MODELLING AND SIMULATION OF SUPPRESSION VIBRATION MECHANICAL SYSTEMS

Jakub Hruška[1], Jiří Vondřich[1], Evžen Thőndel[1]

[1]Czech Technical University in Prague, Faculty of Electrical Engineering,
166 27 Prague 6. Technická 2, Czech Republic

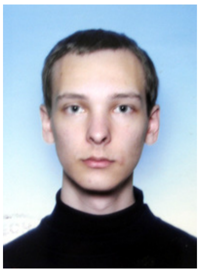
vondrich@fel.cvut.cz, thonde@fel.cvut.cz

Abstract

By attaching the absorber to the mechanical system, which is modelled as a one degree of freedom system, the new system becomes a two degree of freedom system. Depending on the driving frequency of the original system, the absorber needs to be carefully tuned, that is, to choose adequate values of the absorber mass and stiffness, so that the motion of the original mass is a minimum. The tenable vibration absorber is advantageous primarily in that it reduces the amplitude of vibrations in the mechanical system. The control vibration absorber is advantageous primarily in that it reduces the amplitude of the vibrations in the system by an oscillating force $F_{(t)}$. A vibration absorber is basically a spring-mass-damper system that is added to any vibrating system with the aim of reducing the amplitude of vibrations. The present article will discuss the method of online suppression of the vibration control LQR (Linear Quadratic Control) for a mechanical system by using a vibrating tuneable absorber. Several analyses and Matlab m-file for the auto-tuning control have been used. The aim of the paper is to acquaint the reader with the design of the incorporated absorber into the vibration system, which makes the suppression of the vibration of the mechanical system to a minimum possible.

Keywords: Absorber, Vibration, Modelling, Simulation, Matlab, Simulink

Presenting Author's biographies



Jakub Hruška is a Bachelor student in the Department of Computer Science and Engineering and interested in computational modelling with Matlab. Jiří Vondřich is an Assistant Professor in the Department of Mechanics and Materials Sciences and interested in modelling, simulation and numerical solution mechanical systems. Evžen Thőndel is a PhD student on the Department of Mechanics and Materials Sciences and interested in computational modelling and control of the dynamic systems with Matlab and Simulink.

1 Introduction

Consider the vibrating mechanical system as a machine, which is compiled of a driven motor, gearbox and mechanisms with elastic and damping parts and driven parts. Vibration transmitted to the frame of this system is possible due to unbalanced rotors and mechanisms, the crank gear of the engine, clearances in bearings, oscillation of the moving driven parts, transient loading by diverging and coasting of driving motors, etc.
The amplitude of the induced vibration is a function of the applied force and its frequency. An exciting force has the greatest effect when applied at the fundamental frequency of the system. The system is then excited at resonance, and in the case of a lightly damped system, the induced movement can be many times greater than the deflection caused by the equivalent static force. The ratio between the two effects is called the magnification factor.
Vibration in mechanical systems has two effects: First, the very high peak accelerations can mean that the effective weight of the vibrating mechanical systems increases several-fold, and this may cause its destruction. Secondly, people near the mechanical systems feel these accelerations, which can be uncomfortable or even dangerous. A vibration absorber is used to protect the mechanical systems from steady-state harmonic disturbance. The equivalent model of this mechanical system with a reduced main mass m_1, located on a cushion with coefficient of elasticity k_1 and damping coefficient b_1 and the affiliate mass m_2 of the absorber, located on a control spring with coefficient of elasticity k_2 and damping coefficient b_2 is possible to illustrate as a two mass system (Fig. 1).
In the case when the frequency ω of the acting oscillating force $F_{(t)}$ driving the system is the same as the frequency ω_2 of the vibration of the affiliate mass m_2 of the absorber, the displacement of the system is $y_1=0$.

