SIMULATION MODEL OF THE THERMODYNAMIC CYCLE OF A THREE-CYLINDER DOUBLE-ACTING STEAM ENGINE

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Abstract

Today, steam engines are used for special purposes only, for example to reduce steam pressure in pressure reduction stations, where they replace the traditional and inefficient throttling process. Throttling is the most used way to control the pressure in steam reduction stations. This way is unsatisfactory from the economical point of view, because the exergy is lost uselessly. It is a part of heat energy that can perform a work. The better way of the pressure reduction is an expansion in a backpressure turbine or in a steam engine by simultaneous transformation of the heat energy into electricity (cogeneration). This article describes the design and implementation of the mathematical model of the thermodynamic cycle in a steam engine used as pressure regulator in a pressure reduction station. The present model is a part of a comprehensive mathematical model of a cogeneration unit and also a part of the author's doctoral thesis. The model assumes detailed mathematical description of physical processes in a steam engine and implementation in an MATLAB-SIMULINK software environment. Follow-up mathematical models (electrical generator, mechanical model) were presented in other articles that are listed in references.

Keywords: Steam engine, Modeling, Simulation, Cogeneration

Presenting Author's Biography

Evžen Thöndel is a PhD student at the Department of Mechanics and Materials Science, Faculty of Electrical Engineering, Czech Technical University in Prague. He deals with the problematic of modeling and optimization of cogeneration units. The main goal of his work is a mathematical model implementation of cogeneration unit driven by a steam engine and design of optimization methods. Presenting article covers a part of his dissertation thesis. Author cooperates with the Technical University in Munich, where he researched one year into cogeneration technology. Author is supported by a German foundation (Deutsche Bundesstiftung Umwelt).

1 Introduction

Steam engines make use of the heat energy existing in the working liquid, converting it to mechanical work. The working fluid of steam engines is steam admitted to the cylinders of the engine where it expands, thus exerting force which can be harnessed for the work. Lower temperature and pressure steam is discharged out of the cylinder and into the environment or, alternatively, can be used for other purposes (cogeneration). Today, steam engines are used for special purposes only, for example to reduce steam pressure in pressure reduction stations, where they replace the traditional and inefficient throttling process.

In order to allow constant and stable operation of the steam engine, the original conditions of the working fluid (i.e. steam) have to be continuously restored; in other words, there has to be a working cycle. The cycle can be defined as a series of successive states, with the first initial state and the last end state being identical. Typically, the cycle is plotted in a $p - V$ chart, V being the volume and p the pressure in the cylinder. The area delimited by the state-transition lines corresponds to the amount of work performed.

The following paragraphs cover the design and implementation of the mathematical model of the steam cycle in a steam engine used as pressure regulator in a pressure reduction station.

2 System Description and Restricting Conditions

Figure No. 1 describes the function of one cylinder of the steam engine. Steam generated in the steam boiler flows through the constant diameter pipe A_1 into the cylinder. Our model assumes a boiler with a constant steam pressure p_1 and a theoretically unlimited steam consumption $\frac{dm_1}{dt}$. The piston in the cylinder has the diameter A, its momentary position being defined by the x coordinate measured from the top dead centre (TDC). At the end of every cycle, lower pressure and temperature steam is discharged from the cylinder into the external system via the constant diameter pipe A_2. This system can be characterized by its volume V_2 and further consumption determined by the mass flow $\frac{dm_3}{dt}$. The present model shall be subsequently used to design a steam engine control mechanism maintaining steam pressure constant irrespective of current steam consumption. The controlling variable used for this purpose shall be the load torque at the steam engine shaft, regulating its speed according to steam usage. Steam inlet and exhaust is controlled by the ideal valves v_1 and v_2.

The previous paragraph describes the restricting conditions used in designing the mathematical model:

- The steam generator can ensure constant steam pressure irrespective of actual steam consumption.
- The general three-dimensional flow in the inlet and exhaust pipes is replaced by a one-dimensional flow.
- The valves controlling the admission and discharge of steam are ideal valves (i.e. the transient process of opening and closing the valve has not been taken into consideration).

