DEVELOPMENT OF NONLINEAR TRANSFORMER MODEL APPROPRIATE FOR LOSS CALCULATION WHEN HIGHER HARMONICS ARE PRESENT

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Abstract

This work proposes a nonlinear transformer model appropriate for calculating the transformer losses when higher order harmonics in currents and voltages are present. Higher order harmonics in currents and voltages are produced by nonlinear loads. Today, by far of the majority of nonlinear loads are power electronic devices, which are massively used in computer equipment and peripherals. In order to calculate increase of hysteresis and eddy currents losses due to higher order harmonics in currents and voltages, the magnetically nonlinear model of the transformer is derived. The derived transformer model is based on lumped parameters model. It is obtained by coupling the electric and magnetic equivalent circuits. Eddy currents effects and nonlinear behavior of the iron core are accounted for using short circuit winding and Jiles-Atherton (J-A) hysteresis model. The J-A hysteresis model parameters are identified using stochastic search algorithm called differential evolution (DE). The J-A hysteresis model parameters are identified by DE in such a way, that the best possible agreement between the measured and by the model calculated $B - H$ hysteresis loops of the iron core is obtained. Inclusion of eddy currents and hysteresis into the magnetically nonlinear transformer model makes this transformer model appropriate for calculation of eddy currents and hysteresis losses when higher order harmonics in currents and voltages are present. To assure that the calculated losses reflect real eddy currents losses and hysteresis losses, the obtained transformer model was confirmed though the comparison of measured and calculated transformer currents.

Keywords: Nonlinear transformer model, Hysteresis, Higher order harmonic components, Transformer losses.

Presenting Author's Biography

Matej Toman received his B. S. degree from the Faculty of Electrical Engineering and Computer Science, University of Maribor, in 2003. Since then he has been employed with the Faculty of Electrical Engineering and Computer Science as a junior researcher. His field of interest is modelling and control of electromechanical systems. He is currently engaged in research of additional transformer losses due to higher harmonic components in currents and voltages.

1 Introduction

Higher order harmonics in voltages and currents have been present for decades. However, today the number of higher order harmonic producing devices is increasing rapidly. Most of this devices are power electronic devices. Due to their tremendous advantages in efficiency and controllability, power electronic devices can be found at all power levels. The drawback of their massive use is increase of higher order harmonics in voltages and currents. This higher order harmonics in voltages and currents flows through the transformer and causes increase of transformer losses.

The authors of this paper focus on development of a transformer model that is appropriate for loss calculation when higher order harmonics in currents and voltages are present. At the stage of transformer modeling only lumped parameters models are considered since calculations with these models are less time consuming than calculations with finite element models. In calculations hysteresis, copper and eddy currents losses are considered. In order to achieve realistic values of hysteresis losses, behavior of the iron core must be accurately represented. In the stage of transformer modeling three different approaches to account for the behavior of the iron core are often used. The first one uses linear function to represent dependence between the magnetic field H and the magnetic flux density B, while the second one uses nonlinear magnetic curve [1]. In both cases behavior of the iron core is not fully considered since hysteresis of the iron core is neglected. As a result such models can not be used for calculation of the hysteresis losses. Only third approach that includes hysteresis in the transformer model could be used to perform such calculations.

In this work authors use Jiles-Atherton (J-A) hysteresis model to account for hysteresis in the transformer model. The parameters of the J-A hysteresis model were determined using differential evolution (DE). The obtained transformer model was confirmed through the comparison of measured and calculated transformer voltages and currents. This magnetically nonlinear transformer model was then used in simulations to calculate transformer losses in the case of linear and nonlinear load. Based on the results the increase of hysteresis, copper and eddy current losses was determined. Calculated eddy current losses were then used to calculate the corresponding eddy current in a short circuit winding. In tests, higher order harmonics in voltages and currents were produced by the diode rectifier, whose model was also derived for simulation purposes. All simulations were performed in Matlab/Simulink environment [2].

