CONSTRAINED POLE ASSIGNMENT CONTROL FOR SECOND ORDER PLANT WITH STABLE ZERO

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Abstract

This paper discusses the design of the control for the second order plant with stable zero. The control combines the approaches of the minimum time control and the pole assignment control. The dynamical classes of PID controllers are introduced in the paper. The designed control, the P-P controller, is applied to a real plant to verify the control design. The thermal plant is used as the real experiment. It is modeled as the plant with slow and fast mode corresponding to different ways of heat transmission, i.e. second order plant with stable zero. The designed control respects input constraints and can be easily tuned by one parameter, the closed loop poles (we use double closed loop pole). When choosing proper closed loop poles one has to take into account parasitic time constants, etc. Nevertheless, the designed controller is able to give the dynamics from the minimum time control to pure linear one according to the chosen poles. The desired control signal has one interval at the saturation then it converges to steady value with the dynamics given by the closed loop pole.

Keywords: Constraints, Pole assignment control, Zero dynamics, Real experiment.

Presenting Author's biography

Mikuláš Huba received the MSc. and PhD. Degrees in technical cybernetics from Slovak University of Technology in Bratislava in 1974 and 1982, respectively. From 1989 he is a Senior Lecture and Head of the Control Theory Group of the Institute of Control and Industrial Informatics at the Faculty of Electrical Engineering and Information Technology. From 1996 he is also Head of the university Distance Education Centre. He is author of more than 180 papers, monographs on Constrained Control and Constrained PID Control and co-author of two monographs on Flexible and Web-Based Learning.

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1 Introduction

The control signal saturation can be considered as the elementary nonlinearity present in practically each control loop. In the 50's and 60's its effects have been intensively treated within the scope of the minimum time systems. Simultaneously, the demand on smooth solutions and quiet steady states lead to the development of the linear control technique called pole assignment control. Nowadays, the most popular techniques dealing with the input constraints used in practice, are the MPC and anti-windup control. The new concept of the constrained pole assignment control respecting output constraints combines the qualities of both the minimum time and of the pole assignment control.

2 Control design

2.1 Dynamical classes of control

By index of the dynamical class it is understood a non-negative integer denoting number of possible intervals with the limit control signal values that can occur under the limit case of the minimum time control. With respect to the Feldbaum's theorem [1] it is possible to conclude that the PID control corresponds to the dynamical processes from the dynamical classes 0, 1 and 2.

While in the DC0 the ideal control response following a setpoint step has also step character (Fig.1) and no saturation phase (therefore it can be successfully treated by the linear theory), the dynamical classes 1, 2 or higher (e.g. Fig.2) are already typical by a period (periods) with saturated control and so they are already nonlinear.

Processes of the DC0 are typically used in situations, where the dynamics of transients may be neglected, i.e. it is not connected with a reasonable energy accumulation. Such processes can e.g. be met in controlling flows by valves. After constraining the rate of control signal changes after a disturbance step, or also after a setpoint step, the transition to a new control signal value can be an exponential one (Fig.1).

Within the DC1 the control signal reaction to a setpoint step change can involve one control interval with constrained control value (Fig.2) that is later followed by a monotonous transient to the new steady state value . For the initial phase of control it is typical accumulation of energy in the controlled process. This is associated with a gradual increase (decrease) of the controlled output variable that is most rapid under impact of the limit control signal value. E.g. by charging a container with liquid, in the first phase of

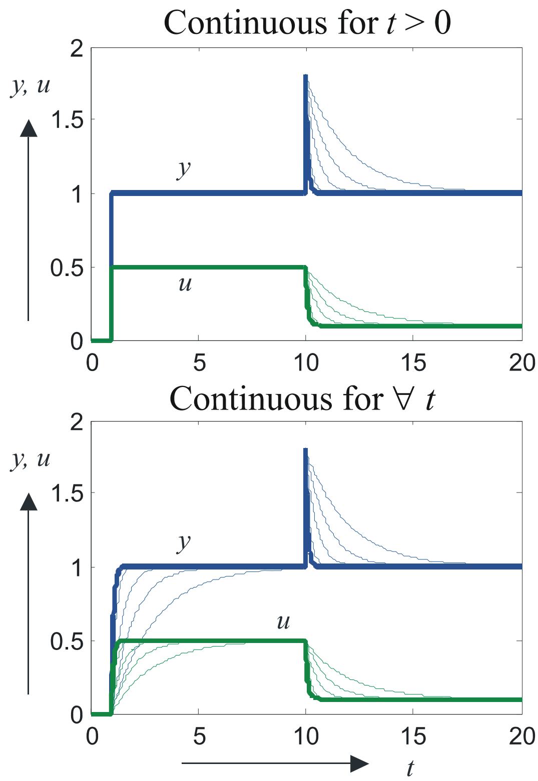

Fig. 1 DC0: Control signal reaction to a setpoint step; up – with the rate constraints just for the disturbance step; down – with a rate constraint also for a setpoint step

