MODELLING DAMPING EFFECTS IN VEHICLE-TRACK INTERACTION

Michele Buonsanti[1], Giovanni Leonardi[2], Francesco Scopelliti[2]

[1]University of Reggio Calabria, Faculty of Engineering
89100 Reggio Calabria, Via Graziella, Italy
Department Mechanics and Materials
[2]University of Reggio Calabria, Faculty of Engineering
89100 Reggio Calabria, Via Graziella, Italy
DIMET Department

michele.buonsanti@unirc.it (Michele Buonsanti)

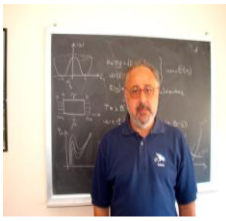
Abstract

It is a well-known fact that railway activities inevitably generate vibrations in the track structure and the sub-ground, which may have negative effects on the surrounding environment and constructions.
Because of the very serious effects that unwanted vibrations can have on dynamic system, it is essential that vibration analysis be carried out as an inherent part of their design; when necessary modifications can easily be made to eliminate vibration or at least to reduce it as much as possible. It is usually much easier to analyze and modify a structure with undesirable vibration characteristics after it has been built.
However, it is sometimes necessary to be able to reduce the vibration of existing structures brought about by inadequate initial design, by changing the function of the structure or by changing the environmental conditions, and therefore techniques for the analysis of structural vibration should be applicable to existing structures as well as to those in the design stage.
In general present-day structures often contain high energy sources which create intense vibration excitation problems. The level of vibration in a structure can be attenuated by reducing either the excitation, or the response of the structure to that excitation or both. It is sometimes possible, in the design stage, to reduce the exciting force or motion by changing the equipment responsible, by relocating it within the structure or by isolating it from the structure so that the generated vibration is not transmitted to the support.
In this paper we analyze the effect of the vertical damper on the global system vehicle-wheel-track when, load and empty conditions appear in vehicle. A final comparison is carried-out among damping and undamping vehicle systems.

Keywords: railway track, damping, vibration.

Presenting Author's biography

Michele Buonsanti. Degree in Architecture, and specialized in Computational Mechanics of Materials and Structures, is Assistant Professor with the Department of Mechanics and Materials of the Faculty of Engineering of the University of Reggio Calabria. He is author and co-author of many scientific papers relative to mechanics of materials particularly, biomechanics, microstructure and variational elasticity.



1 General

In order to optimize the positive aspects of railway transport, it is necessary to conjugate an adequate management of the system with an in depth analysis of the environmental compatibilities with the ecosystem of the closed areas; this also with reference to the aspects of the induced dynamic pressures from the passage of the convoys and the propagation of the acoustic waves in the interested area.

To this aim, in railway transport it is possible to identify three different categories of vibration-acoustic phenomena, according to the principal centre of generation/propagation of the pressure wave: aerial (air-borne), structural (structure-borne) or in the ground (ground-borne).

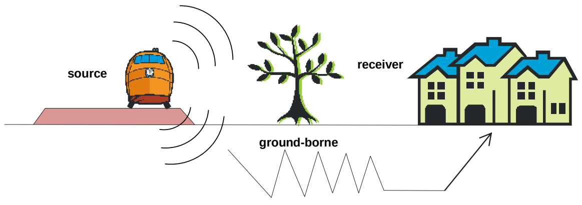


Fig. 1 Ground-borne vibration problem

Although ground–borne vibration from train traffic is very unlikely to cause damage to the buildings and structures, the economical and environmental aspects of the issue deserves careful consideration. Besides high maintenance cost due to excessive vibration in the railway structure, ground-borne vibration may cause annoyance to the people living near the railway or interfere with the operation of sensitive equipment.

Therefore preparing an environmental impact assessment prior to building new railway lines through densely populated areas or upgrading the existing ones to be used by heavier or faster trains is becoming more common nowadays.

Particular attention is set on freight trains transit. Heavy freight trains cause ground vibrations with predominant frequency components in the range of 4-30 Hz. At these frequencies, if the amplitude is great enough, the vibration is felt. This is a cause of disturbance to some line side residents, who may also express concern about possible damage to their property [1].

2 Dynamic Principles

2.1 Theoretical backgound

When considering dynamic aspects of track one should realise that dynamics is in fact the interaction between load and structure. Loads vary in time and the way this happens determines the character of the load. Generally speaking, distinctions can be made between periodic loads, impact loads and general dynamic loads [2].

