CROP PLANNING UNDER RISK AND ENVIRONMENTAL CONSTRAINTS

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Abstract

In the paper is presented an overview of the research in mathematical modeling in agriculture and a mathematical model for crop planning which includes climate risk, market risk and environmental risk. The model is a multi-objective stochastic programming model that contains two types of levels for the application of fertilizers/pesticides: a maximum admissible level and a desirable level. It is based on portfolio theory. The above mentioned levels are introduced in order to find crop plans that comply to environmental constraints. The model considers penalties proportional to the overcoming of the desirable environmental levels and several classes of land quality. Safety-first type constraints on the quantity of crops are considered. If they are approximated with the help of empirical distribution functions of the crop yields then one obtains a problem with cardinality type constraints. The land productivity coefficients are random variables that incorporate weather risks. The crops market prices are random variables that incorporate the market risk. Simulations starting from the historical data or from scenarios randomly generated on land productivity coefficients and crops market prices are performed. Several variants and special cases of the multi-objective model are formulated: the minimum environmental risk problem, the minimum financial risk problem and the maximum return problem.

Keywords: sustainable agriculture, mathematical models, crop planning, risk, portfolio theory, multiobjective programming model.

Presenting Author's biography

Radulescu Marius was born in Bucharest, Romania. He graduated from Faculty of Mathematics, University of Bucharest in 1977. He holds a PhD in mathematics from Centre of Mathematical Statistics, Bucharest in 1985. At present he is a senior research worker at the Institute of Mathematical Statistics and Applied Mathematics "Gheorghe Mihoc-Caius Iacob" in Bucharest. His scientific interests are connected with: mathematical modelling, risk management, multiple criteria decision making, mathematics of finance, optimisation theory, nonlinear functional analysis and its applications to boundary value problems for differential equations, real analysis, numerical analysis, approximation theory. In 1991 he was awarded a prize of the Romanian Academy for contributions to global inversion theorems and applications to boundary value problems.

1 Mathematical models applied in agriculture

Agricultural production is vital for a nation's security and for its economic welfare. It is subjected to various risks. We mention here a few of them: weather risks, pests risks, diseases risks, market risks, the interaction of technology with other farm and management characteristics risks, genetics risks, machinery efficiency risk and the quality of inputs risks.

Increasingly, farmers almost the world over are being exposed to unpredictable competitive markets for inputs and outputs, so that price or market risk is often significant and may increase over time. Market risk includes the risks generated by the unpredictable currency exchange rates.

Sound planning determines a favorable outcome or yield for the farmer and mitigation of the risks.

The great majority of the problems connected to sustainable agriculture have a multi-criteria character. The concept of sustainable agriculture supposes harmonization or simultaneous realization of the objectives connected to economic growth and environment.

The uncertainty from the agriculture problems is modeled with the help of probability theory. Many of the practical problems that occur in agriculture are stochastic programming problems with multiple objectives.

In practice, in the process of mathematical modeling, one cannot take into account all the factors that have an impact to agricultural production. The number of such factors is large and the growth of their number determines the rapid growth of the complexity of the models.

Among the mathematical models applied in agriculture one can quote prediction models, crop planning models, optimal selection of fertilizer/pesticide models, crop rotation models etc.

An important mathematical instrument which was successfully applied to modeling the problems from agriculture was portfolio theory. The above mentioned theory was developed as a result of the research in the domain of financial management. The application of portfolio theory for finding an optimal allocation of agricultural land is popular in the literature. In Newbery and Stiglitz [7], Schaefer [11], Hardaker [4], Hazell and Norton [5] and Blank [1] were presented or applied various variants of portfolio theory to the land allocation decisions. In Collender [2], Romero [9], [10], were studied several models for resources allocation in agriculture that are taking into account specific risks. Mathematical models that take into account farmers decisions and climate change were studied in Lewandrowski and Brazee [6]. Applic ations of portfolio theory to biodiversity conservation were studied in Figge [3]. For other references regarding applications of portfolio theory to agriculture see Radulescu [8].

2 A crop planning model for sustainable agriculture

In the following is formulated a stochastic programming model with multiple objectives and mixed variables, that is continuous variables (real variables) and integer variables for crop planning in agriculture. The model takes into account weather risks, market risks and environmental risks. Input data include historical land productivity data for various crops and soil types and yield response to fertilizer/pesticide application. Some special cases of the model are discussed.

