# MODELING FREEWAY TRAFFIC FLOW: AN LPV APPROACH

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**Abstract**

The problem of modeling the complex behavior of freeway flow leads to a nonlinear macroscopic model. Since this high dimensional non-linear characteristic the control and estimation problems could not be performed easily. From this purpose the paper introduces a new, general modeling formalism for freeway traffic flow. It is well-known form the theory of Linear Parameter Varying (LPV) systems that such models represent a numerically tractable class of complex nonlinear systems. The main idea is to derive some arbitrary time dependent parameters by capturing the nonlinearities in the system. Transformation of the full nonlinear model to affine and quasi Linear Parameter Varying (LPV) system is presented. The paper investigates the problem of selecting the adequate scheduling variables, endogenous parameters and some linear approximations giving a novel way to describe freeway traffic systems. An important aspect of the model selection is the feasibility of the resulted system throughout the controller and observer design. The paper describes the problem of quadratic stabilizability and detectability for LPV flow models. The Linear Matrix Inequality (LMI) conditions are developed to verify these important properties. Finally, a numeric example suggests the application of the LPV structure for a general freeway section with on- and off-ramps. The comparison of the simulation response of the non-linear and the derived nominal LPV model has also been investigated.

**Keywords: quasi-LPV, freeway modeling**

## Presenting Author's Biography

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# 1 Introduction

One of the most progressing research area in traffic modeling is the theory of freeway traffic flow. There is a high demand on creating the most accurate set of models describing the real traffic.

A possible macroscopic technique is based on the analogy between the traffic flow and the streaming fluids or gases. However, the generalized density, speed and traffic volume are commonly used variables, the analogy is not valid for certain specific case. The basic correlation between the traffic variables is described by the well-known fundamental diagram. After Lighthill and Whitham [1, 2] formulated, the theory of kinematic waves were also adapted for freeways. Taking traffic waves into account, the freeway flow model can be extended to a second order macroscopic model. Due to the wave's dynamic, the traffic flow becomes highly nonlinear and complex. The complexity is increased by the segmentation of a freeway therefore a large scale system needs to be considered.

Modern control theory offers the opportunity to handle highway traffic models (and also other road traffic systems [3]) as dynamic systems. Introducing the time dependent freeway model, a more and more complex and liable description is given. Modern, respectively postmodern techniques therefore introduce the states of a freeway dynamical system. The dynamic state equation formulates how the system evolves in time. The state equations describe the variation of the actual states based on the given inputs to the system. Two important questions are arising with respect to the application in freeway traffic. First, the observation of the not measured variables and second, the control of the main flow with variable speed limits and ramp metering. The traffic modeling literature is large enough and contains several solution for traffic analysis and synthesis [1, 2, 4, 5, 6, 7].

The control objective on freeways could be stated as keeping the main flow volume near to the maximal capacity of the given stretch. Based on the fundamental diagram, this is equivalent to keep the density of the stretch around the its critical value. Since the problem is formulated with nonlinear equations, there is a need for the application of nonlinear control techniques. Denote, linear controller can also be used to assure the control performance. Unlike the nonlinear freeway control theory, linear control system design is elaborated [4]. While the nonlinear formalism is used to describe global behavior, linear systems are applied only to reflect local characteristics around a given operation point. Linearizing or simplifying the complete nonlinear plant always leads to loose certain and sometimes important information. Hence, there is a trade-off between the accurate model description and the simplicity of the controller. Since, the final goal is always to achieve an optimal performance level with the appropriate control algorithm, the realization of the closed-loop system needs to be taken into consideration.

As previously it was mentioned, the first part of the freeway model analysis and synthesis is the observation of the real flow. The problem is stated in a linear or in a nonlinear way. The state estimation of non-linear systems is an existing problem. The estimation technique of the Extended Kalman Filter (EKF) is widely applied [8, 6, 5] in the industry. The EKF is based on the linearization of the nonlinear system around the given operation point depending on the state trajectory. The convergence of the estimation has been investigated and it has been showed that EKF gives a suboptimal solution of the filtering problem. Even if the convergence of the EKF is not guaranteed, it is often used as a "nonlinear" observer. State estimation on freeways could multiple the available set of traffic information, by estimating the non-measured variables.