2 Model of the mechanical system with an absorber

Consider a model of the mechanical system with absorber consisting of a mass m_1 suspended on a spring with k_1 and damping b_1 on which a force varying harmonically in time with frequency ω and maximum amplitude F is acting. The vibration absorber consists of a second mass m_2, a spring of stiffness k_2 and damping b_2 (Fig. 1).
The tuneable absorber connected with the vibrating mechanical system is advantageous primarily in that it reduces the amplitude of the vibrations in the system by an acting oscillating force $F_{(t)}$ (alternative 1, 2 and 3):

1) $F_{(t)}=me\omega^2 sin\omega t$, where m is the mass of the unbalanced rotor of mechanical system, e is the eccentricity of the unbalanced rotor and ω is the angular velocity of the unbalanced rotor - alternative 1,
2) a square wave course of the acting force $F_{(t)}$ with an amplitude force F and frequency ω - alternative 2,
3) the course of the impact load $F_{(t)} = F$ at the start of the mechanical system -alternative 3 (Fig. 10).

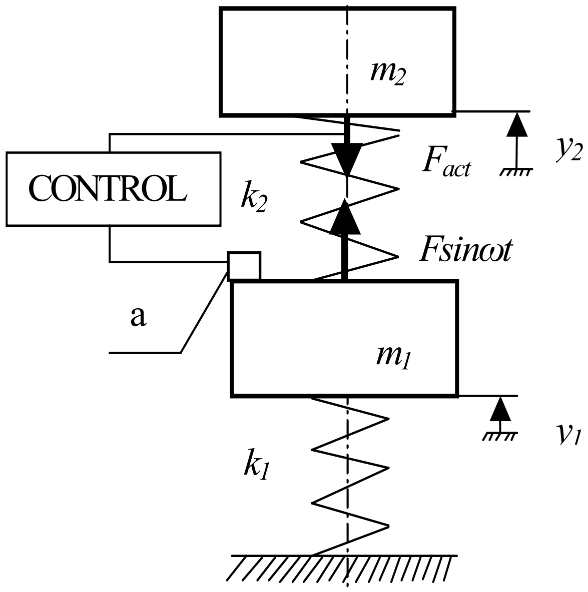


Fig. 1 The model of the mechanical system with an affiliate control absorber m_2

3 Design of the absorber

A system without an absorber is a system with one degree of freedom. The system has a circular natural frequency given by $\omega_1^2 = k_1 / m_1$. The equation of motion with forcing is (the influence of the damping $b1$ on the spring is very small):

$$m_1 \ddot{y}_1 = -k_1 y_1 - b\dot{y}_1 + F_{(t)}, \tag{1}$$

where $F_{(t)} = Fe^{i\omega t}$. The response is harmonic, with $y_1 = Y_1 e^{i\omega t}$ and:

$$Y_1 = \frac{F}{m_1(\omega_1^2 - \omega^2)}. \tag{2}$$

If the excitation frequency ω_1 is close to the natural frequency ω, the system will resonate – we get a very large response. The system will vibrate at any excitation frequency, but the amplitude of the response is largest when the excitation frequency is close to the natural frequency. The concept of the vibration absorber is that we want to reduce the motion of the mass m_1 to zero. To do this, first let us modify the SDOF to make it a system with 2 degrees of freedom, as shown below. The equations of motion

ISBN 978-3-901608-32-2

for the model of the mechanical system with the absorber (Fig. 1) are:

$$m_1\ddot{y}_1 = -k_1 y_1 + k_2(y_2 - y_1) - b_1\dot{y}_1 + b_2(\dot{y}_2 - \dot{y}_1) + F_{(t)},$$
$$m_2\ddot{y}_2 = -k_2(y_2 - y_1) - b_2(\dot{y}_2 - \dot{y}_1). \quad (3)$$

Substituting $y_1 = Y_1 e^{i\omega t}$, $y_2 = Y_2 e^{i\omega t}$ and $F_{(t)} = Fe^{i\omega t}$, yields two simultaneous equations (the influence of the damping *b1, b2* on the spring is very small):

$$Y_1 = \frac{(k_2 - m_2\omega^2)}{K}F, \quad Y_2 = \frac{k_2}{K}F, \quad (4)$$

with

$$K = (k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2.$$

These equations define the dynamics of the system with one-degree of freedom after it has been modified by attaching the secondary mass/spring system. The extra mass and spring are the absorber. Ideally, we want to completely stop the vibration of the primary mass m_1. We can do this by setting $Y_1=0$ in the first equation (2). This yield:

$$\omega = \sqrt{\frac{k_2}{m_2}}, \text{ or } \omega = \omega_2. \quad (5)$$

That is, if the natural frequency of the added mass-spring system by itself is the same as the excitation frequency, the primary mass will stop moving. What this means is that we can tune the absorber to a single excitation frequency.