2 Nonlinear transformer model and Jiles-Atherton hysteresis model

Voltage balances in a single phase transformer, whose iron core behaves magnetically nonlinear are described with voltage equations (1):

$$\begin{aligned} u_1 &= i_1R_1 + \frac{d\psi_1}{dt} = i_1R_1 + L_{\sigma 1}\frac{di_1}{dt} + N_1\frac{d\phi}{dt} \\ u_2 &= i_2R_2 + \frac{d\psi_2}{dt} = i_2R_2 + L_{\sigma 2}\frac{di_2}{dt} + N_2\frac{d\phi}{dt} \end{aligned} \tag{1}$$

In (1) u_1, u_2 and i_1, i_2 are the primary and the secondary voltages and currents, R_1 and R_2 are the primary and the secondary resistances, ψ_1 and ψ_2 are the primary and the secondary flux linkages, $L_{\sigma 1}$ and $L_{\sigma 2}$ are the primary and the secondary leakage inductances, N_1 and N_2 are the primary and the secondary number of turns, while ϕ is the magnetic flux. Derivative of the magnetic flux $d\phi/dt$, which appears in (1) is expressed with (2), where magnetomotive force (MMF) is defined as $\theta = N_1i_1 - N_2i_2$.

$$\frac{d\phi}{dt} = \frac{d\phi}{d\theta}\frac{d\theta}{dt} = \frac{d\phi}{d\theta}\left(N_1\frac{di_1}{dt} - N_2\frac{di_2}{dt}\right) \tag{2}$$

By combining (1) and (2), the voltage balances in a nonlinear single phase transformer becomes (3):

$$\begin{aligned} u_1 &= i_1R_1 + L_{\sigma 1}\frac{di_1}{dt} + N_1\frac{d\phi}{d\theta}\left(N_1\frac{di_1}{dt} - N_2\frac{di_2}{dt}\right) \\ u_2 &= i_2R_2 + L_{\sigma 2}\frac{di_2}{dt} + N_2\frac{d\phi}{d\theta}\left(N_1\frac{di_1}{dt} - N_2\frac{di_2}{dt}\right) \end{aligned} \tag{3}$$

where therm $d\phi/d\theta$ represents nonlinear behavior of the transformer. In this work nonlinear behavior of the transformer is accounted for by the J-A hysteresis model. In original form J-A hysteresis model is used to account for nonlinear behavior of the iron core, but here it is suggested to be used to account for nonlinear behavior of the transformer as a device. In order to include J-A hysteresis model into nonlinear transformer model some simplifications must be considered. This simplifications define the use of the mean path length of the magnetic flux l and the average area of the iron core A. By using this simplifications in voltage balance equations (3) and knowing that the magnetic flux density and the magnetic field are equal to $B = \phi/A$ and $H = \theta/l$, (3) becomes (4).

$$\begin{aligned} u_1 &= i_1R_1 + L_{\sigma 1}\frac{di_1}{dt} + N_1A\frac{dB}{dH} \\ &\left(\frac{N_1}{l}\frac{di_1}{dt} - \frac{N_2}{l}\frac{di_2}{dt}\right) \\ u_2 &= i_2R_2 + L_{\sigma 2}\frac{di_2}{dt} + N_2A\frac{dB}{dH} \\ &\left(\frac{N_1}{l}\frac{di_1}{dt} - \frac{N_2}{l}\frac{di_2}{dt}\right) \end{aligned} \tag{4}$$

Term dB/dH represents nonlinear behavior of the transformer and can now be determined using J-A hysteresis model.