There is a permanent need to control the motorway traffic in order to avoid traffic jams respectively increase the safety level of a given section. Two main possibilities are applied to directly influence freeways traffic. First, the ramp metering, i.e the freeway on-ramp flow is controlled by signaling. On the other hand, the display of different speed limits throughout Variable Massage Sings (VMS). Traffic control synthesis is based on the results of control engineering [4, 9, 10, 11].

In recent years, a promising approach for nonlinear control theory is certainly the Linear Parameter Varying (LPV) formalism in state space [12, 13, 14, 15, 16]. The LPV class is a specific formulation of the nonlinear systems using measured, computed or estimated parameters. Parameter dependency is given under the time (parameter) dependent variation of the coefficient matrices. The linear represents the casual structure of the dynamic problem in state space where the dynamic and the output equations are the linear combination of the states and the inputs. The LPV description preserves the linear time invariant (LTI) structure, the only difference stays at the computation of the coefficients. The parameter vector is a continuously time dependent known function. It has been showed that non-linear systems could be cast into an LPV form by several ways. Therefore, the LPV model is not unique. In the particular case when the parameter vector coincides (partially or entirely) with the state vector the system is called quasi Linear Parameter Varying (qLPV) system.

The goal of the paper is the development of a control oriented LPV model of freeway flow. This model should contain the complex behavior of traffic flow and should be able to reproduce traffic phenomenons. Moreover the LPV structure will make it possible to apply the LPV design methodology which is an effective way to control and observe non-linear systems.

The paper is divided into 5 sections. After the introductory section, in the problem statement part, the paper describes briefly the freeway traffic model and formulates the problem. The forthcoming part presents the proposed solution for parameter dependent modeling of the freeway flow. Analytic questions are answered in the next section. Finally simulation results illustrate the accuracy of the qLPV model.

 

## 2 Problem setup

Recent traffic researches are mainly based on the second order macroscopic traffic flow model [11, 6, 9]. This model uses aggregated traffic variables, such as traffic density, space mean speed and traffic flow to describe freeway flow. Fig. 1. illustrates a freeway stretch.

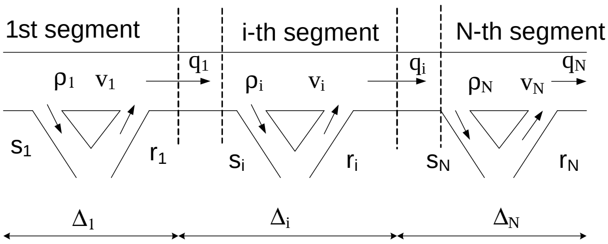


Fig. 1 Freeway division and traffic variables

Due to the complex behavior, the model is discretized in space; the stretch is subdivided into $N$ segments with length $\Delta_i$, $i = 1 \dots N$ and each segment is given by its traffic variables denoted by the subscript as follows:

- $\rho_i(k)$ denotes the density of the *i*-th segment at time step *k*
- $v_i(k)$ denotes the space-mean speed of the *i*-th segment at time step *k*
- $q_i(k)$ denotes the traffic flow leaving the *i*-th segment at time step *k*
- $s_i(k)$ denotes the off ramp flow of the *i*-th segment at time step *k*
- $r_i(k)$ denotes the on ramp flow of the *i*-th segment at time step *k*

After introducing these variables, the nonlinear difference equations of the second-order macroscopic traffic flow model for a segment $i$ are written by:

$$\begin{aligned}
\rho_i(k+1) &= \rho_i(k) + \frac{T}{\Delta_i n}\left[q_{i-1}(k) - q_i(k)\right] \\
&+ \frac{T}{\Delta_i n}\left[r_i(k) - s_i(k)\right] \qquad (1)\\
s_i(k) &= \beta_i(k) \cdot q_{i-1}(k) \qquad (2)\\
v_i(k+1) &= v_i(k) + \frac{T}{\tau}\left[V(\rho_i(k)) - v_i(k)\right] \\
&+ \frac{T}{\Delta_i} v_i(k)\left[v_{i-1}(k) - v_i(k)\right] \\
&- \frac{\nu}{\tau}\frac{T}{\Delta_i}\frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \kappa} \\
&- \frac{\delta T}{\tau \Delta_i}\frac{r_i(k) v_i(k)}{\rho_i(k) + \kappa} \qquad (3)\\
V(\rho) &= v_f exp\left[-\frac{1}{a}\left(\frac{\rho}{\rho_{cr}}\right)^a\right] \qquad (4)\\
q_i(k) &= \rho_i(k) \cdot v_i(k) \cdot n \qquad (5)
\end{aligned}$$

where $T$ denotes the sample time, $n$ is the number of lanes and $a$, $v_f$, $\rho_{cr}$, $\kappa$, $\tau$, $\delta$, $\nu$ are additional constant parameters. The macroscopic model was shown to work accurately with segment lengths in the order of 500 meters (or less) [7]. Longer sections could be built up by the interconnection of several segments through the boundary relations (i.e. $\rho_{i+1}$, $v_{i-1}$). The second order macroscopic model is used as a basis of different problems regarding the freeway control and surveillance.

The most challenging problem in freeway control engineering is the state (density, speed and volume) observation. Special inductive loop-detectors are installed at distinct locations (usually 4-5 kilometers far from each other) in a freeway's pavement, not in the entire stretch of freeway. These detectors collect traffic data from a single point i.e. no information is available between their installation points. Using the dynamical equations of freeway flow and the theory of state estimation one could design a freeway estimator which filters out the measurement and process noises and gives a suboptimal estimation of the traffic variables between detector stations. This technique multiplies the available set of data, and this additional information could be used for better freeway control and incident detection.

## 3 Derivation of the qLPV model

Complex systems require complex mathematical techniques to be described. Complex usually covers the nonlinear effect and the large number of state variables.

A certain class of nonlinear systems might be rewritten into linear but parameter dependent plants [12]. The LPV methodology keeps the linearity with respect to the state variables but it could contains nonlinear parameter in the coefficient matrices. By definition, parameters are needed to be available for the description. One of the biggest advantage of the technique is to handle nonlinearities. More, the LPV concept extends the linear estimation and control aspects towards the reformulated nonlinear world.

An n-th order LPV model is defined as:

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(p(t)) & B(p(t)) \\ C(p(t)) & D(p(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \qquad (6)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the input vector, $y(t) \in \mathbb{R}^{n_y}$ is the output vector and $p(t) \in \mathcal{P}$ is the parameter vector over a given compact set $\mathcal{P}$. Two alternate class of the parameters exists. Exogenous and endogenous variables can be used. Quasi linear parameter-varying (qLPV) systems are defined whenever any of the scheduling parameters becomes a state of the system as well.

The affine dependency with $d(p(t)) = N$ in the system

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can be given by:

$$A(p(t)) = A_0 + p_1(t)A_1 + p_2(t)A_2 + ... + p_N(t)A_N \quad (7)$$

$$B(p(t)) = B_0 + p_1(t)B_1 + p_2(t)B_2 + ... + p_N(t)B_N \quad (8)$$

$$C(p(t)) = C_0 + p_1(t)C_1 + p_2(t)C_2 + ... + p_N(t)C_N \quad (9)$$

$$D(p(t)) = D_0 + p_1(t)D_1 + p_2(t)D_2 + ... + p_N(t)D_N. \quad (10)$$

As it was described in the previous section, the control objectives are usually defined on traffic density. Nevertheless, highly non-linear speed equations are used to give a more accurate description of the conservation equation. The main idea is to treat these speed variables as scheduling parameters. Since, the speed equation (Eq. 3) is a nonlinear term and might be parametrized by $v_i$ to gain an LPV system. One could design feedback controllers stabilizing the system (i.e. using ramp-metering) and estimating state variables (i.e. freeway estimator) whenever the parameter vector is in the pre-defined magnitude bounded set. Denote, parameter rate variation can be taken into consideration as well. In order to derive an LPV model from the non-linear equations (1)-(5) one reformulates by applying a linear approximation of (4).