This is associated with a gradual increase (decrease) of the controlled output variable that is most rapid under impact of the limit control signal value. E.g. by charging a container with liquid, in the first phase of control the input valve will be fully opened and only close to the required level the control the input flow will decrease to a steady state value keeping the required level. Similar transients can be frequently met in speed control in mechatronic systems, in the temperature, pressure and concentration control, etc.

After limiting rate of changes during the transients, the span of the limit control action decreases, but the total length of transient to the new steady state increases. When constraining also the control signal increase after a setpoint change, the control signal does not catch to reach the limit value, since the necessary control decrease to the steady state has to start yet before it – the length of transient growths further.

With respect to one possible interval with constrained controller output for dealing with this dynamical class it is usually not enough to remain within the linear control. Typical solutions for this dynamical class are frequently achieved with different anti-windup (aw) controllers.

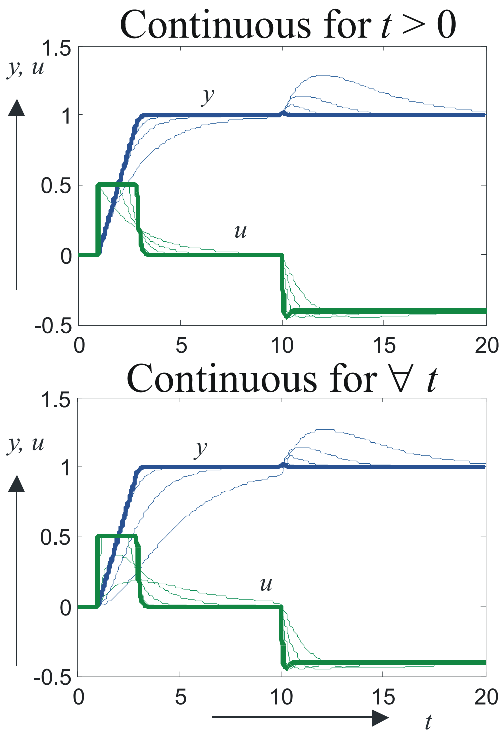

Fig. 2 DC1: Control signal reactions to a setpoint step parametrized by the closed loop poles; left – without rate constraints for t=0; right – with a rate constraint for t=0

For more information on the Constrained PID control or constrained pole assignment control see e.g. [6], [7], [8]. However the constrained PID control is not the only way how to deal with constraints. There can be used e.g. Fundamental and "ad hoc" solutions inspired by the works of Åström et al. [2],[3] that tried to develop general parameterized solutions which can be relatively easily adjusted to a particular situation by building on parameterizations as the sensitivity functions, or the complementary sensitivity functions related to the robust control. Having clear-cut physical interpretation of the effect of such tuning parameters and clear picture of its appropriate default values, the tuning should be much simpler and reliable.

However, from the point of a more general point of view of the constrained control the sensitivity and complementary sensitivity functions do not represent and optimal solution. They e.g. do not mach the natural expectation that by decreasing the range of possible parameter fluctuations, the effect of the non-modelled dynamics (parasitic delays) and the amplitude of the measurement noise - when there are no other specifications on the control quality - the achieved solutions would converge to the results of the minimum time control.

Such a requirement was obviously followed using another way of the closed loop parameterization – the pole assignment method by Glattfelder and Schaufelberger [4]. The anti-windup PI controller they have analyzed was very close to the ideal control signal step reaction converging to the one pulse of the minimum time control. But not completely.

In order to introduce an effective controller classification, it is further important to introduce new notion of "fundamental" controllers. Such a controller has to have following properties:

1. For the nominal dynamics $S(s)$ it must yield transient responses reaching from the fully linear up to the time optimal ones that can be simply scalable by the closed loop poles, (or other equivalent parameters as the time constants).
2. For a reliable controller tuning that guarantees monotonous responses the choice of the poles has to be restricted by identifying the perturbation (parasitic) dynamics $\delta S(s)$.

