DETERMINING THE OPTIMAL OPERATING POINT OF AN INDUCTION GENERATOR

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Abstract

The work deals with a wound rotor induction machine operating as a generator. A wind or water turbine is normally mounted to its shaft. The rotor winding is fed by a voltage source inverter while the stator winding is connected to the electric grid. Optimal operating points of the system consisting of an induction generator and voltage source inverter are evaluated by a stochastic search algorithm called differential evolution. The optimization objective is to find operating points with the best efficiency of the system as a function of the shaft speeds and delivered mechanical power. In order to perform the optimization a magnetically nonlinear two-axis induction machine model oriented with the magnetizing current vector is applied. The same mathematical model of induction machine and the same optimization method is used for determining the optimal operating points of the same machine operating as an induction motor. The proposed method is appropriate to be applied in the systems, where the same induction machine feed by the voltage source inverter is applied as a generator and also as a motor. The magnetic nonlinear behaviour of the discussed induction machine, operating as a generator and as a motor, is accounted for by the current dependent flux linkage characteristic. Voltage and current limits of the induction machine and voltage source inverter are used as optimization bounds. The optimization is performed in the program package Matlab.

Keywords: Induction machine, generator, Nonlinear two-axis model, Optimization, Differential evolution.

Presenting Author's biography

Silvo Ropoša received his diploma degree in Electrical Engineering from the University of Maribor, Faculty of Electrical Engineering and Computer Science, in 2004. Since 2004 he has been working as network planner in public company for electricity distribution Elektro Maribor d.d. Currently, he is a postgraduate student at the University of Maribor, Faculty of Electrical Engineering and Computer Science. His research interests include electromagnetic phenomena in nonlinear circuits, optimization of generators for small power plants and connecting small power plants to the public networks. He is involved in the research work of the research group for Electromechanical systems control at the Faculty of Electrical Engineering and Computer Science in Maribor.

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1 Introduction

Determining the optimal operating points of an induction motor is a very frequent method, used all around the world. Another method, which is much more unknown, deals with determination of optimal operating points of induction generators. When we have to deal with an induction machine feed by a voltage source inverter, it is useful and recommendable to operate in an optimal operating point in both regimes of the machine: motor regime and generator regime.

This work focuses on determining optimal operating points of an induction generator. In order to perform the optimization an appropriate machine model and an appropriate optimization tool are needed. In this work, a stochastic search algorithm called differential evolution is applied to perform optimization, while the induction machine is represented by its two-axis magnetically nonlinear model. For the same induction machine, operating as a motor and represented with the same mathematical model, determining of the optimal operating points with the same optimization method is performed. In this case the wound rotor induction motor operates with short circuited rotor windings. The results of determined optimal operating points of an induction machine operating as a generator and as a motor are presented in this work.

2 Induction machine model

In the case of determining the optimal operating point of an induction motor a magnetically nonlinear two–axis induction machine model oriented with the rotor flux linkages vector is very often applied [1,2,3,4,5]. This orientation is not suitable to be used in the model of wound rotor induction machine operating as a generator. Therefore, the two–axis induction machine model oriented with the magnetizing current is applied.

A magnetically nonlinear two–axis induction machine model is given by (1) describing voltage balances in the stator and rotor winding in the d– and q–axes of an arbitrary rotating reference frame d-q:

$$\begin{aligned} u_{sd} &= i_{sd}R_s + \frac{d}{dt}\psi_{sd} - \dot{\Theta}_s\psi_{sq} \\ u_{sq} &= i_{sq}R_s + \frac{d}{dt}\psi_{sq} + \dot{\Theta}_s\psi_{sd} \\ u_{rd} &= i_{rd}R_r + \frac{d}{dt}\psi_{rd} - \dot{\Theta}_r\psi_{rq} \\ u_{rq} &= i_{rq}R_r + \frac{d}{dt}\psi_{rq} + \dot{\Theta}_r\psi_{rd} \end{aligned} \quad (1)$$

where u_{sd}, u_{sq}, u_{rd}, u_{rq} and i_{sd}, i_{sq}, i_{rd}, i_{rq} are the stator and rotor voltages and currents in the d– and q–axes, R_s and R_r are the stator and rotor resistances Ψ_{sd}, Ψ_{sq}, Ψ_{rd}, Ψ_{rq} are the stator and rotor flux linkages in the d– and q–axes, while $\dot{\Theta}_s$ and $\dot{\Theta}_r$ are the speeds of the rotation of stator (α–β) and rotor (γ–δ) reference frames [1,2] with respect to the d-q reference frame.