The simplest dynamic model is the so-called discrete one-mass spring with mass m, spring constant k, and damping c. The single degree of freedom system is governed by the differential equation

$$mu_{tt} + cu_t + ku = F(t) \qquad (1)$$

A particular class of dynamic loading in the single degree of freedom system can be considered the impulsive loads [2]. Such a load consists of a single principal impulse, generally of relatively short duration. Impulsive or shock loads frequently are of great importance in the design of still class of structural systems. The procedure for approximating the response of a structure to a short duration impulse may be used as the basis for developing a formula for evaluating response to a general dynamic loading. Consider the arbitrary general loading $F(t)$, specifically the intensity of loading $F(\tau)$ acting a time $t = \tau$. This load acting during the short interval of time $d\tau$ produces a short duration impulse $F(\tau)d\tau$ on the structure. It should be noted carefully that although the procedure is only approximate for impulse of finite duration, it becomes exact as the duration of loading approaches zero. Thus for the differential time interval $d\tau$ the response produced by the loading $F(\tau)$ is exactly $(t > \tau)$.

$$du(t) = \frac{p(\tau)d\tau}{m\omega} sen\omega(t-\tau) \qquad (2)$$

In (2), the term $du(t)$ represents the differential response to the differential impulse over the entire response history for $t > \tau$. The entire loading history may be considered to consist of a succession of such short impulses. For this linearly elastic system, then, the total response can be obtained by

$$u(t) = \frac{1}{m\omega}\int_0^t F(\tau) sen\,\omega(t-\tau)d\tau \qquad (3)$$

The (3), defined as *Duhamel integral*, represents the solution of equation (1) that can be found in the time domain using the convolution integral involving the unit impulse response. Then (3) has the form

$$u(t) = \int_0^t F(\tau)h(t-\tau)d\tau \qquad (4)$$

where

$$h(t-\tau) \equiv \frac{1}{m\omega} sen\omega(t-\tau) \qquad (5)$$

is called convolution integral. The function, on the left hand side of (5) is generally referred to as unit-impulse response in undamped system. In the damped systems, the Duhamel integral is similar to the undamped analysis except that the free vibration response initiated by the differential load impulse $F(\tau)$ $d\tau$ is subjected to exponential decay, namely

$$du(t) = e^{-\xi\omega(t-\tau)}\left[\frac{p(\tau)}{m\omega} sen\,\omega(t-\tau)\right] \qquad (6)$$

In fact it is more convenient to perform an analysis in the frequency domain, because this approach involves expressing the applied loading in terms of harmonic components, evaluating the response of the structure to each component, and then superposing of the harmonic response to obtain the structural response. A better way to resolve (1) is, even in simple case, to use the Fourier transform to determine the so-called frequency response function or transfer function H of the system. This function type describes the relationship between response and excitation in the frequency domain. When the force as function of time is known, a Fourier transform can be made and the response then simply follows from a multiplication of the transformed force by the transfer function. To find the frequence domain response, for non periodic and arbitrary function $F(t)$, we utilize the classical Fourier transform FT in the form

$$\mathcal{F}(\omega) = \int_{-\infty}^{+\infty} F(t)e^{-i\omega t} \tag{7}$$

under condition that

$$\int_{-\infty}^{+\infty} |F(t)| dt < \infty \tag{8}$$

The formula (7) represents the harmonic components distribution of $F(t)$ or, as currently the $F(t)$ spectre. Putting the canonical form of the (1)

$$u_{tt} + 2\xi\omega_o u_t + \omega_o^2 u = F(t) \tag{9}$$

applying the FT we find the algebraic formulation

$$(-\omega^2 + 2\xi\omega_o\omega + \omega_o^2)U(\omega) = \mathcal{F}(\omega) \tag{10}$$

with the positions

$$\mathcal{F}(u(t)) = U(\omega)$$

$$\mathcal{F}(u_t) = i\omega U(\omega) \tag{11}$$

$$\mathcal{F}(u_{tt}) = -\omega^2 U(\omega)$$

Introducing the $H(\omega)$ function in the formula (10) we have

$$U(\omega) = H(\omega)\, F(\omega) \tag{12}$$

$H(\omega)$ is currently called the *transfer function* and in the unit impulse load we have

$$H(\omega) = U(\omega)/F(\omega) = \frac{1/k}{1 - \dfrac{\omega^2}{\omega_n^2} + i2\xi\dfrac{\omega}{\omega_n}} \tag{13}$$

in which

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad and \quad \xi = \frac{c}{2\sqrt{km}}$$

are the undamped natural frequency and the damping ratio, respectively. When modal analysis is carried out, the formulation problem assumes the general form

$$\mathbf{u}(t) = \mathbf{\Phi}\mathbf{q}(t) \tag{14}$$

where, according with [3] $\mathbf{q}(t)$ represents the solution of uncoupled differential system

$$\mathbf{q}_{tt}(t) + \mathbf{\Xi}\mathbf{q}_t(t) + \mathbf{\Omega}^2\mathbf{q}(t) = \mathbf{\Phi}^T\mathbf{F}(t) \tag{15}$$

in which $\mathbf{\Xi}$, $\mathbf{\Omega}$, $\mathbf{\Phi}$ are, respectively, the dissipation, spectral and modal matrix. Applying the FT in (15) hold

$$\mathbf{Q}(\omega) = \int_{-\infty}^{+\infty} \mathbf{q}(t)e^{-i\omega t}\, dt = \mathbf{H}(\omega)\,\mathbf{\Phi}^T\mathbf{F}(\omega) \tag{16}$$