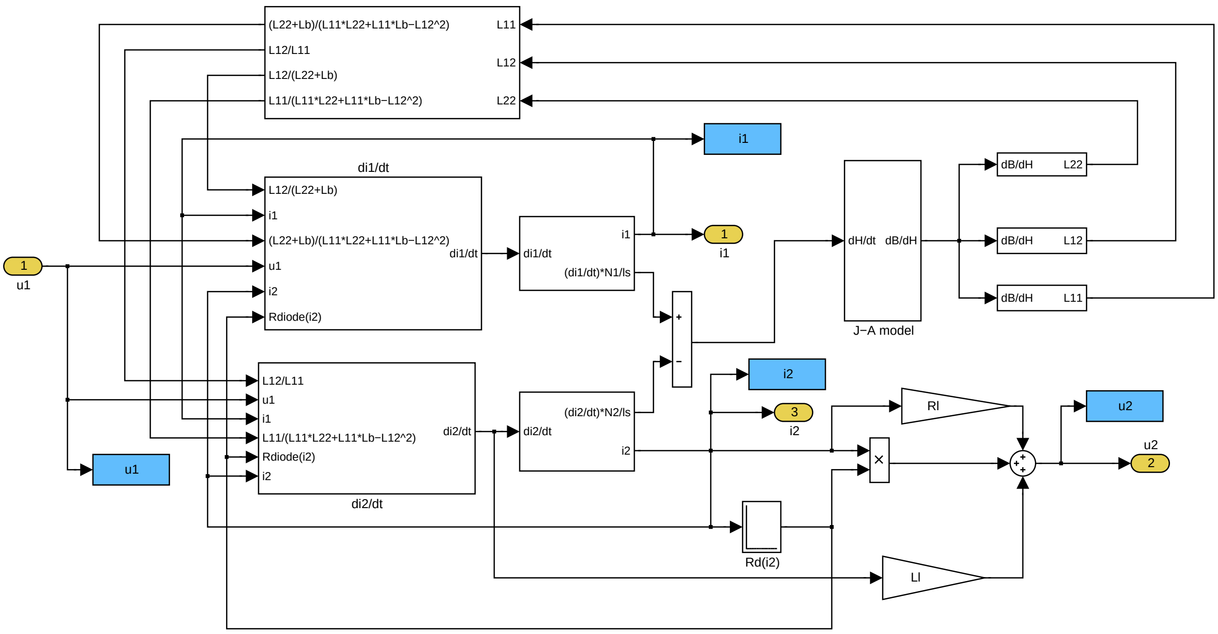

Fig. 1 Block scheme of a nonlinear single phase transformer with nonlinear load

2.1 Nonlinear model of a single phase transformer with nonlinear load

Formulation (4) shows voltage balances in nonlinear single phase transformer at no load. If load formed by the resistor, the inductor and the diode rectifier in series is considered, then the second equation from (4) should be combined with (5):

$$u_2 = i_2 R_l + L_l \frac{di_2}{dt} + i_2 R_d(i_2) \tag{5}$$

where R_l and L_l are the resistance and the inductance of the load and $R_d(i_2)$ is the resistance of the diode rectifier, which is secondary current dependent. Obtained equations are not appropriate for use in simulations, therefore they must be rewritten in the form of time derivatives of the primary and the secondary currents as shown in (6):

$$\begin{aligned}
\frac{di_1}{dt} &= \frac{L_{22}+L_l}{L_{11}L_{22}+L_{11}L_l-L_{12}^2} \\
&\left(u_1 - i_1R_1 - \frac{L_{12}}{L_{22}+L_l} i_2 \left(R_2+R_l+R_d(i_2)\right)\right) \\
\frac{di_2}{dt} &= \frac{L_{11}}{L_{11}L_{22}+L_{11}L_l-L_{12}^2} \\
&\left(\frac{L_{12}}{L_{11}}u_1 - \frac{L_{12}}{L_{11}}i_1R_1 - i_2\left(R_2+R_l+R_d(i_2)\right)\right)
\end{aligned} \tag{6}$$

where L_{11}, L_{12} and L_{22} are defined as:

$$L_{11} = L_{\sigma 1} + N_1^2 \frac{A}{l}\frac{dB}{dH}, \quad L_{12} = N_1N_2\frac{A}{l}\frac{dB}{dH},$$
$$L_{22} = L_{\sigma 2} + N_2^2 \frac{A}{l}\frac{dB}{dH} \tag{7}$$

Formulations (6) and (7) give us basic equations of the nonlinear model of a single phase transformer with nonlinear load. Based on this equations a block scheme in Matlab/Simulink environment was composed for simulation purposes. It is presented in Fig. 1. As it could be seen from the figure, the nonlinear behavior of the transformer is determined by J-A hysteresis model.