The relations among stator and rotor currents in the d– and q–axes, that are appear in Eq. (1) describing voltage balances and magnetizing currents in the d– and q–axes i_{md} and i_{mq} are given by (2).

$$\begin{aligned} i_{md} &= i_{sd} + i_{rd} \\ i_{mq} &= i_{sq} + i_{rq} \end{aligned} \quad (2)$$

The norm of magnetizing current i_m is defined by (3).

$$i_m^2 = i_{md}^2 + i_{mq}^2 \quad (3)$$

Stator end rotor flux linkages, which are used in (1), are given by (4):

$$\begin{aligned} \psi_{sd} &= \Psi_m + L_{\sigma s}i_{sd} = L_m i_{md} + L_{\sigma s}i_{sd} \\ \psi_{sq} &= \Psi_m + L_{\sigma s}i_{sq} = L_m i_{mq} + L_{\sigma s}i_{sq} \\ \psi_{rd} &= \Psi_m + L_{\sigma r}i_{rd} = L_m i_{md} + L_{\sigma r}i_{rd} \\ \psi_{rq} &= \Psi_m + L_{\sigma r}i_{rq} = L_m i_{mq} + L_{\sigma r}i_{rq} \end{aligned} \quad (4)$$

where Ψ_m is the flux linkage due to the main flux, L_m is the magnetizing inductance [3] while $L_{\sigma s}$ and $L_{\sigma r}$ are the stator and rotor leakage inductances. The magnetizing inductance L_m is link between the magnetizing current i_m and the flux linkage Ψ_m.

By using (2) in (4) expressions (5) can be obtained:

$$\begin{aligned} \psi_{sd} &= L_m i_{md} + L_{\sigma s}i_{sd} \\ \psi_{sq} &= L_m i_{mq} + L_{\sigma s}i_{sq} \\ \psi_{rd} &= (L_m + L_{\sigma r})i_{md} - L_{\sigma r}i_{sd} \\ \psi_{rq} &= (L_m + L_{\sigma r})i_{mq} - L_{\sigma r}i_{sq} \end{aligned} \quad (5)$$

In the steady state operation all time derivatives of flux linkages in (1) equal zero. When in such conditions flux linkages in (1) are replaced by the right hand side expressions in (5), equation (6) is obtained.

$$\begin{aligned} U_{sd} &= R_s i_{sd} - \dot{\Theta}_s L_{\sigma s} I_{sq} - \dot{\Theta}_s L_m I_{mq} \\ U_{sq} &= R_s i_{sq} + \dot{\Theta}_s L_{\sigma s} I_{sd} + \dot{\Theta}_s L_m I_{md} \\ U_{rd} &= -R_r i_{sd} + \dot{\Theta}_r L_{\sigma r} I_{sq} + R_r i_{md} - \dot{\Theta}_r (L_m + L_{\sigma r}) I_{mq} \\ U_{rq} &= -\dot{\Theta}_r L_{\sigma r} I_{sd} - R_r i_{sq} + \dot{\Theta}_r (L_m + L_{\sigma r}) I_{md} + R_r i_{mq} \end{aligned} \quad (6)$$

In the next step the d–axis can be aligned with the magnetizing current vector which leads to (7):

$$\begin{aligned} I_m &= I_{md} \\ I_{mq} &= 0 \end{aligned} \quad (7)$$

and voltage equation (6) can be simplified to (8).