The amplitude of the acting force on the mechanical system is $F = 15N$ (Fig. 1). Assume there is a maximum permissible absorber deflection of $Y_2 = 10$ mm. The motion of the secondary mass m_2 is given by the second equation (2). Assuming we use the entire clearance for the motion of the absorber, we can calculate the absorber stiffness as:

$$k_2 = \frac{F}{Y_2} = \left|\frac{15}{0.01}\right| = 1500\ kNm^{-1}. \quad (6)$$

Recall that for the absorber to work, its natural frequency (before it is fastened to the vibrating system) is the same as the excitation frequency (3):

$$m_2 = \frac{k_2}{\omega^2} = \frac{1500}{5^2} = 60\ kg, \quad (7)$$

where $\omega = 5\ s^{-1}$ is the excitation frequency.

The result of the solution equation (1) is shown in the graph of the function in dimensionless variables of the amplitude characteristic dependence displacements y_1/y_2 and angular velocities ω/ω_2 of the absorber m_2 (Fig. 2). The affiliate mass m_2 of the absorber is not moveable in the case when the ratio $\omega/\omega_2=1$. The next result is the phase characteristic: The change of the motion of the affiliate mass m_2 of the absorber is 180° in the area when the ratio $\omega/\omega_2 = 1$ (Fig. 3).

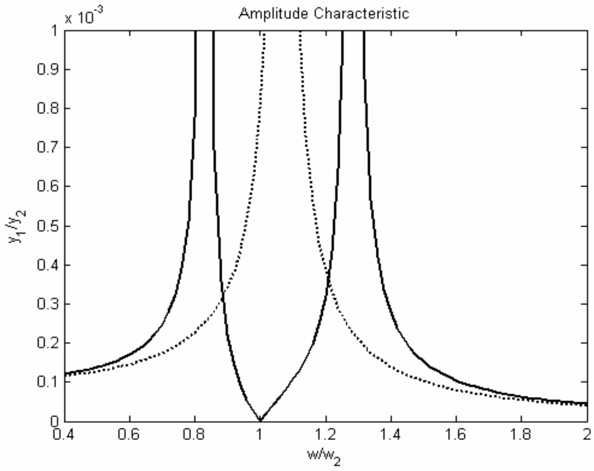


Fig. 2 The amplitude characteristic compared to dimensionless variables (the dotted line is without the absorber)

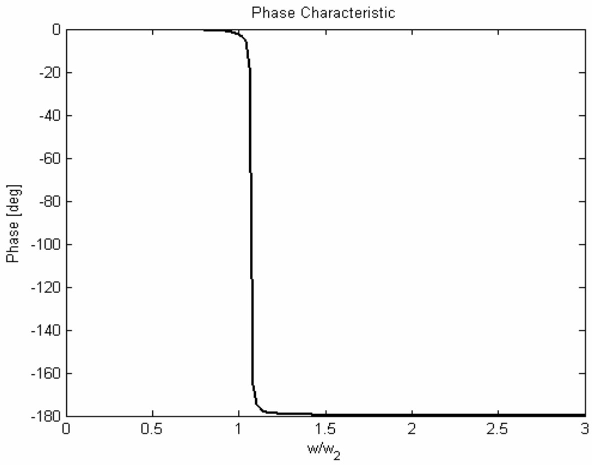


Fig. 3 Phase characteristic compared to dimensionless variables

4 Design of the tuneable absorber

The next possibility to reduce the adjustable vibration in the transient state is with the control absorber, where an air-operated spring with a changed coefficient of elasticity k_2 through the changed pressure supply of the air is incorporated. The model with affiliate mass m_2 of the control absorber's changed coefficient of elasticity k_2, an accelerometer *a* located on mass m_1 and control unit is shown on Fig. 1. It is possible to write the state description system and model (Fig. 1) in the form:

$$\dot{x} = Ax + B_{act}F_{act} + B_{tech}F_{tech}, \dot{y}_1 = Cx + Du,$$
$$\dot{x}_m = Ax_m + B_{act}F_{act} + B_{tech}F_{tech} + LC,$$
$$\dot{y}_{1m} = Cx_m + Du_m, \quad (8)$$

where A is the state matrix, C is the state matrix of the output, D is the matrix of the coupling between the input and output, F_{act} is the control force in the air-operated spring, F_{tech} is the spurious force from the technological process, B_{act} is the matrix of the control input, B_{tech} is the matrix of the spurious force input, x_m is the state vector of the model, y_{1m} is the displacement of the model and the vector of the input u is:

$$F_{act} = -G.x_m, \quad (9)$$

where G is the control matrix.

It is possible to obtain this state bond with the help of the minimization of the integral criterion on the LQR (Linear Quadratic Regulator) control [1]:

$$J = \int_0^\infty (x^T \cdot Q \cdot x + F_{act}^T \cdot R \cdot F_{act})dt, \quad (10)$$

where Q and R are balance matrices. The LQR is the control, which minimizes the accuracy of the control (the first part of Equation (10)) opposite to the power excitation (the second part of Equation (10)). The LQR control radiates from complete vector states, which in real life must not be in the feedback position. In our case, we have to dispose the output parameters from the accelerometer *a* (Fig. 1). One way, in which this problem can be solved, is use of the so-called state observer, where the parameters from the accelerometer are used to reconstruct the state of the system.

5 Results and conclusion

Vibrations of the mechanical system vanish perfectly at a certain frequency when they have a vibration absorber with small damping. But if forced frequencies vary from the anti-resonance frequency, their vibration amplitudes increase significantly. Then, the absorber with small damping cannot be applied to the structure subjected to variable frequency loads or to the loads having high frequency components. The present article discusses a method of vibration control LQR for a structure by using the vibration absorber with small damping. In the method, a variable stiffness vibration absorber is used for controlling the principle mode. The stiffness is controlled by the accelerometer *a* under the auto-tuning algorithm for creating an anti-resonance state. The optimal vibration absorber with damping with the air-operated spring is also utilized for controlling higher modes. A method to obtain the optimal parameters has been presented for the vibration absorber, which controls higher modes. In order to validate the control method and the analysis, experimental tests will be carried out in the next phase of research.

The Matlab solution Equations (3) for the parameters of the system are in the m-file on Fig. 4:

```
The constants:%---//
w = 5; %exciting frequency [rad/sec]
m1 = 350; % mass m1 [kg]
k1 =10000;   %coefficient of elasticity k1 [N/m]
m2 =60;   % affiliate masse m2 [kg] of absorber
k2 = w^2*m2;   %coefficient of elasticity k2 [N/m]
b1 = 1;       %damping coefficient b1 [Ns/m]
b2 = 1;       %damping coefficient b1 [Ns/m]
%x1 = dy1      velocity [m/s] of the  system
%x2 = dy2      velocity  [m/s] of the absorber
%x3 = y1       position y1 [m] of the system
%x4 = y2       position y2 [m] of the absorber
%The system with damping %-------------------//
A = [  -(b1+b2)/m1   b2/m1   -(k1+k2)/m1   k2/m1;
       b2/m2        -b2/m2   k2/m2         -k2/m2;
       1            0        0             0;
       0            1        0             0   ];
B = [   1/m1;
       0;
       0;
       0  ];
C = [  0  0  1  0  ];
D = [  0  ];
sys = ss(A,B,C,D);
%The system without damping%------//
A = [  -b1/m1  -k1/m1;
       1     0     ];
B = [   1/m1;
       0  ];
C = [  0  1  ];
D = [  0  ];
sys1 = ss(A,B,C,D);
%Bode characteristic%------//
W = 0.1*w:0.1:10*w;
[mag phase] = bode(sys, W);
[mag1 phase] = bode(sys1, W);
```