The matrix $\mathbf{H}$ is a dynamics property transformation matrix, over the forcing response, and can be put as

$$\mathbf{H}(\omega) = [\mathbf{\Omega}^2 - \omega^2\mathbf{I} + i\omega\mathbf{\Xi}]^{-1} \tag{17}$$

the *j-th* component coincides with the transfer function of the *j-th* single oscillator. Finally, after solving (16) the FT for modal response appears as

$$\mathbf{U}(\omega) = \mathbf{\Phi}(\mathbf{H}(\omega)\mathbf{\Phi}^T\mathbf{F}(\omega)) = \mathbf{H}(\omega)\,\mathbf{F}(\omega) \tag{18}$$

Studying a complex oscillating system with damping, in the final paragraph we report the closed form solution for single system excitation. In particular we consider:

A-applied jump (time and intensity finite)

For our argumentation, for example this represents a typical defect in rail join. In this case we have two-distinct response namely, the first described by means

$$u(t) = \nu\,[1 - \varepsilon_2(t) - (\xi_o\omega_o/\varpi_o)\varepsilon_1(t)] \quad 0 < t \le t^* \tag{19}$$

in wich

$$\nu = F_o / k$$

$$\varepsilon_1(t) = exp(-\xi_o\omega_o t)sen(\varpi_o t) \tag{20}$$

$$\varepsilon_2(t) = exp(-\xi_o\omega_o t)cos(\varpi_o t)$$

where ϖ_o is called damped frequency, F_0 intensity of the jump in the (0-t^*) interval time. Successively we have the latter response when $t \ge t^*$

$$u(t) = \nu\{\varepsilon_2\Delta t - \varepsilon_2(t) + (\xi_o\omega_o/\varpi_o)[\varepsilon_1\Delta t - \varepsilon_1(t)]\} \tag{21}$$

B-Periodic forcing

This aspect can be regarded as surface irregularity on top of the rail. By [3], for any periodic forcing we can decompose in the harmonic components

$$F(t) = a_o + \Sigma_{1-\infty}\, a_n cos(n\omega_1 t) + \Sigma_{1-\infty}\, b_n sen(n\omega_1 t) \tag{22}$$

where a_o, a_n, b_n are the Fourier coefficients. Now, we consider a typical forcing over the time interval $(0$-$t_f)$

$$F(t) = F_o \quad 0 < t < t_{f/2} \tag{23}$$

$$F(t) = -F_o \quad t_{f/2} < t < t_f$$

Then, the periodic forcing assumes the form where a_o, a_n are equal to zero and b_n unlike to zero for n uneven.

$$F(t) = \frac{4F_o}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} sen(n\omega_f t) \qquad n = 1,3,5 \tag{24}$$

The system response assumes the form

$$u_n(t) = v\frac{4}{n\pi}\frac{\beta_n}{(1-\beta_n^2)^2+(2\xi_o\beta_n)^2} + \left[2\xi_0\varepsilon_2 + \frac{2\xi_o^2-1+\beta_n^2}{\sqrt{1-\xi_0^2}}\varepsilon_1 + \frac{1-\beta_n^2}{\beta_n} sen(n\omega_f t) - 2\xi_o \cos(n\omega_f t) \right] \tag{25}$$

Generally speaking, we apply early considerations to vehicle-track interaction and develop successively the particular response. In this case the interaction excitations are induced by irregularities in the wheel rail interface. In the most simplified form the equation of free motion appear as (plane *x*-*y*)

$$mu_{tt} + cu_t + ku = cy_t + ky \tag{26}$$

In order to determine the transformation function with *y(t)* as input and *u(t)* as output function, we use the Fourier transformation with the initial form

$$H_{y\to u}(\omega) = \frac{k + ci\omega}{k + ci\omega - m\omega^2} \tag{27}$$

and final form

$$H_{y\to u}(\omega) = -\frac{1 + i2\xi\frac{\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2} + i2\xi\frac{\omega}{\omega_n}} \tag{28}$$