$$\begin{bmatrix} U_{sd} \\ U_{sq} \end{bmatrix} = R_s \begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix} + \dot{\Theta}_s \begin{bmatrix} 0 \\ 1 \end{bmatrix} I_m L_m + \dot{\Theta}_s L_{\sigma s} \begin{bmatrix} -I_{sd} \\ I_{sq} \end{bmatrix}$$
$$\begin{bmatrix} U_{rd} \\ U_{rq} \end{bmatrix} = R_r \begin{bmatrix} I_m \\ 0 \end{bmatrix} - R_r \begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix} + \dot{\Theta}_r \left(L_m + L_{\sigma r} \right) \begin{bmatrix} 0 \\ I_m \end{bmatrix} + + \dot{\Theta}_r L_{\sigma r} \begin{bmatrix} -I_{sd} \\ I_{sq} \end{bmatrix} \tag{8}$$

The electromagnetic torque T_e can be in this case expressed by (9).

$$T_e = p_p L_m I_m I_{sq} \tag{9}$$

The two–axis model of a wound rotor induction machine given by (8) and (9) is suitable to be used for determining optimal operating points by the differential evolution in the generator and motor regime.

3 Magnetically nonlinear characteristic

In order to achieve better agreement between the measured and calculated results, the magnetically nonlinear induction machine characteristic $\Psi_m(I_m)$ shown in Fig 1 can be accounted for in the model (8), (9) by the magnetizing inductance $L_m = \Psi_m / I_m$.

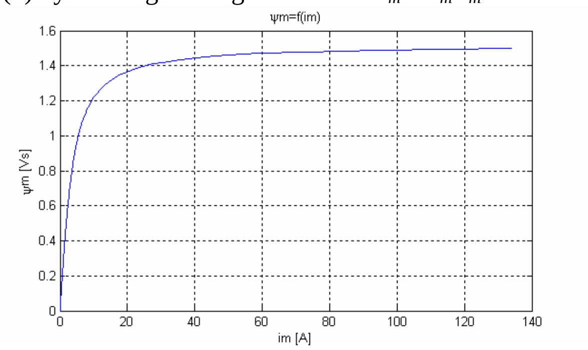

Fig. 1 Magnetically nonlinear characteristic

4 Differential evolution

Differential evolution is a method for mathematical optimization of multidimensional functions and belongs to the class of Evolution strategy optimizers. Differential evolution finds the global minimum of a multidimensional, multimodal (i.e. exhibiting more than one minimum) function (mathematics) with good probability.

The crucial idea behind differential evolution is a scheme for generating trial parameter vectors. Differential evolution adds the weighted difference between two population vectors to a third vector. In this way no separate probability distribution has to be used which makes the scheme completely self-organizing [6,7].

5 Determining the optimal operating points of an induction generator by differential evolution

Fig. 2 schematically shows a wound rotor induction generator connected to the electric grid. A source of power in the form of wind or water turbine is normally mounted to generator shaft. The rotor winding of generator is fed by the second source of power in the form of a voltage source inverter while its stator winding is connected to the electric grid.

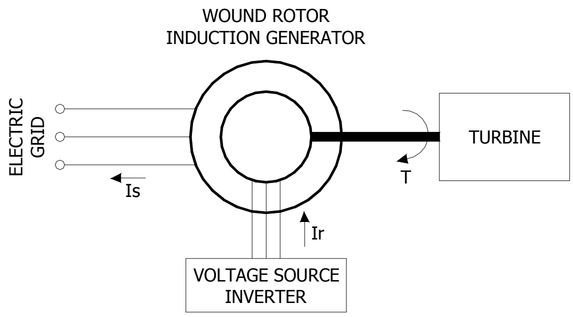

Fig. 2 Induction generator on the grid

Optimal operating points of the induction generator are determined by the differential evolution as a function of mechanical power and rotor speed, which are given in several points. In each operating point the currents I_{sd} and I_{sq} have to be determined by differential evolution considering voltage and current limits of the voltage source inverter and motor. The goal of optimization is to determine currents I_{sd} and I_{sq} that assure the best efficiency, which is defined by (10)

$$\eta_{generator} = \frac{P_{\text{stator}}}{P_{\text{insert}}} = \frac{P_s}{P_{mec} + P_r} \tag{10}$$

where P_s is generated electric power on the stator (11), P_r is electric power delivered to the rotor from the voltage source inverter (12) while P_{mec} is the mechanical power delivered to the generator shaft.

$$P_s = U_{sd} I_{sd} + U_{sq} I_{sq} \tag{11}$$

$$P_r = U_{rd} I_{rd} + U_{rq} I_{rq} \tag{12}$$

The schematic course of optimization for generator operating regime is presented on Fig.3.