ISBN 978-3-901608-32-2

```
mag = squeeze(mag(1,1,:));
mag1 = squeeze(mag1(1,1,:));
phase = squeeze(phase(1,1,:));
figure(1);
plot(W./w,mag,'-k',W./w,mag1,':k');
axis([0.4 2 0 0.001]);
xlabel('w/w_h');
ylabel('y/y_h');
title('Amplitude Characteristic');
figure(2);
plot(W./w, phase,'-k');
axis([0 3 -180 0]);
xlabel('w/w_h');
ylabel('Phase [deg]');
title('Phase Characteristic');
```

Fig. 4 The Matlab m-file for the solution of the tuneable absorber

If we use the air-operated spring with the changed coefficient of elasticity k_2 with the possibility to regulate the pressure p_{act} for the air in the operated spring in dependence to the displacement y_1, it is possible to reduce this displacement y_1 to a minimum (Fig. 5, 7 and 9).

In the case when the frequency ω of the acting force $F_{(t)}$ (alternative 1, 2 and 3) driving the mechanical system is the same as the frequency ω_2 of the vibration of the absorber, the displacement y_1 of the mechanical system after starting (t=10 sec) of LQR control is shown on Fig. 5 for alternative 1, on Fig. 7 for alternative 2 and on Fig. 9 for alternative 3.

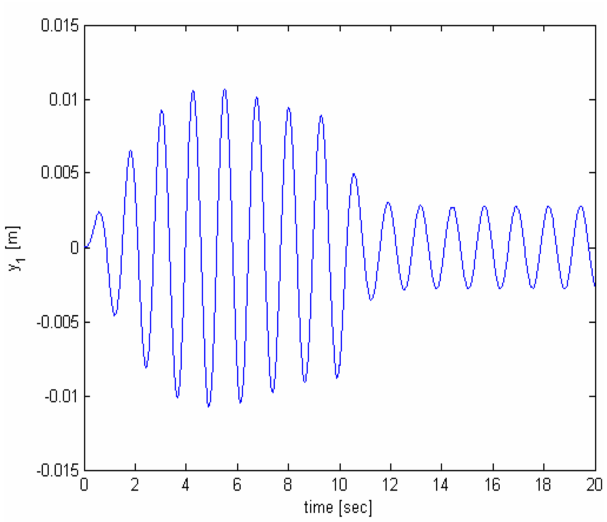


Fig. 5 Displacement y_1 of the mechanical system after starting (10 sec) the LQR control (alt. 1)

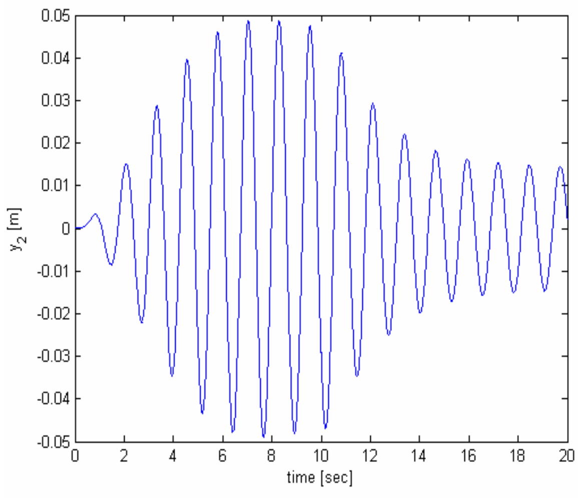


Fig. 6 Displacement y_2 of the absorber after starting (10 sec) the LQR control (alt. 1)

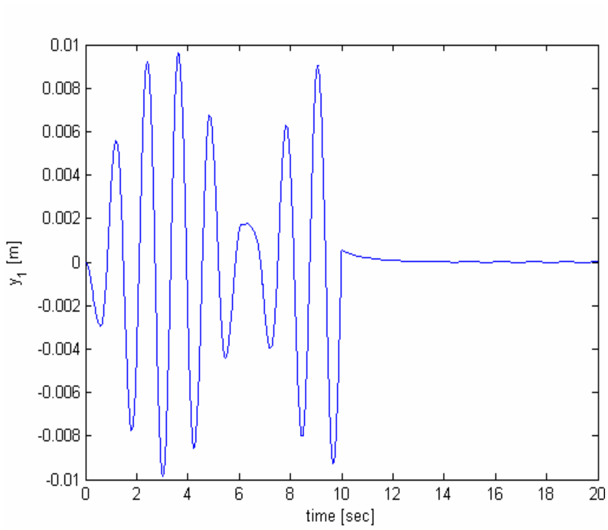


Fig. 7 Displacement y_1 of the mechanical system after starting (10 sec) the LQR control (alt. 2)