Again, another interesting fact regards the transfer function among wheel force $\mathbb{F}^w$ in the rail

$$\mathbb{F}^w(t) = k\Delta + c\Delta_t \tag{29}$$

where $\Delta = u - y$, and in this case the transfer function hold

$$H_{\Delta\to\mathbb{F}}(\omega) = k + ci\omega \tag{30}$$

Following the transfer function between, input displacement *y* and output force $\mathbb{F}^w$ in the form

$$H_{y\to\mathbb{F}}(\omega) = \frac{m\omega^2(k + ci\omega)}{k + ci\omega - m\omega^2} \tag{31}$$

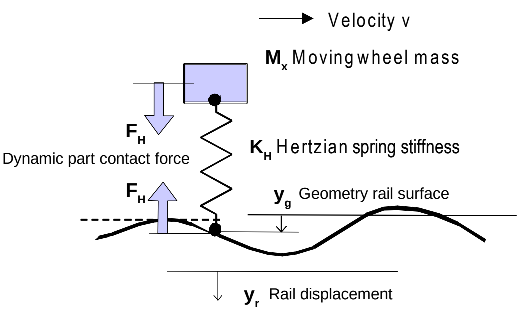


Fig. 2 Wheel-rail interaction (from [4])

Now, it's possible to consider the transfer function among wheel and rail, simulated by Hertzian spring and such that the relationship holds

$$\mathbb{F}^h = k_h\,[y_w - y_r - y_g] \tag{32}$$

as represented in Fig. 2.

Finally the global transfer function has the form

$$H_y(\omega) = -\,M_w[H_w(\omega) - H_r(\omega) - 1/k_h] \tag{33}$$

The (33) is the basis to show global interaction among subsystems as in Fig. 3

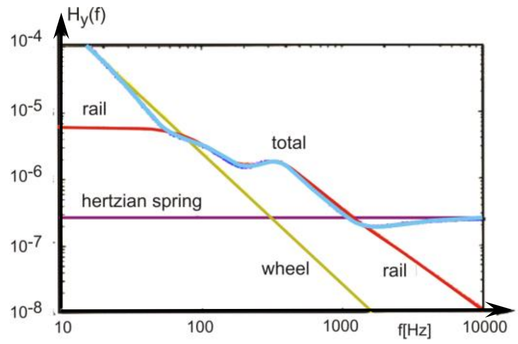


Fig. 3 Global transfer function (from [4])

2.2 Vehicle model

The main parts of the train, from a vibration generation point of view, are schematically shown in Fig. 4.

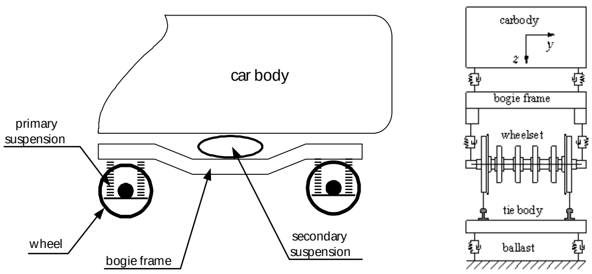


Fig. 4 Main parts of a train bogie

The car body is connected to the bogie by the secondary suspension; the weight of the car body is then transferred to the wheels via a bogie frame that is connected to the wheels by the primary suspension system. According to [4] the vehicle is modelled as a multibody system where the elements are considered rigid bodies joined by elastic and visco-elastic restraint. The equation of motion for the system can be written as

$$Mu_{tt} + Cu_t + ku = F_e(t) + \mathbb{F}^h \tag{34}$$

Where *M*, *C*, and *K* correspond to the mass, viscous damping and stiffness matrices of the car body, F_e and $\mathbb{F}^h$ are, respectively, the external forces and the contact force vectors. Particularly in this paper, a schematic representation, of the proposed model, follows in next Fig. 5

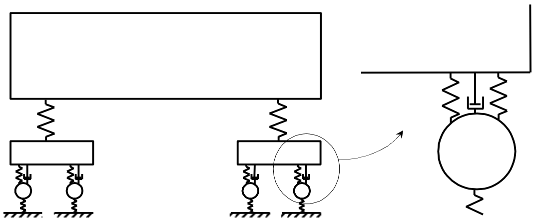


Fig. 5 Total freight models

The main core of the proposed modelling processes is the prediction of the dynamical forces at wheel and the consequential track response. Particularly, we model a *Shmms* freight car supposed to be travelling in rectilinear plane with high velocity. Initially two excitation conditions are considered namely, firstly a jump of 5 mm among two track sections; latter wiggles track with variable wave-length with amplitude equal to 1 mm. After this, consider us a complex condition for variable rail vertical displacement and we extend the analysis on the damping variable effects derived by wheel-rail exchange load. Another, vibrations effect is caused by the large forces between wheels and rails. These forces fluctuate in response to wheel and rail roughness over a wide range of frequencies. In fact, according to [1], irregularities in wheel and rail generate sharp peak response in the track-wagon global system.