The optimization was performed for a four pole, three phase laboratory induction machine with the data P_s=3*kW*, U_s=380*V*, U_r=85.5*V*, I_s=7.5*A*, I_r=23*A*, R_s=1.976*Ω*, R_r=2.91*Ω*, L_s=2.335*H*, L_r=2.335*H*, L_m=0.223*H*, T_n=15*Nm* and n_n=1328min^{-1}.

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For known variables: P_{mec}, ω, U_s and Θ_s

1.) Currents I_{sd} and I_{sq} have to be determined by differential evolution!

2.) $U_{sd} = R_s I_{sd} - \dot{\Theta}_s L_{\sigma s} I_{sq}$

$\Downarrow$

3.) $U_{sq} = \sqrt{U_s^2 - U_{sd}^2}$

$\Downarrow$

4.) $\Psi_m = \dfrac{U_{sq} - R_s I_{sq} - \dot{\Theta}_s L_{\sigma s} I_{sd}}{\dot{\Theta}_s}$

$\Downarrow$

5.) I_m (from $\Psi_m(I_m)$ characteristic)

$\Downarrow$

6.) $L_m = \dfrac{\psi_m}{I_m}$

$\Downarrow$

7.) $\dot{\Theta}_r = \dot{\Theta}_{grid} - \dot{\Theta}_s$

8.) $U_{rd} = R_r\left(I_m - I_{sd}\right) - \dot{\Theta}_r L_{\sigma r} I_{sq}$

9.) $U_{rq} = -R_r I_{sq} + \dot{\Theta}_r\left(\left(L_m + L_{\sigma r}\right) I_m + L_{\sigma r} I_{sd}\right)$

(Equations for considering voltage and current limits of machine and voltage source inverter!)

$\Downarrow$

10.) $P_s, P_r, \eta, T_e, P_{mec}$

(Values for optimization!)

$\Downarrow$

11.) Objective function value: $\eta \rightarrow \max$

(IF

number of iterations < max. *number of iterations*

THEN

return to step 1

ELSE

end of optimization)

Fig. 3 Schematic course of optimization for generator regime working

6 Determining the optimal operating points of an induction motor by differential evolution

For determining the optimal operating points of the motor the same mathematical model of induction machine as for generator is applied. The model is given by Eq. (8) and Eq. (9). Optimal operating points are determined by the differential evolution as a function of mechanical torque (T_{mec}) and rotor speed (n), which are given in several points. In the steady state operation of the motor the mechanical torque and electric torque are equal. The mechanical load is normally mounted to the motor shaft. The stator winding of the motor is fed by a voltage source inverter while the rotor windings are short circuited (Fig. 4). In each operating point the currents I_{sd} and I_{sq} have to be determined by differential evolution considering voltage and current limits of the voltage source inverter and motor. The goal of optimization is to determine currents I_{sd} and I_{sq} that assure the best efficiency. Efficiency of the motor is defined by (13).

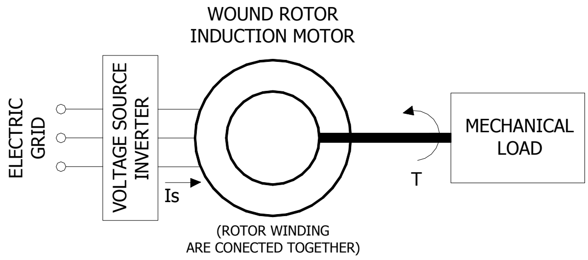

Fig. 4 Induction motor on the grid

$$\eta_{motor} = \frac{P_{mec}}{P_s} \tag{13}$$

In Eq. (13) is P_s the electric power delivered from the voltage source inverter (11), while P_{mec} is the mechanical power on the motor shaft, defined by the demanded torque T_e and rotor speed ω (14).

$$P_{mec} = T_e \omega = T_e \frac{2\pi n}{60} \tag{14}$$

The number of revolutions per minute is denoted by n.