The application of fertilizers/pesticides is desirable since they contribute to the growth of agricultural production. On the other side the application of fertilizers/pesticides in great quantities, over some levels bring damages to environment and human health.

In order to protect the environment and of course the people's health one considers two kinds of levels: desirable levels and maximum admissible levels for the application of fertilizers/pesticides.

Consider a farm which has an agricultural land divided into several plots. Let $P_1, P_2, ..., P_m$ be the plots from the farm's land. We consider that if a plot is cultivated then it is cultivated with the same crop. Also we consider that the soil quality of a plot is homogeneous. Denote by S_j the area of the plot P_j j=1, 2,..., m. We consider that the farmer have to choose a crop plan from n crops $C_1, C_2, ..., C_n$. In order to obtain high yields the farmer uses fertilizers/pesticides. Denote by k the number of fertilizers and pesticides used by the farmer. For the fertilizer/pesticide r denote by q_{1ir} (respectively q_{2ir}) the desirable level for the application of the fertilizer/pesticide r for a quantity of one unit of crop C_i (respectively the maximum admissible level for the application of the fertilizer/pesticide r for a quantity of one unit of crop C_i). Obvious $0 \le q_{1ir} \le q_{2ir}$. Let J_i be the cartesian product of intervals $[0, q_{2ir}]$, that is:

$$J_i = [0, q_{2i1}] \times [0, q_{2i2}] \times \ldots \times [0, q_{2ik}].$$

Consider the probability space (Ω, K, P). Denote

$I=\{1,2,...,n\}$, $J=\{1,2,...,m\}$, $K=\{1,2,...,k\}$. For every $i\in I$, $j\in J$ we define the plot productivity functions $c_{ij}:\Omega\times J_i\to\mathbf{R}_+$ and the market price functions $b_i:\Omega\to\mathbf{R}_+$. Thus if $\mathbf{q}$ is a vector from J_i then the function $\omega\to c_{ij}(\omega,\mathbf{q})$ is a random variable. Analogous all the functions b_i are random variables. $c_{ij}(\cdot,\mathbf{q})$ represents the quantity of crop C_i that can be produced on a unit area of plot P_j and b_i the market price for a quantity of one unit of crop C_i. Let a_{ij} be the sum of money used by the farmer in order to cultivate crop C_i without fertilizers/pesticides on the plot P_j. For every $i\in I$, $j\in J$, $r\in K$ denote by:

- d_r - the cost of a quantity of one unit from the fertilizer/pesticide *r*

- c_{ijr} - the plot productivity coefficient, that is the quantity of crop C_i obtained from one unit area of plot P_j when a quantity of one unit of fertilizer/pesticide *r* is used.

- c_{ij0} the quantity of crop C_i that it is obtained from one unit of area of plot P_j

- w_{ijr} - the penalization coefficient for overcoming the desirable level q_{1ir}.

- y_{ijr} - the decision variable representing the quantity of fertilizer/pesticide *r* used for the cultivation of one unit area of plot P_j with crop C_i. Denote $\mathbf{y}_{ij}=(y_{ij1},y_{ij2},\ldots,y_{ijk})$.

- x_{ij} - the decision variable that takes the value 1 if the crop C_i is cultivated on plot P_j and takes value zero if the crop C_i is not cultivated on the plot P_j.

- $[M_1,M_2]$ the range for the sum of money available for investment

- Q_i the inferior bound for the yield of crop C_i necessary to be obtained

- ε_i a safety coefficient for obtaining at least the yield Q_i of crop C_i

The yield of crop C_i obtained from the plot P_j when the decisions are given by the matrices $\mathbf{x}=(x_{ij})$ and $\mathbf{y}_{ij}=(y_{ij1},y_{ij2},\ldots,y_{ijk})$, $i\in I$, $j\in J$ is equal to $x_{ij}S_j\left(c_{ij0}+\sum_{r=1}^{k}c_{ijr}y_{ijr}\right)$.

Take $y_{ij0}=1$ for every $i\in I$, $j\in J$.

The yield of crop C_i is equal to $\sum_{j=1}^{m}\sum_{r=0}^{k}c_{ijr}(\cdot)S_jx_{ij}y_{ijr}$, $i\in I$.

In the model we consider safety-first type constraints on the crop yields.