2.2 Jiles-Atherton hysteresis model

The J-A hysteresis model is widely used for modeling the nonlinear behavior of the magnetic materials. It is based on known ideas of the domain walls bending and on its translation. In this work the authors propose the use of the J-A hysteresis model for modeling the nonlinear behavior of the transformer. Although it is used for different purpose the J-A hysteresis model is not subjected to any modifications and can be used as presented in [3], [4] and [5]. The J-A hysteresis model is given by (8) - (11):

$$\frac{dM}{dH} = \frac{(1-c)\frac{dM_{irr}}{dH_e} + c\frac{dM_{an}}{dHe}}{1-\alpha c\frac{dM_{an}}{dH_e} - \alpha(1-c)\frac{dM_{irr}}{dH_e}} \tag{8}$$

$$\frac{dM_{an}}{dH_e} = \frac{M_s}{a}\left[1-\coth^2\left(\frac{H_e}{a}\right)+\left(\frac{a}{H_e}\right)^2\right] \tag{9}$$

$$\frac{dM_{irr}}{dH_e} = \frac{\gamma\left(M_{an}-M_{irr}\right)}{k\delta} \tag{10}$$

$$\gamma = \begin{cases} 1; & (M_{an}-M_{irr})\,dH_e \geq 0 \\ 0; & (M_{an}-M_{irr})\,dH_e < 0 \end{cases},$$
$$\delta = \begin{cases} +1; & \frac{dH}{dt} \geq 0 \\ -1; & \frac{dH}{dt} < 0 \end{cases} \tag{11}$$

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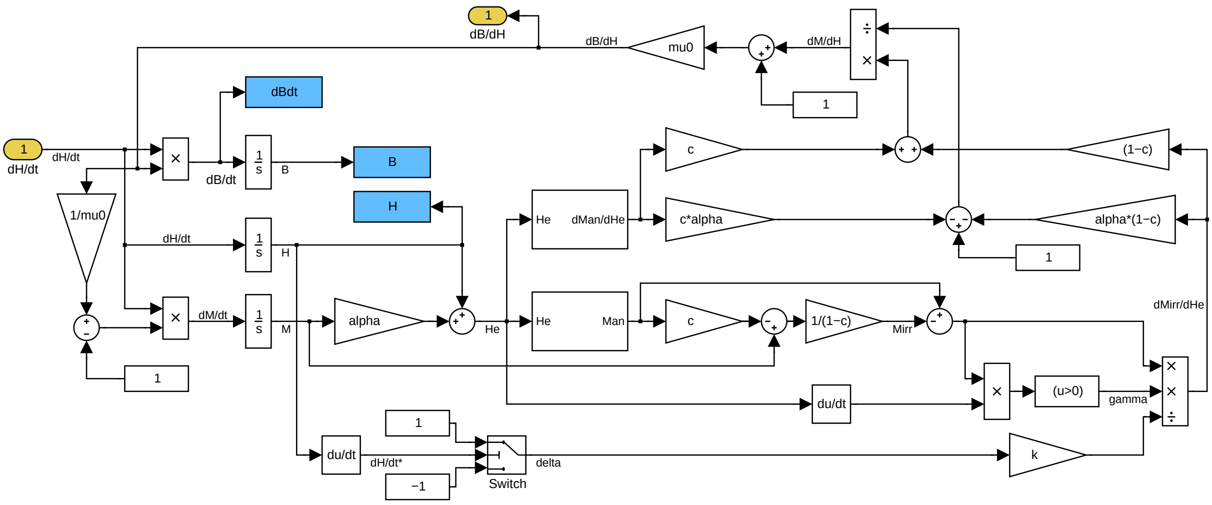

Fig. 2 Block scheme of the J-A hysteresis model

where M_{an} and M_{irr} are the anhysteretic and irreversible magnetizations, μ_0 is the permeability of vacuum, a is the the anhysteretic behavior, α is the main field parameter, M_s is the saturation magnetization, c is the parameter which is proportional to the hysteresis loop width and domain flexing constant, k is the pinning parameter and H_e is denoted as $H_e = H + \alpha M$.