The optimization was performed for the laboratory induction machine, presented in the previous section.

7 Results

The results of optimization described in the previous sections are presented in the next two subsections. The first subsection presents results for the induction generator while the second subsection presents results for the induction motor. The results of optimization for generator are presented as a function of mechanical power and rotor speed (P_{mec},ω) while the results of optimization for motor are presented as a function of electric torque and rotor speed (T_e,ω).

7.1 Results for generator

In all operating points given by the rotor speed ω and mechanical power delivered to the generator shaft P_{mec}, optimal set of stator currents I_{sd} and I_{sq} is determined by the optimization considering magnetically nonlinear characteristic of the induction machine and current and voltage limits of the machine and voltage source inverter. The stator terminal voltages are given by the grid and have constant amplitude. The currents I_{sd} and I_{sq} which ensure the best efficiency are set by the voltage source inverter that supplies rotor windings.

Fig. 5 show values for I_s, I_{sd} and I_{sq} for constant mechanical power (P_{mec}=3000W) and different rotor speeds determined by the optimization. I_s is the stator current vector length, given by (15)

$$I_s = \sqrt{I_{sd}^2 + I_{sq}^2} \qquad (15)$$

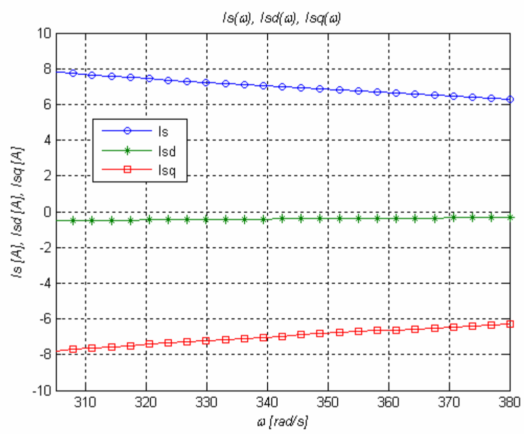

Fig. 5 Optimization determined I_s, I_{sd} and I_{sq}, for different rotor speeds at P_{mec}=3000W

Fig. 6 shows values for Ψ_m, I_m, L_m and T_e for constant mechanical power (P_{mec}=3000W) and different rotor speeds determined by the optimization.

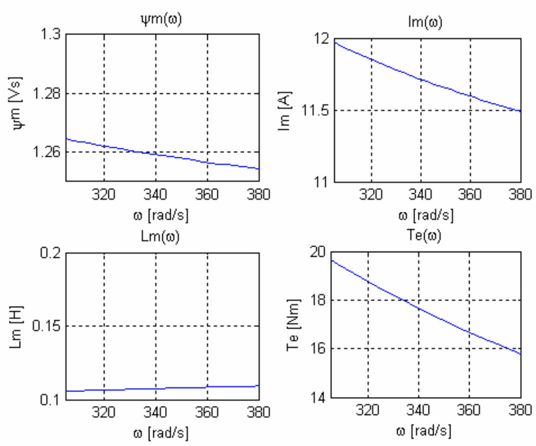

Fig. 6: Optimization determined Ψ_m, I_m, L_m and T_e, for different rotor speeds at P_{mec}=3000W

Figs. 7 and 8 show two–dimensional presentations of efficiency characteristics for discussed induction generator. They are given in Fig. 7 as function of P_{mec} at different constants speeds. Fig. 8 shows the same characteristics given as function of rotor speed at different constant P_{mec}. The three–dimensional presentation of induction generator efficiency characteristic is shown in Fig 9. It is actually composed from results shown in Fig. 7 and Fig. 8.