The casual irregularities can be product of alternate excitation, in overload terms, defined as impulsive loads. Analytically, these actions are represented by *cos*-functions, and the harmonic excitation sources are defined. In previous cap.2 theoretical aspects to Hertzian contact forces has been carried-out. Now, we specify the questions to modelling the sub-system. The wheels in turn transfer the load to the rails as shown in Fig. 6.

Fig. 6 Contact forces between wheel and rail

The wheel-track contact model shows two type of non linearity, derived by means the following relationships

$$\mathbb{F}^h = k_h \, \Delta y_h^{1.5} \quad \Delta > 0 \qquad (35a)$$

$$\mathbb{F}^h = 0 \qquad \Delta < 0 \qquad (35b)$$

Since the (35a) present the 1.5 exponent values and (35b) put the unilateral conditions in the rail wheel contact, the coupled formulation (35 a&b) characterize a non linear formulation. About the damping property, we consider following force-velocity law:

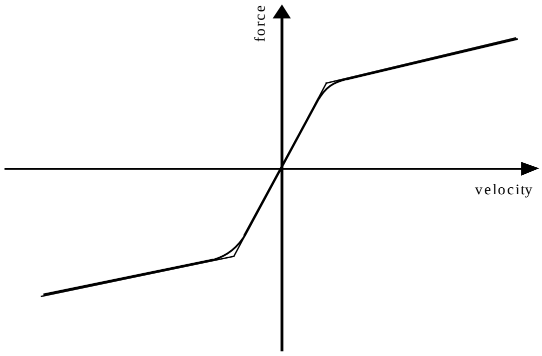


Fig. 7 Damping relation

3 Multi-body model

A complete model of a Shimms freight wagon has been developed. Three different subassemblies compose the model:

- the car-body sub-system;
- the front bogie sub-system;
- the rear bogie sub-system.

The car-body and the bogie frame are treated as rigid bodies and defined giving their mass characteristics, which are obtained taking into account the presence of auxiliary elements. The front and rear bogies are equal.

The single bogie is basically composed by the bogie frame, two wheelsets, suspensions and dampers connecting the bogie frame to the wheelsets and to the carbody. Masses of the components constituting the bogie such as auxiliary elements, suspensions and dampers, are reduced to the bogie frame except those of the wheelset and those of arms and axleboxes connecting bogie and wheelsets.

Primary and secondary suspensions are represented with elastic linear elements while the vertical and lateral dampers are treated as viscous non-linear elements; in particular the vertical dampers have the behaviour of Fig. 8.

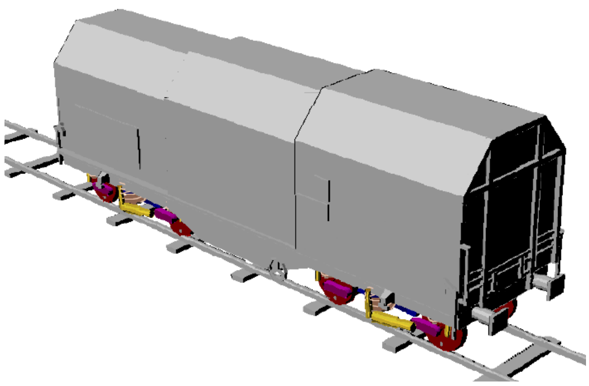


Fig. 8 Complete model of the Shimms wagon

In the following tables are reported characteristics of the model

Tab. 1 Mass data

Body	[Kg]
Wagon mass (tare)	22000
Wagon mass (laden)	100000
Bogie frame	2070
Wheelset	1500

Tab. 2 Stiffness data

	K [N/m]
primary suspension	$6.17 \cdot 10^5$
secondary suspension (tot.)	10^{10}

3.1 Simulation analysis

Simulations were done under two conditions:

Preload analysis was done to check for the static equilibrium of the vehicle and for checking that the vehicle has been modelled correctly. This was also done to confirm that since the vehicle is a symmetric vehicle, the various loads and deflections observed were equal and that the vehicle is balanced correctly.

Dynamic analysis of the Shimms was done at a speed of 160 km/h i.e. 44.4 m/s for 20 seconds of simulation time i.e. 889 meters of straight run.

The model was assembled and run successfully. There were no errors in the simulation. We assume that the dynamic wheel-rail forces are induced by irregularities in the wheel-rail interface.

The rail irregularities might include dipped joints and corrugations as well as general undulation in the track top. The wheel irregularities can be wheel flats, surface irregularities and wheel eccentricity. The variations in the vertical profiles of either surface (wheel and rail) introduce a relative displacement input to the system. The process is assumed to be linear, so that for a given wavelength λ, a displacement input is generated at the passing frequency $f = c/\lambda$, where c denotes the train speed [5].