The first point involves the requirement to generalize the two limit solutions – the linear pole assignment control and the relay minimum time control to a compact set of responses that can be simply modified by the closed loop poles by offering properties that combine basic features of both limit solutions.

The second point is related to a reliable controller tuning. It tells that the system has to be approximated in such a way that besides of the nominal dynamics it is also determined the always present parasitic time delay (perturbation dynamics) that determines borders for the closed loop poles choice guaranteeing the expected properties.

Many of the known approaches do not fulfil the requirements on the fundamental solutions, since they do not enable to approach the minimum time transient responses, or they do not involve free design parameters at all. These approaches do not guarantee strictly optimal results and so they have reasonably contributed to the inflation of different "optimal" controller tuning. They further survive due to the conservativeness of practice despite the fact that the new digital controllers enable an easy dead time modeling and compensation. Of course, it has no sense to fight against their use, but it should be shown that they do not represent optimal solutions. In such a way, all the ambiguity of solutions reported e.g. by O'Dwyer [5] can be reasonably reduced.

Let us consider a plant with 2 modes (slow and fast). Its transfer function is

$$F(s) = \frac{K_1}{1+T_1 s} + \frac{K_2}{1+T_2 s} = K\frac{1+T_0 s}{(1+T_1 s)(1+T_2 s)}\,;$$
$$K = K_1 + K_2\ ;\ T_0 = \frac{K_1 T_2 + K_2 T_1}{(K_1 + K_2)} \tag{1}$$

This transfer function describes e.g. the thermal plant with two ways of heat transmission:

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Heat radiation (fast mode)

Heat conduction via body of the plant (slow mode)

The output of both the blocks can be described by the following differential equations

$$\dot{y}_1 = \frac{K_1 u - y_1}{T_1}\ ;\ \dot{y}_2 = \frac{K_2 u - y_2}{T_2} \tag{2}$$

For the output of the system

$$y = y_1 + y_2 \tag{3}$$

using (1), (2), (3) one can write

$$\dot{y} = \dot{y}_1 + \dot{y}_2 = \overline{K}u - \frac{1}{T_1 y} - \tau\, y_2; \qquad \overline{K} = \left(\frac{K_1}{T_1} + \frac{K_2}{T_2}\right);\ \tau = \left(\frac{1}{T_2} - \frac{1}{T_1}\right) \tag{4}$$

$$e = w - y\ ;\ \dot{e} = -\dot{y} \tag{5}$$

The value of control signal u_w which maintains the output of the system at the value $w = const$ is

$$u_w = \frac{w}{K_1 + K_2} \tag{6}$$

Using the control signal (5) the outputs of the single channels are

$$y_{1\infty} = K_1 u_w = \frac{K_1}{K_1 + K_2} w\ ; \qquad y_{2\infty} = K_2 u_w = \frac{K_2}{K_1 + K_2} w \overset{!}{=} w_2 \tag{7}$$

The pole assignment control is given by

$$\dot{e} = \alpha e \tag{8}$$

where α is the chosen closed loop pole. Substituting in (5) one gets

$$-\overline{K}u + \frac{y - w + w}{T_1} + \tau(y_2 - w_2 + w_2) = \alpha(w - y) \tag{9}$$

which leads to the control design as the cascade structure of the P-P controller

$$u = \frac{1}{K} w + K_R e + K_{R2} e_2\ ;\ e_2 = w_2 - y_2\ ; \qquad K_R = -\frac{\alpha + 1/T_1}{\overline{K}}\ ;\ K_{R2} = -\frac{T_1 - T_2}{K_1 T_2 - K_2 T_1} \tag{10}$$

3 Thermal plant

The plant used for real experiment was Measurement and Communication System uDAQ28/LT (fig.3.). This product of several years of development offers measurement of 8 process variables (controlled temperature and its filtered value, ambient temperature, controlled light intensity, its filtered value and its derivative, the ventilator speed of rotation and its motor current). The temperature and the light intensity control channels are interconnected by 3 manipulated variables: the bulb voltage (the heat & light source), the light-diode voltage (the light source) and the ventilator voltage (the system cooling). The plant can be easily connected to standard computers via USB, when it enables to work with the sampling periods 40-50 ms and larger.

Within a Matlab/Simulink scheme the plant is represented as a single block, limiting use of costly and complicated software package for the real time control. So, the usual process-computer communication based on standard converter cards (that is also supported) is necessary just for more demanding applications requiring higher sampling frequencies.