The fundamental relation can be rewritten by:

$$V(\rho) = v_f \left[1 - \frac{\rho}{\rho_{op}}\right] \quad (11)$$

where $\rho_{op}$ is the traffic density value corresponding to the scope of freeway control. The Fig. 2. shows the linear aprroximation of the fundamental equation. First order curve is fitted on a set of measurement. Data had been collected on a 4.5 km long highway section($M3$) in Hungary. The linear approximation is valid only up to a given density (50 veh/km). Piecewise linear or higher order fitting generates more accurate models.

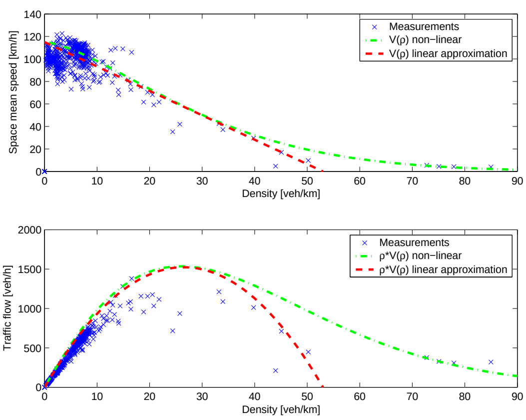


Fig. 2 The theoretical and the fitted fundamental relationship

The first order approximation can be thought as an off-line model calibration on the actual highway segments. On the other hand, the replacement of $\rho_i(k) + \kappa$ with $\rho_{cr} + \kappa$ in Eq. 5 might be considered as a modification of the speed evolution. However, this assumption is clearly valid in accident-free case, in a non-accident free case, the speed term is just slightly depend on those (wave propagation) terms where the nominator contains $\rho_i(k) + \kappa$, see 5.

Applying the above simplifications, one might reformulate the Eq. (3) by:

$$\begin{aligned} v_i(k+1) &= v_i(k) + \frac{T}{\tau}\left[v_f\left[1 - \frac{\rho_i(k)}{\rho_{op}}\right] - v_i(k)\right] \\ &+ \frac{T}{\Delta_i} v_i(k)\left[v_{i-1}(k) - v_i(k)\right] \\ &- \frac{\nu}{\tau}\frac{T}{\Delta_i}\frac{\rho_{i+1} - \rho_i(k)}{\rho_{cr} + \kappa} \\ &- \frac{\delta T}{\tau \Delta_i}\frac{r_i(k)v_i(k)}{\rho_{cr} + \kappa} \end{aligned} \quad (12)$$

There is no need for the density approximation. Since, one could rewrite it by substituting equation (2) into (1), than equation (5) into (1) and than factor out the speed variables:

$$\begin{aligned} \rho_i(k+1) &= \rho_i(k) + v_{i-1}(k)\frac{T}{\Delta_i}\rho_{i-1}(k) \\ &- v_i(k)\frac{T}{\Delta_i}(1+\beta_i)\rho_i(k) \\ &+ \frac{T}{\Delta_i n} r_i(k) \end{aligned} \quad (13)$$

Equations (13) and (12) are the affine quasi LPV model equations of the freeway traffic flow.

For sake of simplicity and to illustrate the LPV structure consider a 1.5 km long freeway stretch, with two lanes divided into three segments, each $\Delta = 500$ meters long is given. The middle segment has an on/off ramp. Detectors are installed at the beginning and at the end of each of the stretches.

The dynamic equations of this stretch could be now written as an affine parameter dependent system under the form of an LPV model:

$$x(k+1) = A(p(k))x(k) + B(p(k))u(k) + W(p(k))w(k) \quad (14)$$

$$A(p(k)) = A_0 + p_1A_1 + p_2A_2 + p_3A_3 \quad (15)$$

$$B(p(k)) = B_0 + p_1B_1 + p_2B_2 + p_3B_3 \quad (16)$$

Where:

$$\begin{aligned} x &= \begin{bmatrix} \rho_1 & v_1 & \rho_2 & v_2 & \rho_3 & v_3 \end{bmatrix}^T \\ u &= \begin{bmatrix} r_2 \end{bmatrix} \\ w &= \begin{bmatrix} \rho_0 & v_0 \end{bmatrix}^T \\ p &= \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \end{aligned}$$