Several simulations were performed changing the regularity conditions, in particular we analyse the followings:

- *Ramp irregularities*

In the simulations, the height of the ramp was 5mm and the ramp was outlined through a smoothed curve with an extension of 1m (Fig. 9).

We analyze the maximum load and the only tare conditions.

The simulation results are reported in terms of vertical forces on the wheelset, and because these loads are almost equal on the different wheelsets, only those relative to the front wheelset are illustrated (in fact, given as fixed the running conditions, the load on the wheel is the half of that on the wheelset).

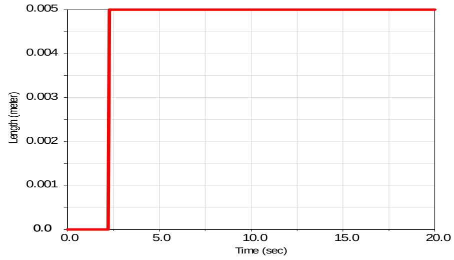


Fig. 9 Ramp irregularity

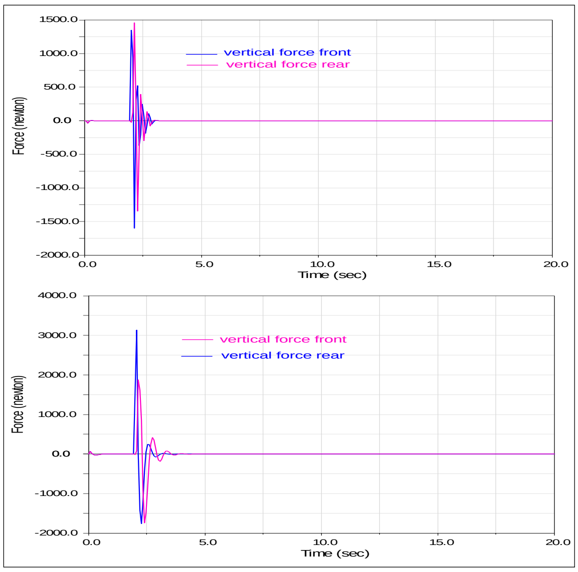


Fig. 10 Vertical force of PS (empty/full)

In Fig. 10 and Fig. 11, are also figured the displacements and the forces transmitted by the wagon with dumper to the principal suspension (PS).

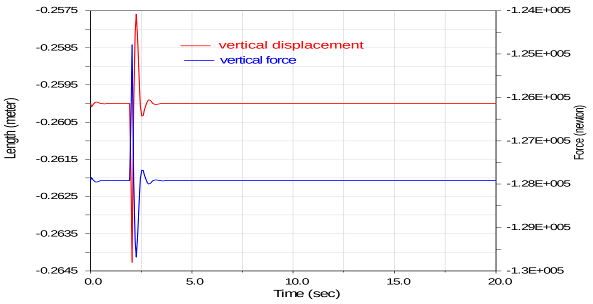


Fig. 11 Vertical force and displacement of PS in full condition

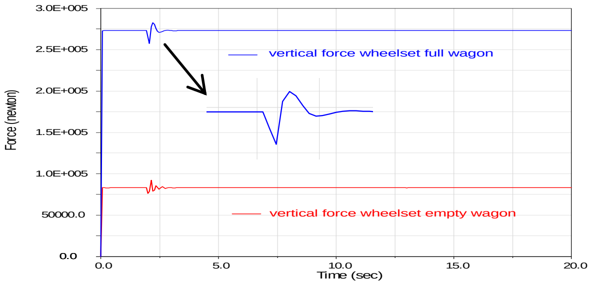


Fig. 12 Vertical force of front wheelset (with dumper)

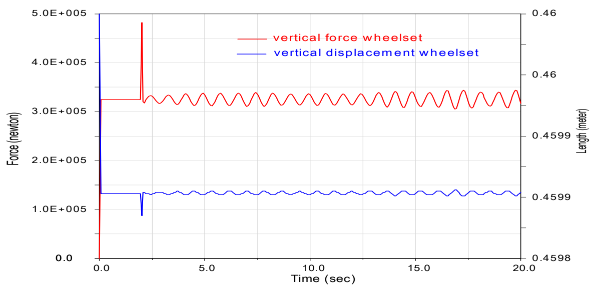


Fig. 13 Vertical force and displacement of front wheelset in full condition (without dumper)

The effects of the introduction of a damper are beneficial and completely obvious: the damper maintains the road–track contact and reduces the solicitations transmitted to the track (Fig. 12 and Fig. 13).