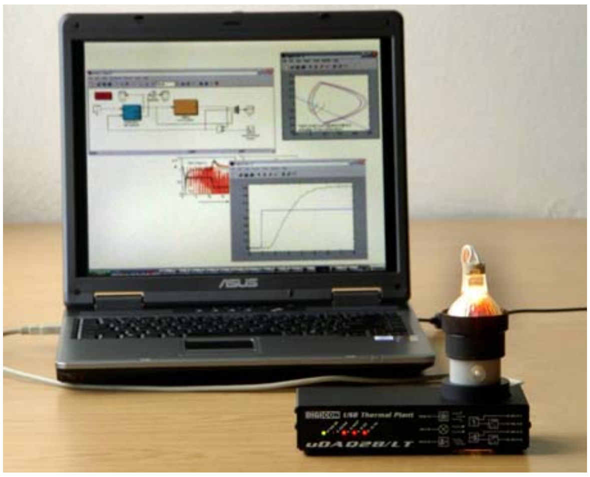

Fig. 3 Thermo-optical plant

4 Real Experiment

4.1 Identification of plant

There are several ways how to identify our plant as two parallel first order systems. We use one step response for the identification (fig. 4).

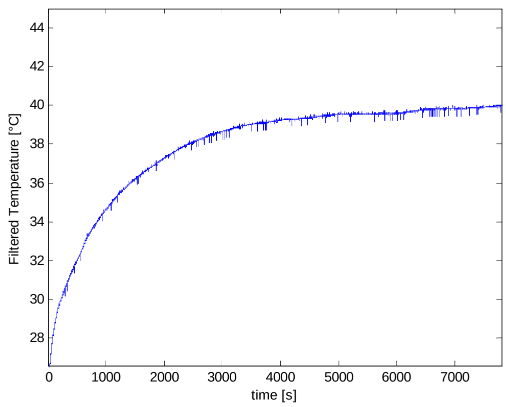

Fig. 4 Step response of the filtered temperature channel

For the system

$$y(t) = K_1\left(1 - e^{-(t-T_d)/T_1}\right) + K_2\left(1 - e^{-(t-T_d)/T_2}\right) \qquad (11)$$

When assuming

$$T_1 >> T_2 >> T_d \qquad (12)$$

One gets

$$T_1 = \frac{t_4 - t_3}{\ln \frac{y(\infty) - y_3}{y(\infty) - y_4}} \qquad (13)$$

$$K_1 = \frac{y_3 - y_4}{e^{-t_4/T_1} - e^{-t_3/T_1}} \qquad (14)$$

$$K_2 = y(\infty) - K_1 \qquad (15)$$

$$K_2 = T_2 \frac{y_2 - y_1}{t_2 - t_1} \qquad (16)$$

The identification depends on the selection of the points, where $t_1, t_2, y_1 = y(t_1), y_2 = y(t_2)$ represent two points from the start of the step response, $t_3, t_4, y_3 = y(t_3), y_4 = y(t_4)$ represent two points from the end of the step response.

The fig. 5 shows, how the identification fits the measured data.

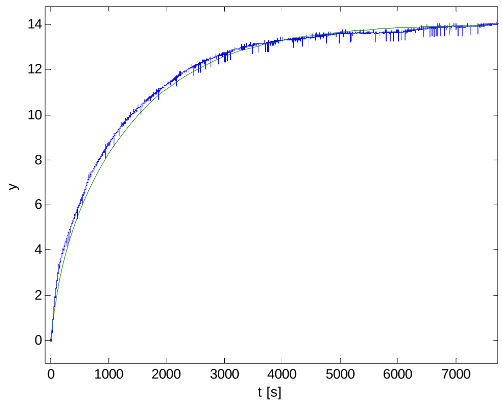

Fig. 5 Step response comparison of the real plant and the model.

4.2 Simulation

The fig. 6 shows the simulation results made in Matlab/Simulink. For the relatively "slow" poles the dynamics of the control signal is similar to the first order plants control. When choosing "fast" poles or when the control is close to the time-optimal one (poles are close to –infinity) the control signal is obviously affected by the zero dynamics of the plant. One interval at the saturation shows that the controller belongs to the dynamical class 1.