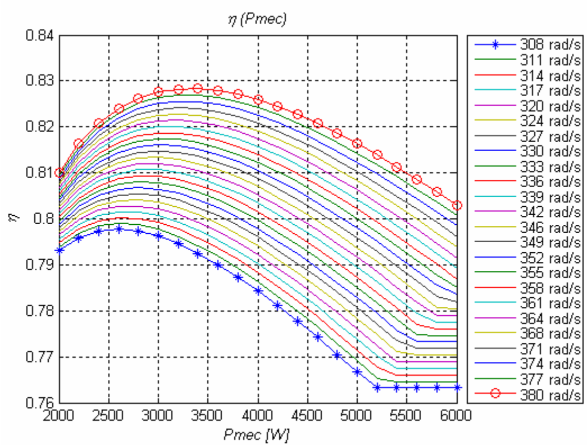

Fig. 7 Characteristics $\eta\,(P_{mec})$ given for different constant rotor speeds

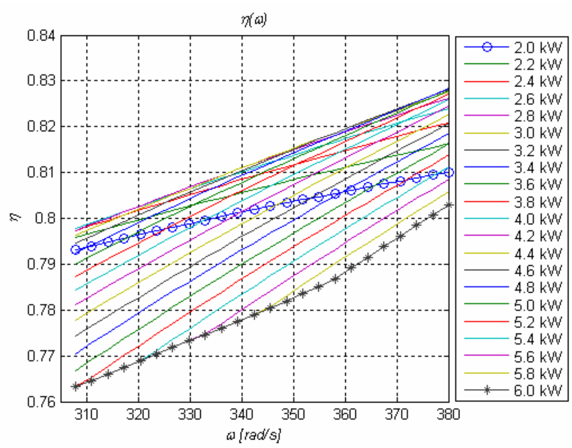

Fig. 8 Characteristics $\eta(\omega)$ given for different constant P_{mec}

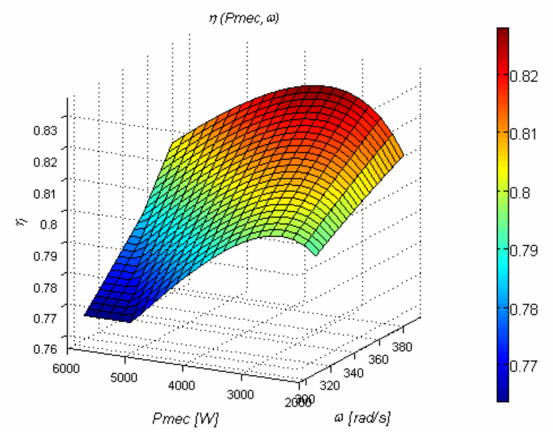

Fig. 9 Efficiency characteristic $\eta\,(P_{mec},\omega)$

Fig. 10 shows the rotor voltage vector length (16) as a function of P_{mec} and ω, while Fig. 11 shows rotor current vector length (17) as a function of P_{mec} and ω.