- *Sinusoidal irregularities*

The excitation due to the undulation of the railroad has been estimated assuming a sinusoidal irregularity with amplitude equal to 1 millimetre and length of 5 m (Fig. 14). Also in this case it was analyzed the maximum weight and size condition and then that one of tare (Fig. 15) and the solicitations transmitted with and without damper (Fig. 16).

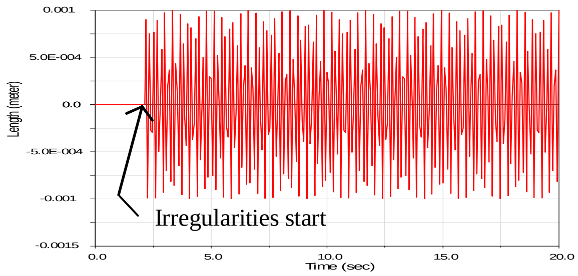


Fig. 14 Sinusoidal irregularity

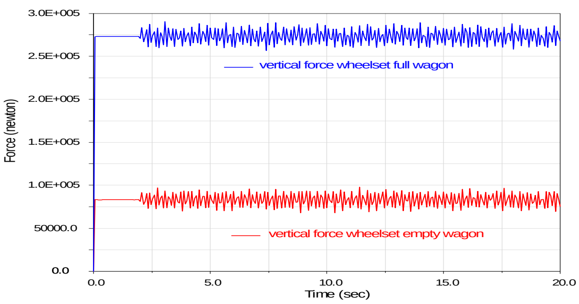


Fig. 15 Vertical force of front wheelset (with dumper)

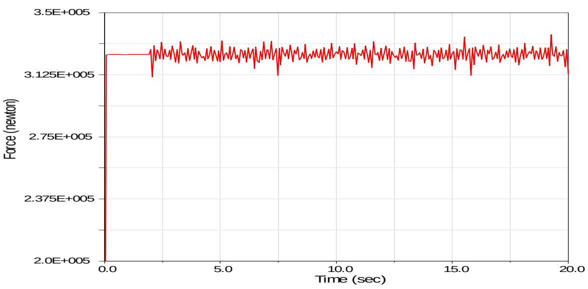


Fig. 16 Vertical force of front wheelset in full condition without dumper

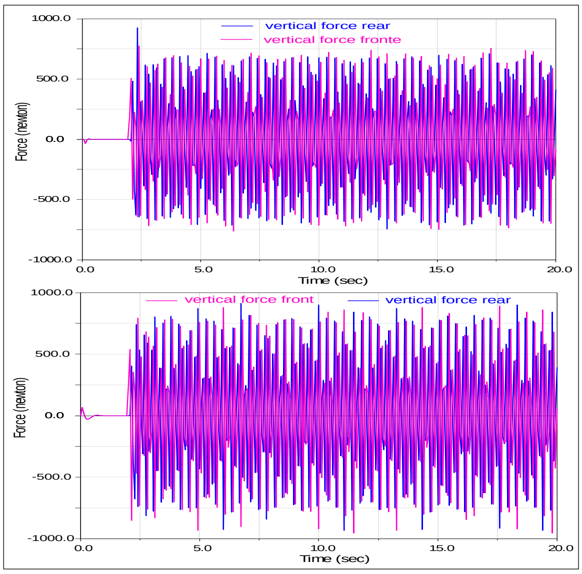


Fig. 17 Vertical force of front wheelset (empty/full) condition with dumper

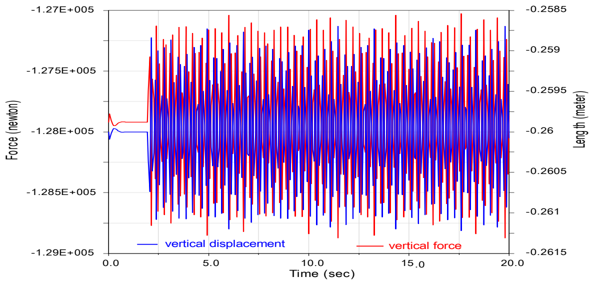


Fig. 18 Vertical force and displacement of PS in full condition

- *PSD irregularities*

Finally, a complex excitation supplied from a stocastic variability of the vertical lowering of the tracks is considered (Fig. 19); also in such case a study was completed on the effects of the introduction of the damper on the load exchanged between wheel and track.