The input to the J-A hysteresis model is the time derivative of the magnetic field, which is given by (12), while the output is the slope of the magnetic flux density dB/dH. It is expressed from term dM/dH by (13).

$$\frac{dH}{dt} = \frac{N_1}{l}\frac{di_1}{dt} - \frac{N_2}{l}\frac{di_2}{dt} \tag{12}$$

$$\frac{dB}{dH} = \mu_0 \left(1 + \frac{dM}{dH}\right) \tag{13}$$

Based on (8) - (13) a block scheme of the J-A hysteresis model was composed in Matlab/Simulink environment as shown in Fig. 2.

Five parameters of the J-A hysteresis model determines the shape of the calculated $B - H$ hysteresis loop from the hysteresis model. These parameters are: a, α, M_s, c and k. The J-A hysteresis model will represent the nonlinear behavior of the transformer only if these parameters are accurately identified. Therefore stohastic search algorithm called differential evolution (DE) was used for their identification [6]. The parameters were identified in such a way that the best possible agrement between measured and from the J-A hysteresis model calculated $B - H$ hysteresis loop was obtained. $B - H$ hysteresis loop needed in parameter identification process was measured according to [7], while using the same simplifications as in the nonlinear transformer model derivation. Using DE the following parameters of the J-A hysteresis model were identified in [8]: $a = 226.25$(A/m), $\alpha = 5.020 \cdot 10^{-4}$, $M_s = 1.335 \cdot 10^6$(A/m), $c = 0.724$ and $k = 300.05$(A/m). With inclusion of the parameters in J-A hysteresis model the

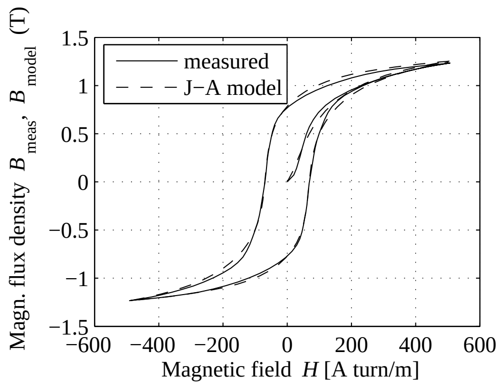

Fig. 3 Measured and calculated hysteresis loops

$B - H$ hysteresis loop was calculated. It is presented in Fig. 3 together with measured hysteresis loop.

3 Confirmation of developed nonlinear transformer model

To assure that the calculated losses reflect real transformer losses, the developed nonlinear transformer model was confirmed through the comparison between measured and calculated transformer voltages and currents. For this purpose experimental setup with a single phase transformer and linear load was composed. A single phase transformer with rated power 2 kVA and rated primary and secondary voltage 230 V was loaded with resistor and inductor in series. The transformer primary winding was supplied with rated sinusoidal voltage of 50 Hz, while the primary voltage and current $u_{1\ \mathrm{meas}}$ and $i_{1\ \mathrm{meas}}$ and the secondary voltage and current $u_{2\ \mathrm{meas}}$ and $i_{2\ \mathrm{meas}}$ were measured. Measured voltage $u_{1\ \mathrm{meas}}$ was then used in nonlinear transformer model with J-A hysteresis model from Fig. 1 and Fig. 2. In simulations the following data were considered: $R_1 = 0.49\ \Omega$, $R_2 = 0.63\ \Omega$, $L_{\sigma 1} = L_{\sigma 2} = 5.8$ mH, $N_1 = 345$ turns,