The boundary relations imply the knownledge of $v_0$, $\rho_0$ and $\rho_4$ by measurement. The variables of the entering flow are supposed to be measured. Though, the density of the next segment is not known, therefore one assumes that $\rho_4 = \rho_3$. The system matrices in (15) and (16) are as given:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_1-a_2 & a_3 & -a_1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & a_1-a_2 & a_3 & -a_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -a_2 & a_3 \end{bmatrix}$$

where:

$$\begin{aligned} a_1 &= \frac{\nu T}{\tau\Delta}\frac{1}{\rho_{cr}+\kappa} \\ a_2 &= \frac{T}{\tau\rho_{jam}} \\ a_3 &= 1-\frac{T}{\tau} \end{aligned}$$

$$A_1 = \begin{bmatrix} -\frac{T}{\Delta} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{T}{\Delta} & 0 & 0 & 0 & 0 \\ \frac{T}{\Delta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{T}{\Delta}(1+\beta) & 0 & 0 & 0 \\ 0 & \frac{T}{\Delta} & 0 & -\frac{T}{\Delta} & 0 & 0 \\ 0 & 0 & \frac{T}{\Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{T}{\Delta} & 0 \\ 0 & 0 & 0 & \frac{T}{\Delta} & 0 & -\frac{T}{\Delta} \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 \\ 0 \\ \frac{T}{\Delta n} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\delta T}{\tau\Delta}\frac{1}{\rho_{cr}+\kappa} \\ 0 \\ 0 \end{bmatrix},$$

$$W = \begin{bmatrix} v_0\frac{T}{\Delta} & 0 \\ 0 & v_1\frac{T}{\Delta} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

## 4 Stabilizability and detectability

This section shows the advantages of the LPV formalism with respect to traffic flow modeling and control.

Before designing a state feedback for control purposes one has to satisfy the stabilizability criteria. Moreover, the dual pre-condition of the state estimator is the detectability that needs to be fulfilled. The stabilizability and detectability check in linear case can be solved by computing and validating Kalman's rank conditions. If the system is non-linear, the equivalent controllability and observability distributions are often hard to compute especially for higher dimensions. One of the main advantage of LPV systems is the simplicity of the above analytic propeties. The notion of quadratic stabilizability and quadratic detectability is known [12, 13]. Just a brief traffic oriented overview is given in the sequel for highway flow modeling purpose.

Quadratic stabilizability means, there exists a feedback controller such that

$$x_{k+1} = (A(p(k)) + B(p(k))K)x_k \tag{17}$$

closed loop is stable for all $p \in \mathcal{P}$ admissible parameter trajectories. The definition involves a single quadratic Lyapunov function, under the form of:

$$V_k = x_k^T P x_k \tag{18}$$

The dissipative condition implies:

$$\begin{aligned} V_{k+1} - V_k = x_{k+1}^T P x_{k+1} - x_k^T P x_k &< 0 \\ ((A+BK)x_k)^T P (A+BK)x_k - x_k^T P x_k &< 0 \\ P - (K^T B^T + A^T) P (A+BK) &\succ 0 \end{aligned} \tag{19}$$

where the last equation is a matrix inequality, $\succ 0$ denotes that the matrix is positive definite.

Using the Schur-complement:

$$\begin{aligned} M_{11} &= P \\ M_{12} &= (A+BK)^T \\ M_{22} &= P^{-1} \end{aligned}$$

it can be written:

$$\begin{bmatrix} P & (A+BK)^T \\ A+BK & P^{-1} \end{bmatrix} \succ 0 \tag{20}$$

Pre- and post-multiplying (20) with

$$\begin{bmatrix} I_n & 0 \\ 0 & G \end{bmatrix} \succ 0 \tag{21}$$

the following equation is given [17]:

$$\begin{bmatrix} P & A^T G^T + K^T G^T B^T \\ GA + BGK & GP^{-1}G^T \end{bmatrix} \succ 0 \tag{22}$$