$$U_r = \sqrt{U_{rd}^2 + U_{rq}^2} \qquad (16)$$

$$I_r = \sqrt{I_{rd}^2 + I_{rq}^2} \qquad (17)$$

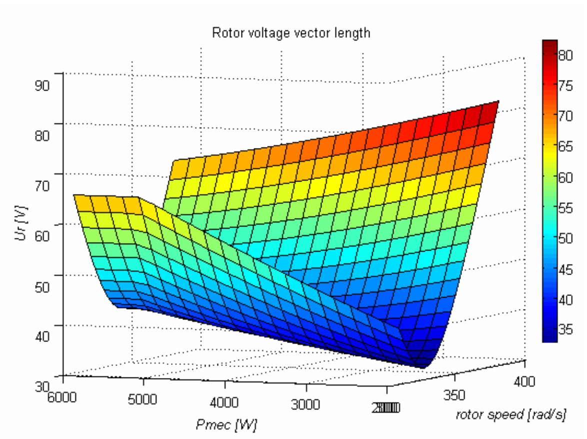

Fig. 10 Rotor voltage as a function of P_{mec} and ω

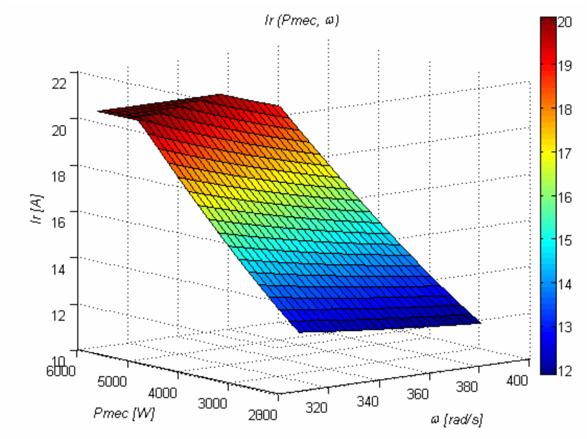

Fig. 11 Rotor current as a function of P_{mec} and ω

Fig. 12 shows the electric power delivered to the rotor from the voltage source inverter (12) as a function of P_{mec} and ω, while Fig. 13 shows the generated electric power on the stator (11) as a function of P_{mec} and ω.

From the results presented, it is obviously clear that for every operating point of the generator (delivered mechanical power to the rotor shaft P_{mec} and given rotor speed ω) there exist such values of input parameters (U_r , I_r), that induction generator is operating with maximal efficiency.

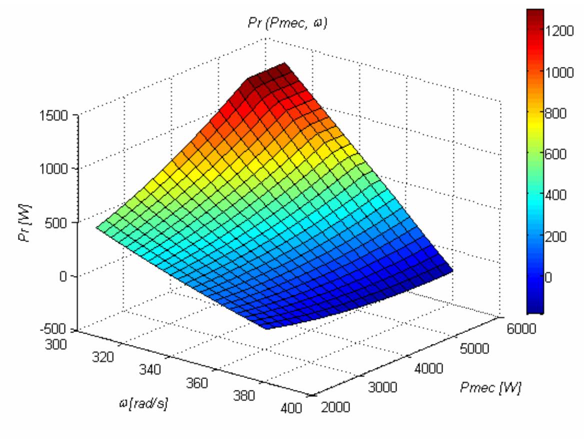

Fig. 12 Electric power delivered to the rotor as a function of P_{mec} and ω

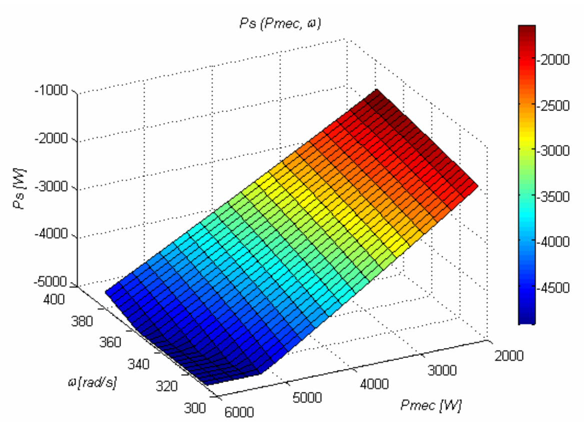

Fig. 13 Generated electric power on the stator as a function of P_{mec} and ω

7.2 Results for motor

In all operating points given by the rotor speed n and mechanical torque T_{mec} demanded on the motor shaft, optimal set of stator currents I_{sd} and I_{sq} is determined by the optimization considering magnetically nonlinear characteristic of the induction machine and current and voltage limits of the machine and voltage source inverter. Due to the short circuited rotor windings the rotor voltages are equal zero. The currents I_{sd} and I_{sq} which ensure the best efficiency are set by the voltage source inverter that supplies stator windings.

