and stochastic scenarios is rather small, due to computational restrictions. The results are illustrated below (see Fig. 7 and Fig.8).

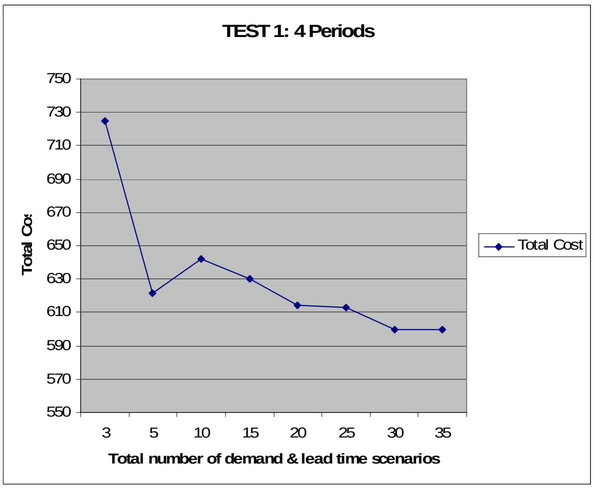

Fig. 7 Influence of the number of scenarios over 4 periods

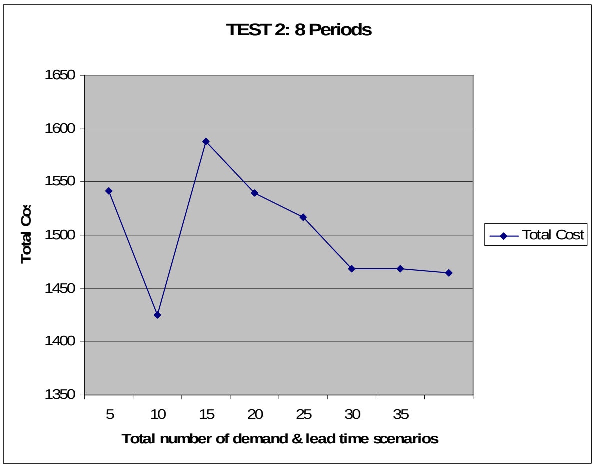

Fig. 8 Influence of the number of scenarios over 8 periods

Both experiments lead once again to the conclusion that for the random approach, the total cost decreases with an increasing number of stochastic scenarios, stabilizing after a certain optimal number. However, in this case the computational restrictions are too high to determine this optimal number of scenarios or to derive any further conclusions. This is therefore once again the major disadvantage of the stochastic modelling approach: the number of scenarios or periods in the time horizon taken into account in reality is much bigger than a regular computer is able to model. However, even with a small number of scenarios or periods, a stochastic approach considering the uncertainty of customer demand and supplier lead times, leads to a far more robust planning solution than any other deterministic equivalent, and therefore results in a superior decision management.

6 Conclusion and future research

This paper has illustrated the application of stochastic approach for the optimisation of the inventory model of the Beer Game under uncertainty of customer demand and lead times, representing an inventory problem in a single-product linear supply chain consisting of 4 echelons. The model was developed and tested in AMPL, as well as for a conventional order policy using optimal order quantities per period, as for POR and SCR order policies. The conducted experiments lead to the conclusion that the stochastic approach increases the total cost output of the model, but nevertheless results in a far more robust planning solution than any other deterministic equivalent, armed against the real life uncertainty. When employing the random approach, a higher number of scenarios results in a lower total cost, reaching a minimum after a variable optimal high number of scenarios. Future enhancements of the beer game model can be obtained by (1) making the inventory and backlog costs dependent of the concerned period in the time horizon, (2) extending the model to an infinite number of supply chain levels.

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