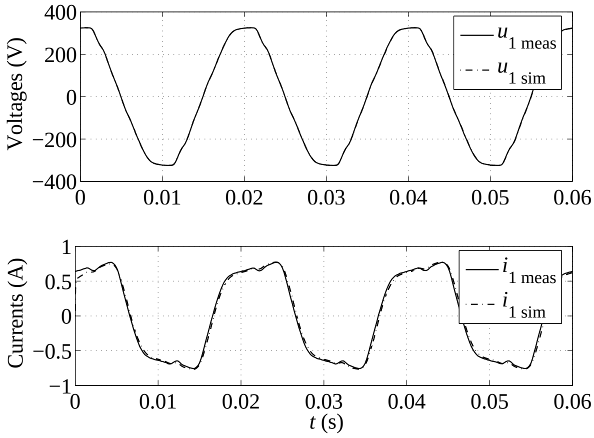

Fig. 4 Measured and calculated primary voltages and currents when linear load is considered

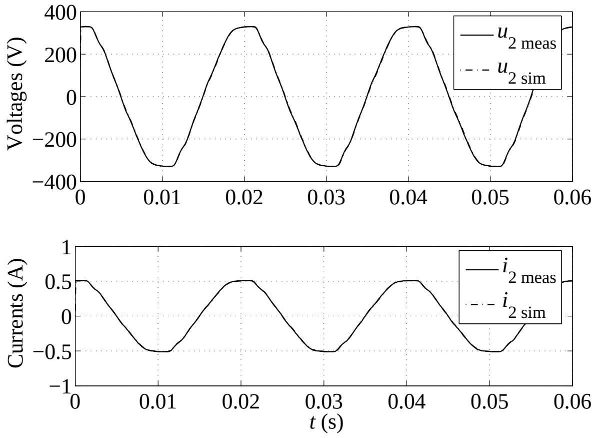

Fig. 5 Measured and calculated secondary voltages and currents when linear load is considered

$N_2 = 352$ turns, $A = 2650 \cdot 10^{-6}$ m^2, $l = 0.6$ m. To simulate linear load, diode rectifier characteristic $R_d(i_2)$ from Fig. 1, was set to zero for all values of the secondary current. Calculated primary voltage and current $u_{1\ \mathrm{sim}}$ and $i_{1\ \mathrm{sim}}$ and secondary voltage and current $u_{2\ \mathrm{sim}}$ and $i_{2\ \mathrm{sim}}$ are shown in Fig. 4 and Fig. 5 together with measured voltages and currents. As can be seen from figures very good agreement between measured and calculated variables is achieved. With some waveforms, agreement is so good that the time plot of measured and calculated voltages and currents are almost the same. This assures that the calculated losses using derived transformer model will reflect real transformer losses. The corresponding calculated $B - H$ hysteresis loop is shown in Fig. 6.

4 Results

Transformer losses are mostly differentiated between those originating in the windings, called copper losses, and those arising from the magnetic circuit, called iron losses. Copper losses are caused by currents flowing through the windings and are sometimes termed as I^2R losses. On the other hand iron losses are composed of

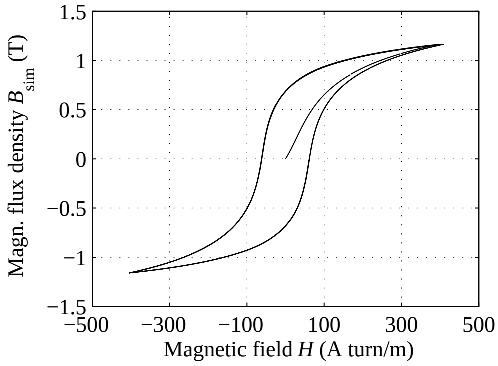

Fig. 6 Calculated $B - H$ hysteresis loop for the case when linear load is used

eddy current losses and hysteresis losses. First copper and hysteresis losses are determined using nonlinear transformer model with linear load. For this purpose results of calculated primary and secondary voltages and currents from previous section are used. The primary and the secondary copper losses, marked with P_{Cu1} and P_{Cu2}, can be calculated with (14):

$$\begin{aligned} P_{Cu1\bullet} &= I_{1\bullet}^2 R_1 = \frac{1}{T}\int_0^T i_{1\bullet} i_{1\bullet}\ dt\ R_1 \\ P_{Cu2\bullet} &= I_{2\bullet}^2 R_2 = \frac{1}{T}\int_0^T i_{2\bullet} i_{2\bullet}\ dt\ R_2 \end{aligned} \tag{14}$$

where $\bullet$ denotes calculated values from simulation, T denotes one period and $I_{1\bullet}$ and $I_{2\bullet}$ denote calculated RMS values of the primary and the secondary currents. Active powers on the primary P_1 and on the secondary side of the transformer P_2 are calculated by (15).