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Let us define $GK = Y$. A lower approximation of $M_{22}$ is written by:

$$\begin{aligned} (G-P)P^{-1}(G^T-P) &\succ 0 \\ GP^{-1}G^T &\succ G+G^T-P \end{aligned} \quad (23)$$

Using the lower bound, finally the following form is computed:

$$\begin{bmatrix} P & A^TG^T+Y^TB^T \\ GA+BY & G+G^T-P \end{bmatrix} \succ 0 \quad (24)$$

which is a Linear Matrix Inequality (LMI). The latter LMI should have a $P = P^T \succ 0$ and $G$ solution for all $p(t) \in \mathcal{P}$. Since, one could determine the lower and upper bound on the parameter values, the last LMI results in a finite number of feasibility test on the corner points of the bounded convex parameter set (see Fig. (4)). It is sufficient to check the feasibility on the extremal point of the vertex due to the affine LPV model.

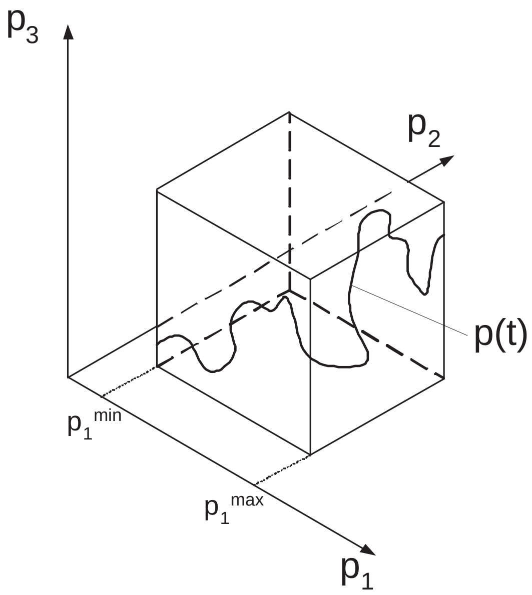


Fig. 3 Bounded parameter set

Whenever the LMI is feasible, a state feedback control might be given by:

$$K = G^{-1}Y$$

In order to determine the lower and upper bound on the scheduling parameters, real data had been collected from Hungarian freeways. The maximum and minimum value of the space mean speed was found:

$$p^{max} = 120\frac{km}{h}$$

$$p_{min} = 0.1 * p^{max} = 12\frac{km}{h}$$

The unknown model parameters were also determined from real traffic data using the parameter identification procedure [7]. With these values one verified the feasibility of the LMIs, so that the LPV model is quadratically stabilizable with ramp metering.

Quadratic detectability is the dual framework of the quadratic stability. The duality permits to drive the problem back to the previous LMIs.

The condition is given:

$$x_{k+1} = (A(p(k)) + LC(p(k))x_k \quad (25)$$

the closed loop with a $L$ observer gain is chosen to be stable. LMIs for the observer design can be written:

$$\begin{bmatrix} P & (A+LC)^T \\ A+LC & P^{-1} \end{bmatrix} \succ 0 \quad (26)$$

$$\begin{bmatrix} P & A^TG_F^T+C^TY_F^T \\ G_FA+Y_FC & G_F+G_F^T-P \end{bmatrix} \succ 0 \quad (27)$$

Using an affine LPV model for highway traffic modeling, LMIs were derived to show the detectability and stabilizability criteria. More, the section clearly describes the set of LMI to solve for a controller and an observer synthesis assuming that the problem might be feasible with a single and not parameter varying Lyapunov function. Studies in traffic flow modeling were found feasible with a simple Lyapunov description.

## 5 Numerical example

This section gives an example to compare the fully non-linear and the qLPV traffic models.

Using the same constant parameter values determined by identification for all segments, the comparison of the non-linear model and the derived qLPV model are carried out. To perform simulation, the above introduced stretch was built in MATLAB/Simulink. The stretch consists three 500 meters long segments, each with two lanes. There is an on-ramp the middle segment. First the two model were compared under slowly varying traffic flow, the typical characteristics of the morning and evening rush hours are represented through changing flow and speed. Simulation response of the models for the case of normal flow and interrupted (accident) flow are shown on Fig. 4.