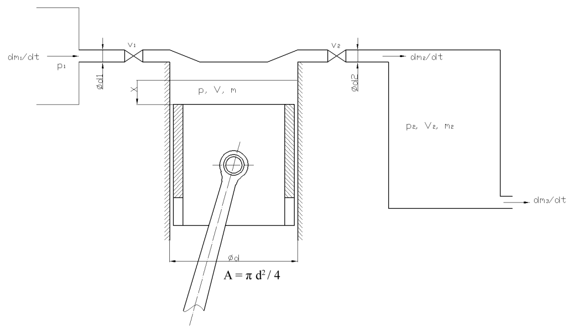

Fig. 1 Steam engine cylinder

3 Mathematical Description of the System

First and foremost, we shall describe the action taking place in the **cylinder**. According to the continuity equation, the difference of the inlet mass flow and the exhaust mass flow is equal to the change in mass accumulated in the cylinder. Expressed mathematically:

$$\frac{dm_1}{dt} - \frac{dm_2}{dt} = \frac{dm}{dt}. \qquad (1)$$

Assuming the diameter is constant, the mass flows can be expressed as:

$$\begin{aligned} \frac{dm_1}{dt} &= \rho_1 A_1 c_1, \\ \frac{dm_2}{dt} &= \rho_2 A_2 c_2, \end{aligned} \qquad (2)$$

with ρ_1, ρ_2, c_1, c_2 being the average density and speed in the inlet and exhaust pipes, assuming the diameters A_1 and A_2 respectively. The equation of the change in the mass accumulated in the cylinder can be expressed in a similar manner; however we have to take into consideration the volume changes caused by the motion of the piston:

$$V = V_Z + Ax = A(b + x), \quad b = \frac{V_Z}{A}, \qquad (3)$$

where V_Z is the residual volume and b the stroke equivalent to the residual volume. After multiplying the equation by the density ρ and differentiating the composite function with respect to time, we gain the following mass change equation:

$$\begin{aligned} \frac{dm}{dt} &= \frac{d}{dt}(\rho V) = \frac{d}{dt}\left[A\rho(x + b)\right] \\ &= A\rho\frac{dx}{dt} + A(x + b)\frac{d\rho}{dt}. \end{aligned} \qquad (4)$$

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Steam compression and expansion in the cylinder is accompanied by state transitions during which heat exchange takes place. The relationship between pressure and the specific volume is characterized by the polytropic process equation:

$$p \cdot v^n = p_0 \cdot v_0^n = const, \tag{5}$$

where n is the polytropic exponent and p_0, v_0 the initial system state for $t = 0$. The residual volume is the reciprocal value of density ($v = \frac{1}{\rho}$). Substituting into the polytropic equation 5, we can ascertain, rewriting the expression in a straightforward manner, the development of density in the cylinder:

$$\rho = \rho_0 \left(\frac{p}{p_0}\right)^{\frac{1}{n}}. \tag{6}$$

The derivative of this function shows the development of density changes in the cylinder:

$$\frac{d\rho}{dt} = \frac{d\rho}{dp}\frac{dp}{dt} = \frac{\rho_0}{p_0^{\frac{1}{n}}}\frac{1}{n}p^{\frac{1}{n}-1}\frac{dp}{dt}. \tag{7}$$

Substituting into equation 4, we can obtain the right side of the continuity equation 1:

$$\frac{dm}{dt} = A\frac{\rho_0}{p_0^{\frac{1}{n}}}\left[p^{\frac{1}{n}}\frac{dx}{dt} + (x+b)\frac{1}{n}p^{\frac{1}{n}-1}\frac{dp}{dt}\right] \tag{8}$$

The flow velocity in the inlet and exhaust pipes can be determined using Bernoulli's equation, which states that the energy in a fluid flowing along a streamline is the same at any two points in that path. Bernoulli's equation can be amended to reflect energy losses (dissipation):

$$\frac{p_1}{\overline{\rho_1}} = \frac{p}{\overline{\rho}} + \frac{c_1^2}{2} + e_{1,dis} \tag{9}$$