$$\begin{aligned} P_{1\bullet} &= \frac{1}{T}\int_0^T u_{1\ \bullet} i_{1\ \bullet}\ dt \\ P_{2\bullet} &= \frac{1}{T}\int_0^T u_{2\ \bullet} i_{2\ \bullet}\ dt \end{aligned} \tag{15}$$

Since hysteresis loop is included into nonlinear transformer model, the diference between the primary and the secondary active powers gives us the sum of hysteresis and copper losses. This is only the case if eddy currents are not considered in simulations. Therefore hysteresis losses are defined as (16).

$$P_{h\ sim} = P_{1\ sim} - P_{2\ sim} - P_{Cu1\ sim} - P_{Cu2\ sim} \tag{16}$$

Using (14) and (15), losses are calculated also for measured values, where $\bullet$ now denotes measured values of voltages and currents when linear load is considered. The results of calculated and measured active powers and losses are shown in Tab. 1. As can be seen from the table, good agreement between measured and from the transformer model calculated active powers and copper losses is achieved.

The diference between measured primary and secondary active powers represent hysteresis loses, copper

Tab. 1 Comparison of the results

Data	Lin. load		Nonlin. load	
	Sim.	Meas.	Sim.	Meas.
P_1 (W)	111.2	115.1	112.8	116.5
P_2 (W)	89.0	89.5	89.1	89.2
$P_1 - P_2$ (W)	22.2	25.6	23.7	27.3
P_{Cu1} (W)	0.173	0.172	0.457	0.461
P_{Cu2} (W)	0.086	0.087	0.171	0.173
P_h (W)	21.9	/	23.1	/
P_e (W)	3.4	/	3.6	/

losses and eddy current losses, with assumption that all other losses are zero. With this assumption the sum of measured hysteresis and eddy current losses could be calculated using (17).

$$\begin{aligned} P_{h\ meas} + P_{e\ meas} &= P_{1\ meas} - P_{2\ meas} \\ &\quad - P_{Cu1\ meas} - P_{Cu2\ meas} \end{aligned} \tag{17}$$

The eddy current losses P_e can now be determined from measured and from the transformer model calculated losses using (18).

$$P_{e\ sim} = P_{h\ meas} + P_{e\ meas} - P_{h\ sim} \tag{18}$$

By using calculated values it can be easily determined that when the eddy current losses represent 13.3 % of all losses when transformer is loaded with linear load.

Transformer losses were also calculated for the case when nonlinear load was used. Nonlinear load was composed of resistor, inductor and a diode rectifier. The resistor and inductor were selected in such a way that the secondary active power was the same as in the case of linear load. Use of diode rectifier causes high order harmonics in currents, which means that the current

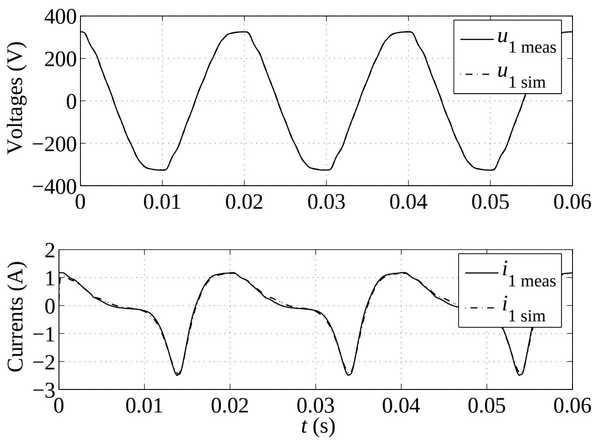

Fig. 7 Measured and calculated primary voltages and currents when nonlinear load is considered