More precisely we demand that the probability that the yield of crop C_i is greater than Q_i, is greater than $1-\varepsilon_i$. This condition is mathematically written as

$$P\left(\sum_{j=1}^{m}\sum_{r=0}^{k}c_{ijr}(\cdot)S_jx_{ij}y_{ijr}\ge Q_i\right)\ge 1-\varepsilon_i$$

The cost of applying the allocation $\mathbf{x}=(x_{ij})$ of crops to plots and the application plans for fertilizers/pesticides $\mathbf{y}=(\mathbf{y}_{ij})$, where $\mathbf{y}_{ij}=(y_{ij1},y_{ij2},\ldots,y_{ijk})$, $i\in I$, $j\in J$ is equal to

$$g(\mathbf{x},\mathbf{y})=\sum_{i=1}^{n}\sum_{j=1}^{m}a_{ij}x_{ij}+\sum_{r=1}^{k}d_r\left(\sum_{i=1}^{n}\sum_{j=1}^{m}S_jy_{ijr}x_{ij}\right).$$

The restrictions $\sum_{i=1}^{n}x_{ij}\le 1$, $j\in J$ show that every plot is cultivated with at most one crop.

In our model we shall make the following assumption:

$$c_{ij}(\omega,\mathbf{q})=c_{ij0}(\omega)+\sum_{r=1}^{k}c_{ijr}(\omega)q_r \quad \text{where}$$

$\mathbf{q}=(q_1,q_2,...,q_k)$

The return (respectively the expected return) obtained when the decision matrices $\mathbf{x}$ and $\mathbf{y}$ are used is equal to

$$\psi(\cdot,\mathbf{x},\mathbf{y})=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k}b_ic_{ijr}x_{ij}S_jy_{ijr} \quad \text{(respectively)}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k}E(b_ic_{ijr})x_{ij}S_jy_{ijr}$$

For every $i,\alpha\in I$, $j,\beta\in J$, $r,\gamma\in K$ denote $\rho_{ijr\alpha\beta\gamma}=E[b_ic_{ijr}b_\alpha c_{\alpha\beta\gamma}]-E[b_ic_{ijr}]\cdot E[b_\alpha c_{\alpha\beta\gamma}]$

Then the variance of the return is equal to $\mathrm{Var}(\psi(\cdot,\mathbf{x},\mathbf{y}))=$

$=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{m}\sum_{\gamma=0}^{k}\rho_{ijr\alpha\beta\gamma}x_{ij}x_{\alpha\beta}y_{ijr}y_{\alpha\beta\gamma}S_jS_\beta$ The variance of the return has the meaning of financial risk. Denote by t_+ the positive part of the real number

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t, that is: $t_+ = \frac{|t|+t}{2} = \max(t,0)$. The environmental risk is equal to $f_2(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{k} w_{ijr}\left(y_{ijr} - q_{1ir}\right)_+$.

One can easily see that there exists a penalization proportional to the amount of fertilizer/pesticide applied in excess over the desirable level.

The environmental risk is a nonsmooth function of $\mathbf{y} = (\mathbf{y}_{ij})$, where $\mathbf{y}_{ij} = (y_{ij1}, y_{ij2}, \ldots, y_{ijk})$, $i \in I$, $j \in J$

The farmer intends to obtain optimal production plans that minimize the environmental penalizations, minimize the financial risk and maximize the expected return. According to the above requirements the multiobjective stochastic programming model for the crop planning is the following:

$$\begin{cases} \min\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{k} w_{ijr}\left(y_{ijr} - q_{1ir}\right)_+\right) \\ \min(\operatorname{Var}(\psi(\cdot,\mathbf{x},\mathbf{y}))) \\ \max\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k} E(b_i c_{ijr})x_{ij}S_j y_{ijr}\right) \\ M_1 \le g(\mathbf{x},\mathbf{y}) \le M_2 \\ P\left(\sum_{j=1}^{m}\sum_{r=0}^{k} c_{ijr}(\cdot)S_j x_{ij} y_{ijr} \ge Q_i\right) \ge 1-\varepsilon_i,\ i \in I \\ 0 \le y_{ijr} \le q_{2ir}x_{ij},\ i \in I,\ j \in J,\ r \in K \\ \sum_{i=1}^{n} x_{ij} \le 1, \text{ for every } j \in J, \\ x_{ij} \in \{0,1\},\ i \in I,\ j \in J \end{cases}$$

If the above safety-first constraints are approximated with the help of empirical distribution functions of the crop yields, we obtain cardinality type constraints.