The pressure energy $\frac{p_1}{\overline{\rho_1}}$ at the start of the inlet pipe is divided into the pressure energy $\frac{p}{\overline{\rho}}$, the kinetic energy $\frac{c_1^2}{2}$ at the end of the inlet pipe and dissipations $e_{1,dis}$. Dissipation can take place for many reasons and a comprehensive analysis thereof is beyond the scope of this work. Therefore, energy dissipation shall be reflected by an empirical formula:

$$e_{1,dis} = \frac{1}{2}\xi_1 c_1^2, \tag{10}$$

ξ_1 being the dimensionless dissipation coefficient. During steam admission, the pressure in the cylinder amounts to $p \doteq p_1$, thus the average steam density $\overline{\rho}_1 \doteq \overline{\rho} \doteq \rho_1$ cannot be used. The flow velocity in the inlet pipe can be calculated by rewriting equation. 9:

$$c_1 = \sqrt{\frac{2}{\rho_1(1+\xi_1)}(p_1 - p)}. \tag{11}$$

A similar procedure can be used for the discharge pipe too:

$$c_2 = \sqrt{\frac{2}{\rho_2(1+\xi_2)}(p - p_2)}. \tag{12}$$

The thermodynamic processes taking place in the **external system** can be ascertained by means of the continuity equation. The change in mass in the system is a result of differences in the incoming and outgoing mass flows:

$$\frac{dm_2}{dt} - \frac{dm_3}{dt} = \frac{dm_2}{dt}. \tag{13}$$

However, in this case system the volume is constant in time:

$$\frac{dm_2}{dt} = V_2\frac{d\rho_2}{dt}. \tag{14}$$

As already pointed out hereinbefore, the pressure in the external system shall be kept constant by means of a mechanism regulating the load torque at the steam engine shaft. In this application, the steam engine replaces the throttling process in the pressure reduction station. Assuming there is liquid steam in the external system, the isobaric and isothermal lines coincide during phase transitions. As for higher steam quality, the following expression can be used to obtain approximate results:

$$p_2 \cdot v_2 = p_{20} \cdot v_{20} = const, \tag{15}$$

p_2 being pressure, v_2 the specific volume of the external system, p_{20} the initial pressure and v_{20} the initial specific volume of the external system for $t = 0$. Rewriting the expression, we can ascertain the relationship of density changes in the external system over time:

$$\frac{d\rho_2}{dt} = \frac{d\rho_2}{dp_2} \cdot \frac{dp_2}{dt} = \frac{\rho_{20}}{p_{20}} \cdot \frac{dp_2}{dt}. \tag{16}$$

And:

$$\frac{dm_2}{dt} = V_2\frac{\rho_{20}}{p_{20}}\frac{dp_2}{dt}. \tag{17}$$

3.1 Mathematical Model Equations

The equations of the mathematical model are differential equations describing the development of the variable(s) in question, in our case the development of the pressure in the steam machine cylinder and the external system.

Substituting expressions 2 and 8 into the continuity equation 1, we obtain, after applying the time-derivative of pressure, the following equation:

$$\frac{dp}{dt} = \left[\frac{1}{A\rho_0}\left(\frac{p_0}{p}\right)^{\frac{1}{n}}(\rho_1 A_1 c_1 - \rho_2 A_2 c_2) - \frac{dx}{dt}\right]\frac{p \cdot n}{x+b} \tag{18}$$

The flow velocities c_1 and c_2 in the inlet and exhaust pipes are determined by the states of the respective valve (see Figure 1). We distinguish three different states:

- **Admission**. The inlet valve is fully open, the flow velocity c_1 being characterized by equation 11. The exhaust valve is closed, the flow velocity c_2 being zero. Steam flows through the inlet tube into the steam engine cylinder.

- **Expansion**. Both valves are closed. The flow velocity $c_1 = c_2 = 0$. The steam expands in the steam engine cylinder, thus exerting force which can be harnessed for the work.
- **Discharge**. The inlet valve is closed, the flow velocity c_1 being zero. The exhaust valve is fully open, the flow velocity c_2 being characterized by equation 12. The steam is discharged from the steam engine cylinder via the exhaust pipe into the external system.