In the second case an accident was simulated in the third segment, by suddenly decreasing the outflow. The responses are given on Fig. 5.

As it could be seen on Fig. 4-5, the nominal qLPV model can accurately simulate the dynamics of freeway flow. Clearly the response of the qLPV model is more like linear under fast variation, due to the linear approximation of the fundamental diagram. Also a small difference between the models appears when the density rises over the critical values, denoted by $\rho_{op}$. On the

 

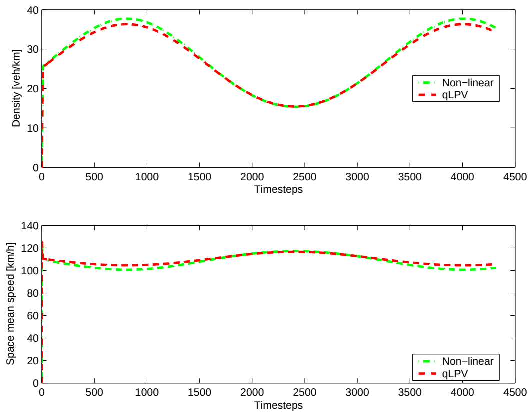


Fig. 4 Simulation results for normal flow

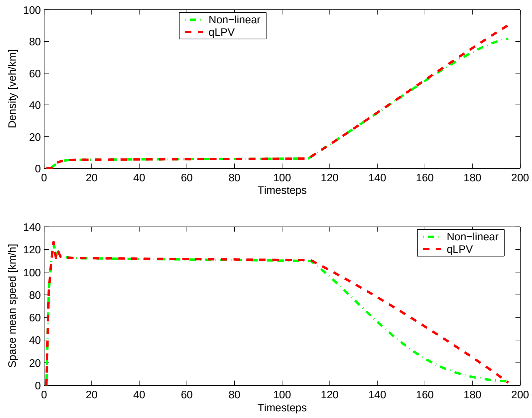


Fig. 5 Simulation results in case of accident

other hand these effects could be take into consideration through robust qLPV framework, which will be in focus of our further research.

Finally Fig. 6 shows the simulation results of on-ramp flow, comparing the uncontrolled and the controlled case.

During the simulation the same slowly varying flow as discussed above and the constant feedback gain $K$ was used, the gain determined through the stabilizability analysis in (24). It could be clearly seen that ramp metering tries to minimize the on-ramp flow during main flow's peak hours. On the other hand the controller allows more vehicle to merge whenever the main flow is weak and the density is under the critical value. Note that also in this simple example a marginal capacity growth has been established with the usage of ramp metering.

## 6 Conclusion and further research

The paper presents a generic model formalism, the Linear Parameter Varying (LPV), in order to handle nonlinearities in a complex highway flow model.

The paper clearly implies the advantages of the modeling technique. Quadratic stabilizability and detectability questions are answered using Linear Matrix Inequalities (LMI). Single Lyapunov function is assumed to make the closed loop system feasible.

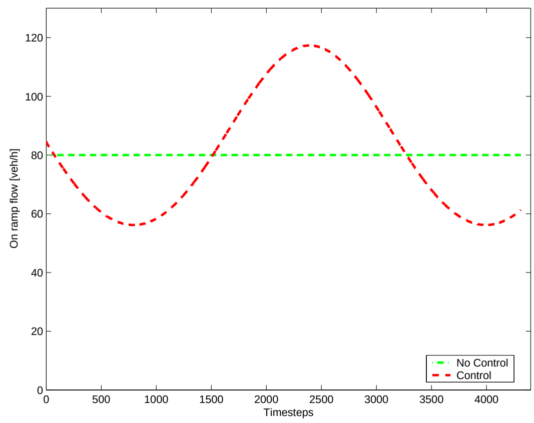


Fig. 6 Simulation results of the on ramp flow

In the near future, the advantage of formulating a parameter dependent Lyapunov function, or parameter dependent gain ($K(p(t))$ or $L(p(t))$) will be given. On the other hand, further works will be carrying on the control of highway flow with the help of LPV systems.

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