Fig. 14 shows the stator current vector length (15) as a function of T_e and n, while Fig. 15 shows the stator voltage vector length (18) as a function of the T_e and n. The three–dimensional presentation of induction motor efficiency characteristic is shown in Fig 16.

$$U_s = \sqrt{U_{sd}^2 + U_{sq}^2} \qquad (18)$$

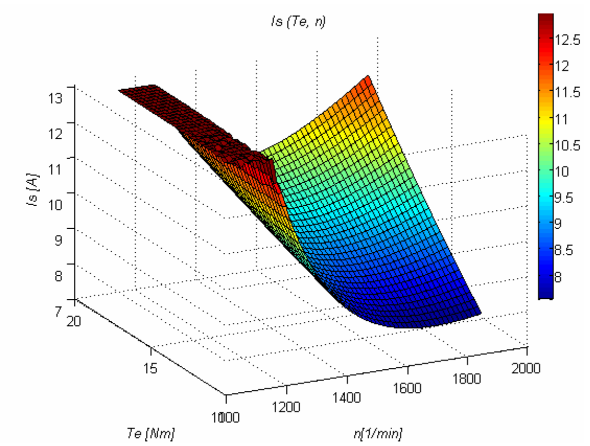

Fig. 14 Stator current as a function of T_e and n

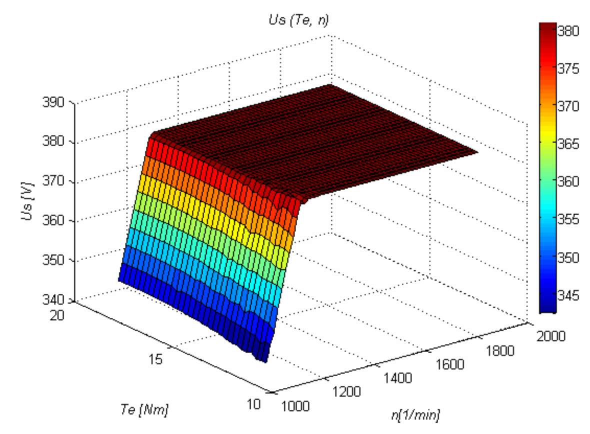

Fig. 15 Stator voltage as a function of T_e and n

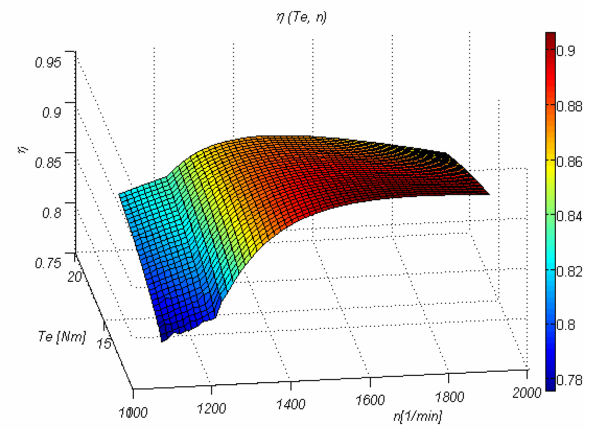

Fig. 16 Characteristic η (T_e,ω)

The results obtained for the motor are similar to those obtained for the generator. For every operating point of the motor (demanded mechanical torque on the motor shaft T and demanded rotor speed n) there exist such values of input parameters (U_s **,** I_s), that induction motor is operating with maximal efficiency.

8 Conclusion

This work shows that optimization method like differential evolution can be used as a tool for determining optimal operating points of an induction machine only when an appropriate magnetically nonlinear machine model is applied. The proposed method can be applied especially in the systems, where the same induction machine feed by the voltage source inverter is applied as a generator and also as a motor [4]. In this case, the same mathematical model of induction machine operating as a generator and as a motor can be used.

In the previous section only one kind of optimization is presented – determining operation points of induction machine with the best efficiency. With the same mathematical model of induction machine and with the same optimization tool, optimal operating points of induction machine with respect to the other optimization objectives can be determined: minimal losses of the system, maximal mechanical or electric power, maximal torque, etc.

The future work will be focused on confirmation of results presented in this work by measurements performed on the laboratory wound rotor induction machine.

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