The time development of the pressure in the external system can be again calculated by substituting equation 17 into continuity equation 13. The expression can be rewritten to show the time development of the pressure p_2:

$$\frac{dp_2}{dt} = \frac{1}{V_2}\frac{p_{20}}{\rho_{20}}\left(\rho_2 A_2 c_2 - \frac{dm_3}{dt}\right). \qquad (19)$$

The flow velocity c_2 is subject to the same consideration as described in the previous paragraph. Being characterized by the consumption of steam from the external system, the mass flow $\frac{dm_3}{dt} = \rho_2 \frac{dV_3}{dt}$ is a known quantity.

4 Model Implementation

By implementing the model, we mean finding, by means of a computer, the solutions of equations 18 and 19 using the given input and initial conditions. The implementation shall take place in an MATLAB-SIMULINK environment. Because of the designed model describing a real three-cylinder steam engine, the system used shall comprise of three double-acting cylinders with the angular offset $\frac{2}{3}\pi$. The dependencies of individual blocks are described in Figure 2.

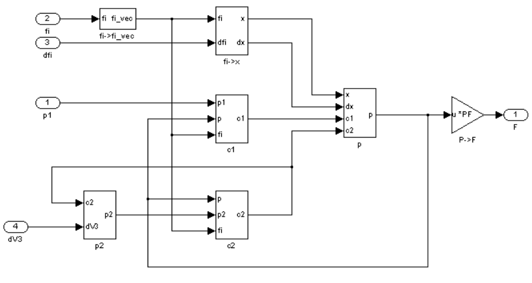

Fig. 2 Basic structure of the thermodynamic model.

The most straightforward and fastest way of calculating the results for a set of three phase-shifted cylinders is to express all quantities as vectors, with each item representing the value of the respective quantity for the cylinder in question. Matlab includes a powerful matrix calculation tool. An alternative implementation using "*for cycles*" is, in terms of calculation time, several times slower.

Because of our model assuming double-acting cylinders, every vector will comprise of six items. The work with vectors starts in the block "$fi \rightarrow fi_{vec}$" with multiplying the angular displacement of the crankshaft φ by the vector $[0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}]\pi$. The resulting vector of the angles $\overrightarrow{\varphi}$ is further transformed to the stroke vectors of the cylinders $\overrightarrow{x}$ in the block $fi \rightarrow x$. The mathematical description of this transformation can be found in [1]. The implementation is described in Figure 3.

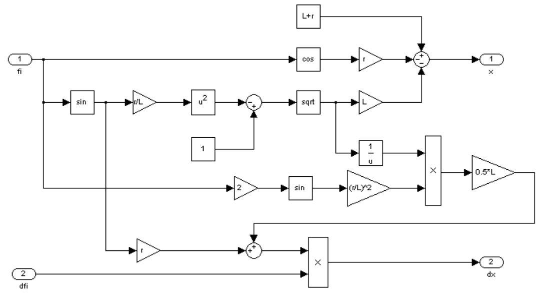

Fig. 3 Transformation of the angular displacement of the crankshaft to cylinder stroke.

Equations of state 18 and 19 are implemented in the blocks p and $p1$. Their detailed structure can be found in Figures 4 and 5.

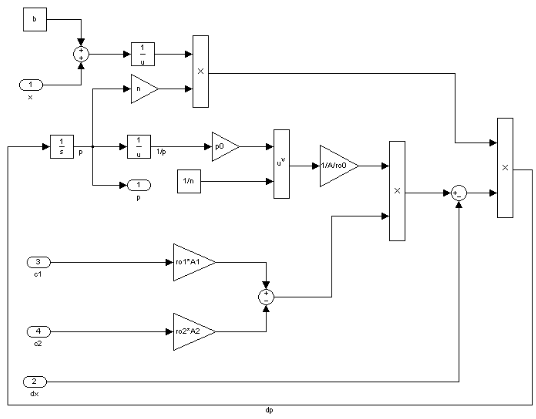

Fig. 4 Equation of state using the relationship 18.

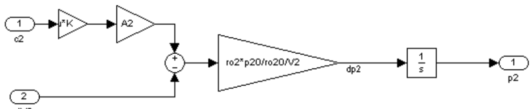

Fig. 5 Equation of state using the relationship 19.