Mathematical programming problems with the above mentioned constraints are very difficult to solve. Recently, simulation-based methods and heuristic algorithms have been successfully used for solving such problems. In our paper we investigate several approaches in order to solve the problems with safety-first constraints. Some numerical results obtained by computer simulation are discussed.

We consider several particular cases of the above stochastic programming model for the crop planning.

More precisely we shall consider a minimum environmental risk problem, a maximum return problem and a minimum return risk problem.

3 The minimum environmental risk problem

In the frame of this problem the farmer tries to find an optimal allocation of crops to plots and an optimal plan for fertilizer and pesticide application that minimize the environmental risk taking into account that

- the financial risk, that is $\operatorname{Var}(\psi(\cdot,\mathbf{x},\mathbf{y}))$, is smaller than a prescribed level τ

- the expected return is greater than a given level W

- the cost of cultivation and application of the fertilizers and pesticides lies in the range $[M_1, M_2]$

- the probability that the yield of crop C_i is greater than Q_i, is greater than $1-\varepsilon_i$

$$\begin{cases} \min\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{k} w_{ijr}\left(y_{ijr} - q_{1ir}\right)_+\right) \\ \operatorname{Var}(\psi(\cdot,\mathbf{x},\mathbf{y})) \le \tau \\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k} E(b_i c_{ijr})x_{ij}S_j y_{ijr} \ge W \\ M_1 \le g(\mathbf{x},\mathbf{y}) \le M_2 \\ P\left(\sum_{j=1}^{m}\sum_{r=0}^{k} c_{ijr}(\cdot)S_j x_{ij} y_{ijr} \ge Q_i\right) \ge 1-\varepsilon_i,\ i \in I \\ 0 \le y_{ijr} \le q_{2ir}x_{ij},\ i \in I,\ j \in J,\ r \in K \\ \sum_{i=1}^{n} x_{ij} \le 1, \text{ for every } j \in J, \\ x_{ij} \in \{0,1\},\ i \in I,\ j \in J \end{cases}$$

With an introduction of some additional variables the objective map can be transformed in a linear map. Thus the minimum environmental risk problem is equivalent to the following problem with safety-first constraints:

$$\begin{cases} \min\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{k} w_{ijr} z_{ijr}\right) \\ y_{ijr} - q_{1ir} \le z_{ijr} \\ \operatorname{Var}(\psi(\cdot,\mathbf{x},\mathbf{y})) \le \tau \\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k} E(b_i c_{ijr})x_{ij}S_j y_{ijr} \ge W \\ M_1 \le g(\mathbf{x},\mathbf{y}) \le M_2 \\ P\left(\sum_{j=1}^{m}\sum_{r=0}^{k} c_{ijr}(\cdot)S_j x_{ij} y_{ijr} \ge Q_i\right) \ge 1-\varepsilon_i,\ i \in I \\ 0 \le y_{ijr} \le q_{2ir}x_{ij},\ i \in I,\ j \in J,\ r \in K \\ \sum_{i=1}^{n} x_{ij} \le 1, \text{ for every } j \in J, \\ x_{ij} \in \{0,1\},\ z_{ijr} \ge 0,\ i \in I,\ j \in J \end{cases}$$

Recall that the land productivity coefficients c_{ijr} are random variables. They incorporate the weather risks. Historical data on productivity coefficients, that is the numbers c_{ijrt} = the land productivity for crop C_i, plot P_j, fertilizer/pesticide r at a moment t can be considered as realizations of the random variables c_{ijr}. The market price of crops coefficients b_i are random variables that incorporate the market risk. Historical data on market price of crops, that is the numbers b_{it} = the market price for one unit of crop C_i at moment t can be considered as realizations of the random variables b_i.

Taking into account the above things, the number

$$\frac{1}{T}\text{card}\left\{\sum_{j=1}^{m}\sum_{r=0}^{k} c_{ijrk} S_j x_{ij} y_{ijr} \geq Q_i\right\}$$

is an empirical estimation of probability that the yield of crop C_i is greater than Q_i.

Here T is the number of moments of time in the time horizon considered.

Taking into account this estimation the above minimization problem can be transformed in a problem with cardinality constraints.