For long wavelength the power spectral density of rail irregularities can be calculated by the expression [3]:

$$S_{yy}(\gamma) = \frac{A_v \cdot \gamma_c^2}{\left(\gamma^2 + \gamma_R^2\right)\cdot\left(\gamma^2 + \gamma_c^2\right)} \qquad (36)$$

with:

λ_c = 0.8246 cycles/m

λ_R = 0.0206 cycles/m

and the parameter A is function of the track conservation state:

$$A_v = \begin{cases} 4.032 \cdot 10^{-7} \quad \text{rad/m} \rightarrow \quad \text{for little irregularities} \\ 1.080 \cdot 10^{-7} \quad \text{rad/m} \rightarrow \quad \text{for great irregularities} \end{cases}$$

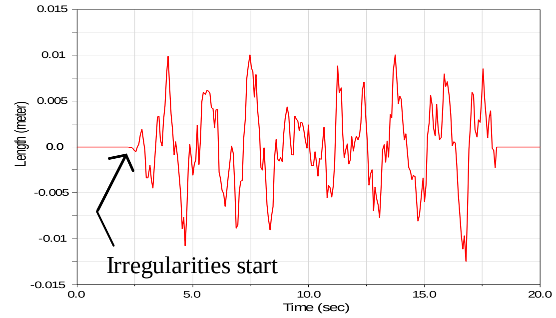


Fig. 19 PSD Irregularities

The following figures show the trend of the vertical load in time in the case of dampened system (Fig. 20) and in the case of damper absence (Fig. 21).

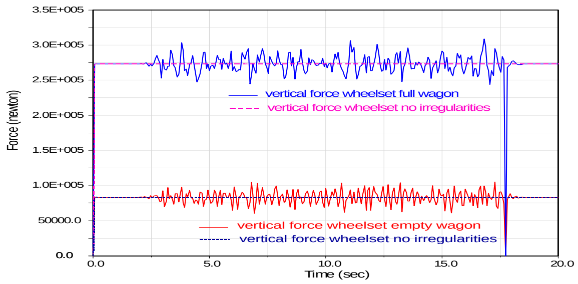


Fig. 20 Vertical force of front wheelset (with dumper)

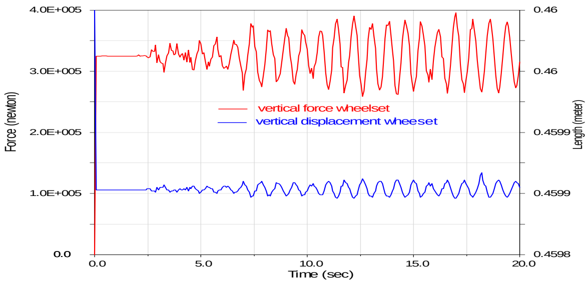


Fig. 21 Vertical force and displacement of PS in full condition (without dumper)

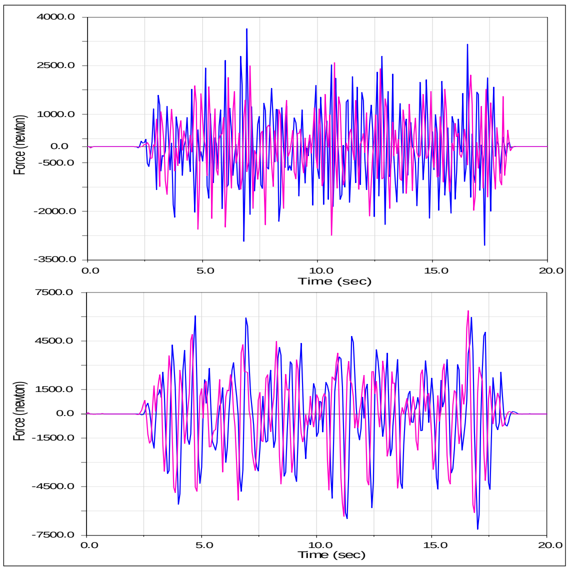


Fig. 22 Vertical force on PS (empty/full)

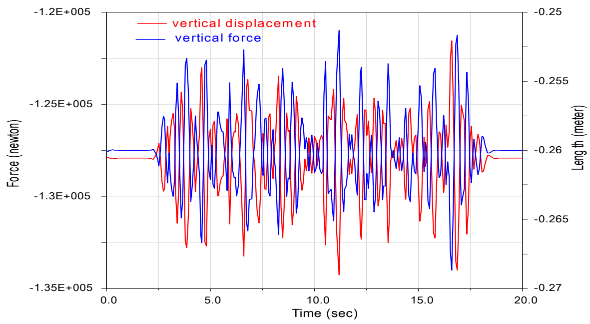


Fig. 23 Vertical force and displacement of PS (full wagon)

4 Conclusions

Considering that the results refer to particular conditions of load and track, from the diagrams of the simulations carried out, the following considerations can be made:

- viscous damping element involves in meaningfully variations of the freight dynamic behavior by either rail-wheel contact force and comfort travel;
- the damping values depend strictly from the load conditions and the damping optimization should be performed by statistic way;
- the simulation has been performed on the max nominal velocity value but real velocity vary respect to the maximum, so the presented studies should be extended to different run conditions in order to evaluate the complete dynamic behavior of the vehicle.

5 References

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