Figures 6 and 7 show the implementation of the flow velocities c_1 and c_2 in the inlet and exhaust pipes. The ideal valves are represented by switch-blocks. However, these blocks introduce into the simulation discontinuities, which slow the process down considerably in the parts near the actuation of the switch. The simulation can be accelerated by deselecting the "*Enable zero* *crossing detection*" option; leaving this option enabled will result in more accurate results at the points near switch actuations. However, it has been proven that this setting has only a minimal impact on the overall accuracy of the simulation.

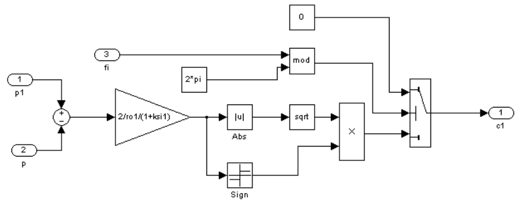

Fig. 6 Flow velocity using the relationship 11 with intake valve control.

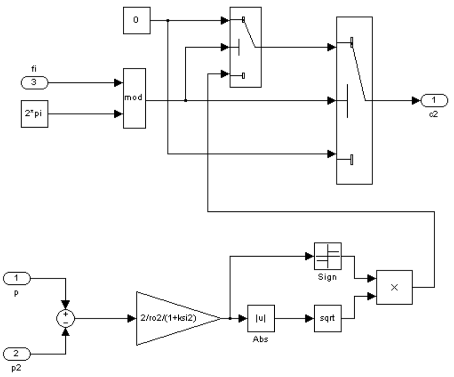

Fig. 7 Flow velocity using the relationship 12 with exhaust valve control.

5 Numerical Identification of Parameters

Numerical identification of parameters means the process of assigning numerical values to the parameters. Table 1 below gives a list of parameters and their respective values. Many parameters can be measured directly or have been provided by the manufacturer of the device, including dimensional parameters (e.g. piston surface or inlet / exhaust pipe surface).

Inlet steam parameters:

$$\begin{aligned} p_1 &= 1\,(MPa), \\ \vartheta_1 &= 190\,(C). \end{aligned}$$

The exhaust area V_2 requires the pressure $p_2 = 0.2MPa$. Steam expansion can be shown for example by means of a $h, s - diagram$ (see Figure 8). Assuming the expansion is ideal, it follows the isentropic line $s = const$ to the point 2_{id}. Real expansion will be irreversible with increasing entropy (point 2). There is also a decrease in the adiabatic cooling $\Delta h_Z = \delta(h_1 - h_{2id})$, δ being the dissipation factor. Enthalpy values at the end of expansion can be characterized by the relationship:

$$h_2 = h_{2id} + \delta(h_1 - h_{2id}). \quad (20)$$

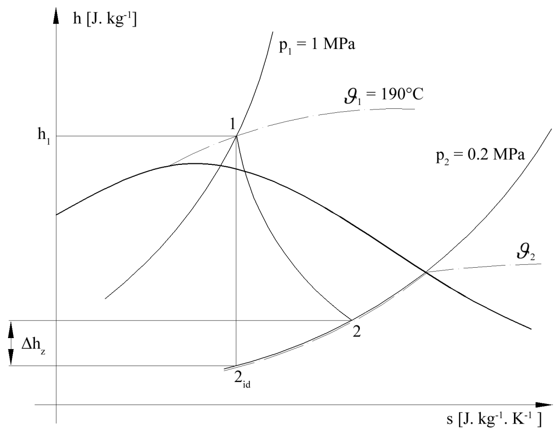

Fig. 8 h,s-diagram for steam.

In case of steam engines, the typical case is $\delta \doteq 0.3$ (see e.g. [2]). Using the Mollier diagram for steam, we can ascertain the enthalpy values and find the final state 2:

$$\begin{aligned} h_1 &= 2802\,(kJ \cdot kg^{-1}), \\ h_{2id} &= 2514\,(kJ \cdot kg^{-1}), \\ h_2 &= 2602\,(kJ \cdot kg^{-1}). \end{aligned}$$

We shall further subtract the specific volumes in points 1 and 2:

$$\begin{aligned} v_1 &= 0.2002\,(m^3 \cdot kg^{-1}), \\ v_2 &= 0.8436\,(m^3 \cdot kg^{-1}). \end{aligned}$$

These parameters can be used to determine the mean value of the polytropic exponent according to:

$$n = \frac{ln\frac{p_1}{p_2}}{ln\frac{v_2}{v_1}} = 1.119. \quad (21)$$

6 Simulation Results

Figures 9 and 10 show the results of the simulation using the parameters given in table 1. For the sake of clarity, the figures show the development of pressure for a single cylinder only. These results have been compared with the real development measured in the cylinder.