We have obtained a very complex problem for which simulation is a common approach. We made simulations for various scenarios regarding land productivity and crop market prices. The result of the simulations show that small land productivities (this is the case when drought occurs), imply an intensive use of fertilizers and pesticides, which determine high environmental penalizations for the decisions that are taken.

4 The maximum return problem

Denote $f_1(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k} E(b_i c_{ijr}) x_{ij} S_j y_{ijr}$

$$f_2(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{k} w_{ijr}\left(y_{ijr} - q_{1ir}\right)_+$$

$$f(\mathbf{x},\mathbf{y}) = f_1(\mathbf{x},\mathbf{y}) - f_2(\mathbf{x},\mathbf{y})$$

In the frame of this problem the farmer tries to find an optimal allocation of crops to plots and an optimal plan for fertilizer and pesticide application that maximize the difference between the expected return and the monetary penalization for the environmental risk taking into account that

- the financial risk, that is $\text{Var}(\psi(\cdot,\mathbf{x},\mathbf{y}))$, is smaller than a prescribed level τ

- the cost of cultivation and application of the fertilizers and pesticides lies in the range $[M_1, M_2]$

- the probability that the yield of crop C_i is greater than Q_i, is greater than $1-\varepsilon_i$

$$\begin{cases} \max(f(\mathbf{x},\mathbf{y})) \\ \text{Var}(\psi(\cdot,\mathbf{x},\mathbf{y})) \leq \tau \\ M_1 \leq g(\mathbf{x},\mathbf{y}) \leq M_2 \\ P\left(\sum_{j=1}^{m}\sum_{r=0}^{k} c_{ijr}(\cdot) S_j x_{ij} y_{ijr} \geq Q_i\right) \geq 1-\varepsilon_i,\ i \in I \\ 0 \leq y_{ijr} \leq q_{2ir} x_{ij},\ i \in I,\ j \in J,\ r \in K \\ \sum_{i=1}^{n} x_{ij} \leq 1, \text{ for every } j \in J, \\ x_{ij} \in \{0,1\},\ i \in I,\ j \in J \end{cases}$$

5 The minimum financial risk problem

In the frame of this problem the farmer tries to find an optimal allocation of crops to plots and an optimal plan for fertilizer and pesticide application that Minimize the financial risk taking into account that

- the environmental risk is smaller than a prescribed level η

- the expected return is greater than a given level W

- the cost of cultivation and application of the fertilizers and pesticides lies in the range $[M_1, M_2]$

- the probability that the yield of crop C_i is greater than Q_i, is greater than $1-\varepsilon_i$

$$\begin{cases} \min(\text{Var}(\psi(\cdot,\mathbf{x},\mathbf{y}))) \\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{k} w_{ijr}\left(y_{ijr} - q_{1ir}\right)_+ \leq \eta \\ \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=0}^{k} E(b_i c_{ijr}) x_{ij} S_j y_{ijr} \geq W \\ M_1 \leq g(\mathbf{x},\mathbf{y}) \leq M_2 \\ P\left(\sum_{j=1}^{m}\sum_{r=0}^{k} c_{ijr}(\cdot) S_j x_{ij} y_{ijr} \geq Q_i\right) \geq 1-\varepsilon_i,\ i \in I \\ 0 \leq y_{ijr} \leq q_{2ir} x_{ij},\ i \in I,\ j \in J,\ r \in K \\ \sum_{i=1}^{n} x_{ij} \leq 1, \text{ for every } j \in J, \\ x_{ij} \in \{0,1\},\ i \in I,\ j \in J \end{cases}$$

6 Conclusions

The instability (risk) in the agricultural production is one of the most important problems in the development dynamics of the human society. Wide fluctuations in crop output not only affect prices and bring about sharp fluctuation in them but also results in wide variations in disposable income of the farmers. The magnitude of fluctuations depends on the nature of crop production technology, its sensitivity to weather, economic environment, availability of material inputs and many other factors. The present paper proposes a multiobjective model based on portfolio theory for an optimal crop planning. The objectives of the model are connected to expected return maximization, environmental risk minimization and financial risk minimization. Starting from the multiobjective model three single objective optimization problems are defined. All the problems formulated have safety first type constraints. These kinds of constraints are approximated with cardinality type constraints.

The problems that are obtained are very complex and simulation is a common approach in order to solve them. We report some simulation results regarding the minimum environmental risk problem.

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