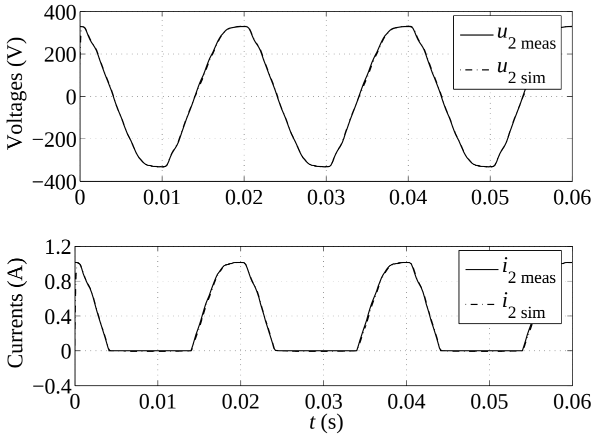

Fig. 8 Measured and calculated secondary voltages and currents when nonlinear load is considered

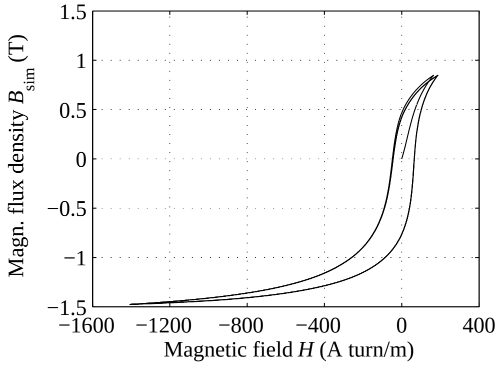

Fig. 9 Calculated B-H hysteresis loop for the case when nonlinear load is used

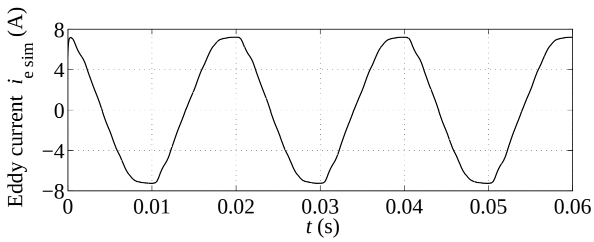

Fig. 10 Eddy current waveform

waveform are highly distorted. Again measurements and simulations were performed and results were compared. In simulations nonlinear transformer model with J-A hysteresis model from Fig. 1 and 2 was used. Diode rectifier was included by inserting nonlinear characteristic $R_d(i_2)$ in the model. Results of calculated voltages and currents are shown together with measured ones in Fig. 7 and Fig. 8. Again good agreement between measured and calculated values are noticed. For given case $B - H$ hysteresis loop is shown in Fig. 9. Using simulation results and measurements, losses were calculated according to (14) - (18) and presented in Tab. 1. Eddy current losses were again calculated as previously determined percentage of all losses. In this way calculated eddy current losses are $P_{e\ sim} = 3.63$ W, which is similar as presented in the table. In order to determine the eddy current that causes corresponding losses, the short circuit winding with one turn $N_e = 1$ and resistance $R_e = 0.1304\ \Omega$ was considered. Eddy current was then calculated using (19):

$$i_{e\ sim} = A \frac{N_e}{R_e} \frac{dB}{dt} \tag{19}$$

where term dB/dt was captured from simulations as could be seen in Fig 2. Time plot of the eddy current is shown in Fig. 10.

5 Conclusion

This work focuses on development of a nonlinear transformer model appropriate for calculating the hysteresis, eddy current and copper losses that arises in the transformer. To assure that from the model calculated losses reflect real transformer losses, calculated voltages and currents were compared with measured ones. This was done for the case when transformer was loaded with linear and nonlinear load. For both cases transformer losses were calculated and compared with measured losses. Based on the results, the increase of transformer losses due to nonlinear load was also determined. From the model calculated increase of transformer losses is 6.3 %, while measured increase of losses is 6.2 %. These results show that the obtained transformer model is appropriate for loss calculation in the case of linear loads and even in the case of nonlinear loads where higher order harmonics in voltages and currents are present.

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