The second chart (Figure 10) shows the development of pressure in the external system. The simulation assumes that no steam was extracted from the system, resulting in a gradual pressure increase. If the pressure in the steam generator reaches the pressure in the external system, the whole process stops. In future articles, this model shall be extended by a model of a control mechanism ensuring that the steam pressure in the external system remains constant.

7 Conclusion

In this article, we have developed and implemented a mathematical model of the thermodynamic processes

Tab. 1 Model parameters

Item	**Code**	**Value**	**Unit**
Piston surface	A	$6 \cdot 10^{-3}$	m^2
Inlet pipe surface	A_1	10^{-3}	m^2
Exhaust pipe surface	A_2	$0.5 \cdot 10^{-3}$	m^2
Initial density of steam in cylinder	ρ_0	5	$kg \cdot m^{-3}$
Average density of steam in inlet pipe	ρ_1	3	$kg \cdot m^{-3}$
Average density of steam in exhaust pipe	ρ_2	3	$kg \cdot m^{-3}$
Initial density of steam in system	ρ_{20}	1.25	$kg \cdot m^{-3}$
Initial pressure of steam in cylinder	p_0	10^6	Pa
Initial pressure of steam in system	p_{20}	$0.2 \cdot 10^6$	Pa
Polytropic exponent	n	1.119	$-$
Residual stroke (related to total piston stroke)	b	10	%
External system volume	V_2	$5 \cdot 10^{-2}$	m^3
Inlet pipe dissipation factor	ξ_1	10^4	$-$
Exhaust pipe dissipation factor	ξ_2	10^4	$-$
Admission end angle	φ_{pln}	65	$grad$
Discharge start angle	φ_{vy1}	160	$grad$
Discharge end angle	φ_{vy1}	310	$grad$

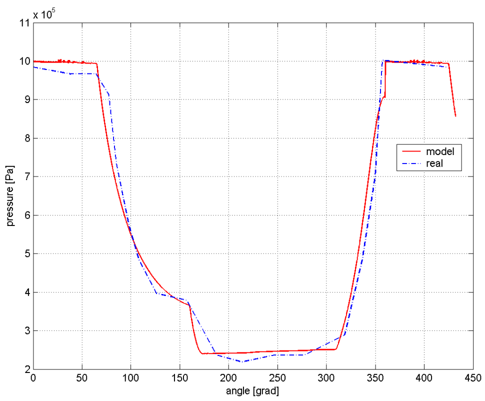

Fig. 9 Pressure development in the cylinder.

taking place in the cylinder of a piston steam engine. The model has been implemented in a MATLAB-SIMULINK environment. The model covered herein follows on from the previous article describing a model of the crank gear developed and implemented in [1]. In the future, it is envisaged to combine these two models and extend them by a pressure control system adjusting the pressure in the external system based on the changes of the load torque. The mechanical load torque will be generated using an asynchronous machine described

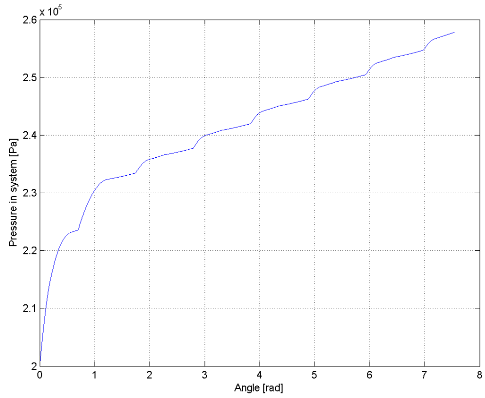

Fig. 10 Steam pressure development in the external system.

in [3]. The ultimate goal is to design a simulation model of a cogeneration unit intended as a pressure regulator in a pressure reduction station.

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