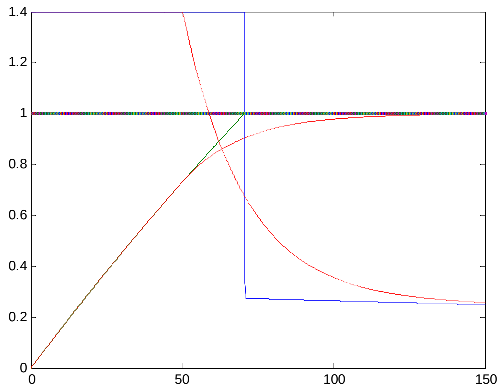

Fig. 6 Control signal and the output for the "fast & slow" poles where the plant is K1=1.0; K2=5.3; T1=184; T2=931

4.3 Using real plant in simulation

The experiment using the real plant described above has been used to verify the control design. Fig. 8, 9 shows the results, where the chosen closed loop pole is $\alpha = -0.015$. Fig. 7 shows the Simulink model. The plant has been identified as

$$G(s) = \frac{1.009}{180.17s + 1} + \frac{4.599}{1450.9s + 1}$$

Several steps of desired value have been used in the experiment.

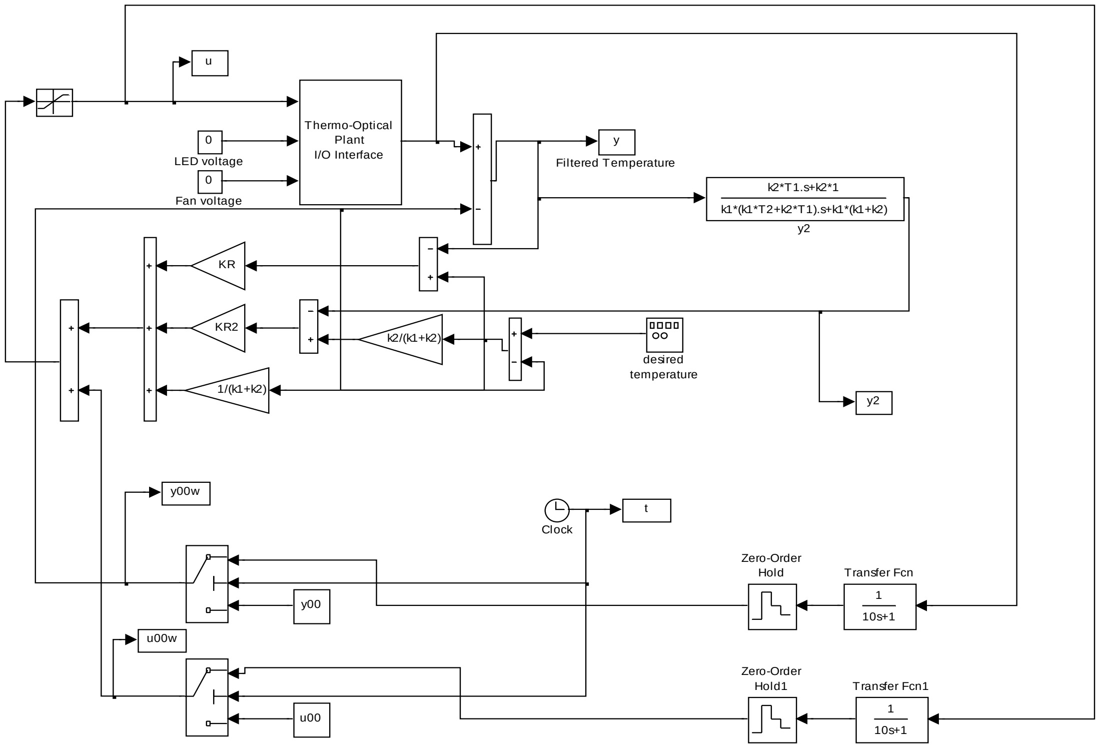

Fig. 7 Simulink model

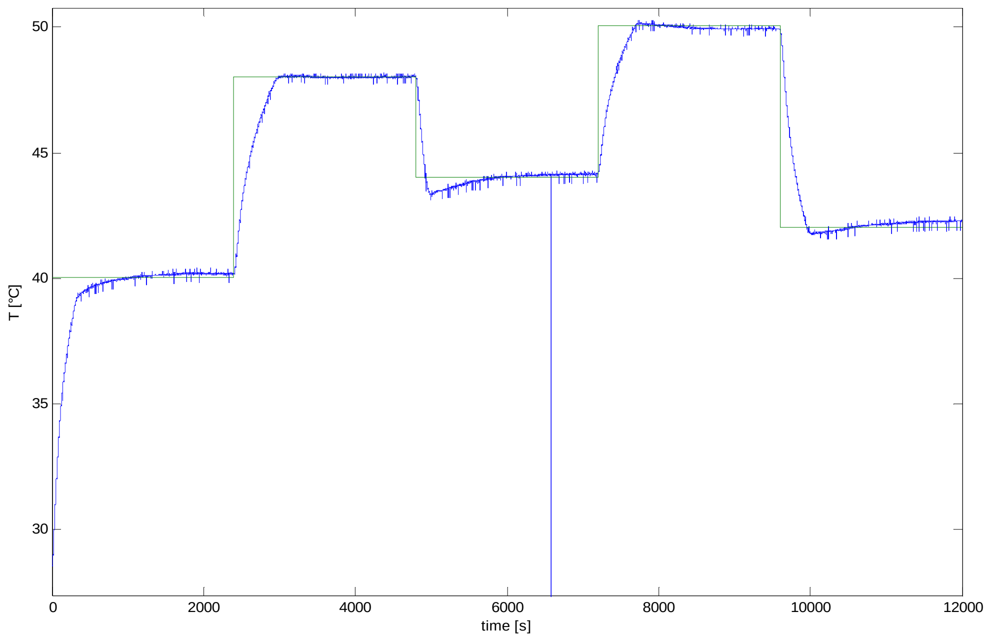

Fig. 8 The output of the system and the desired temperature

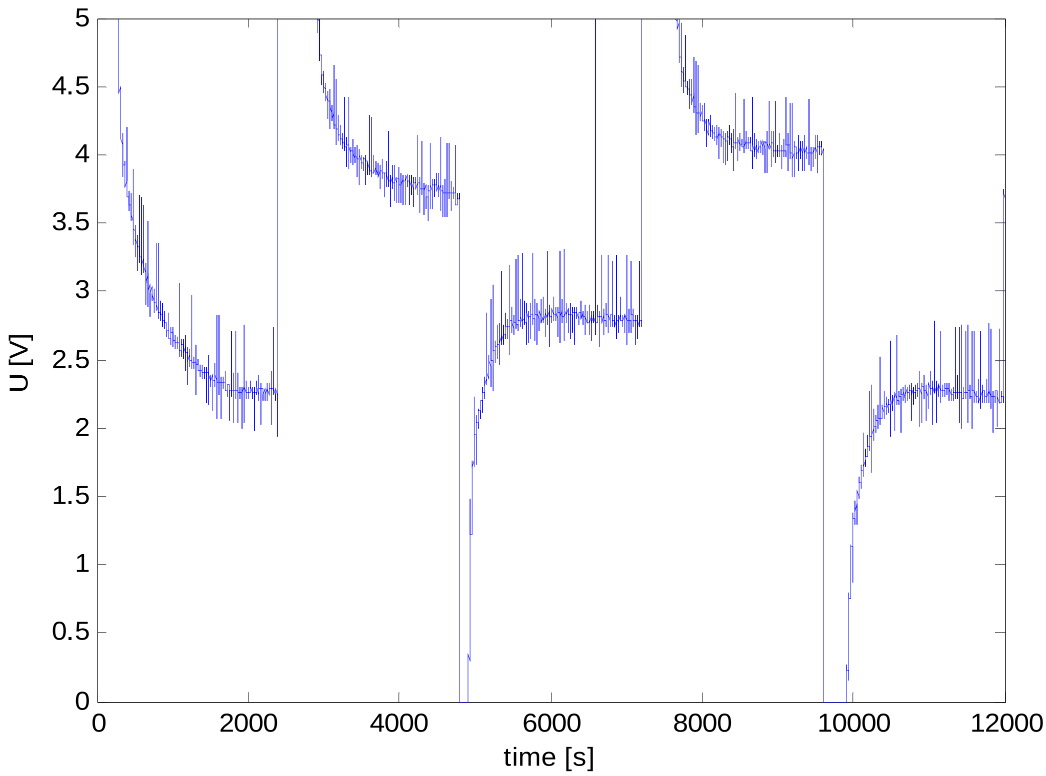

Fig. 9 Control signal

The experiment shows that the designed control works with the real plant as well. There is a little overshoot when cooling the plant, because the cooling has a different dynamics as the heating of the plant. It can be suppressed by choosing "slower" closed loop poles. Nevertheless, the zero dynamics is suppressed and the control signal has one interval at the saturation then it converges to desired value with the dynamics given by the closed loop poles.

5 Conclusion

The P-P controller respecting input constraints has been shown. The dynamics of the closed loop is determined by the chosen closed loop poles. The real experiment has proven the simulation results. The control signal starts at one of the constraint and then it decreases/increases with the dynamics given by the closed loop pole, so it has one interval at the constraint and then it remains within the limits.

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