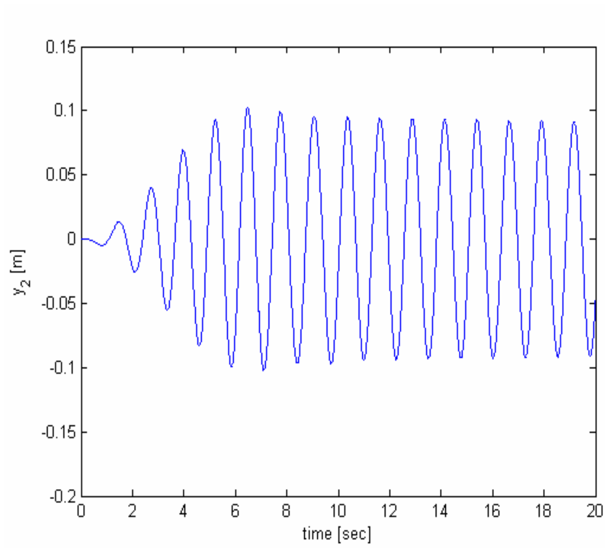


Fig. 8 Displacement y_2 of the absorber after starting (10 sec) the LQR control (alt. 1)

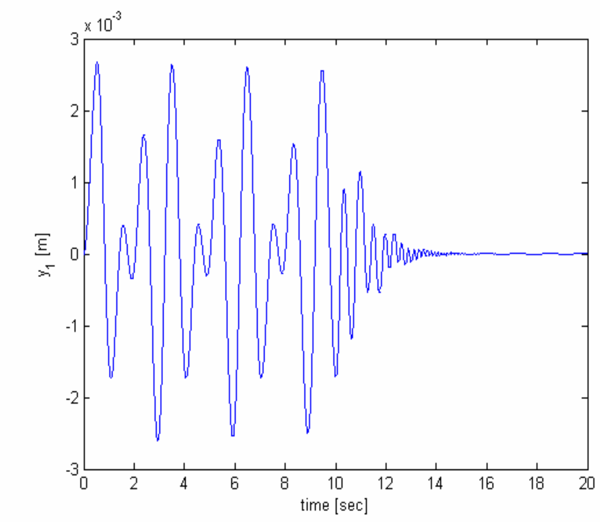


Fig. 9 Displacement y_1 of the system after starting (10 sec) the LQR control (alt. 3)

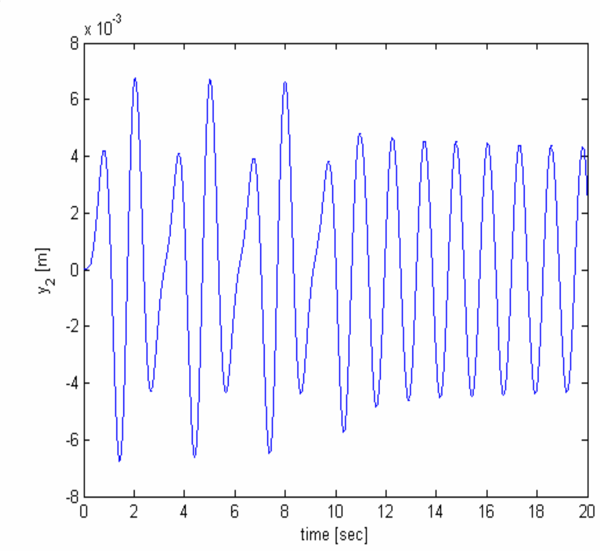


Fig. 10 Displacement y_2 of the absorber after starting (10 sec) the LQR control (alt. 3)

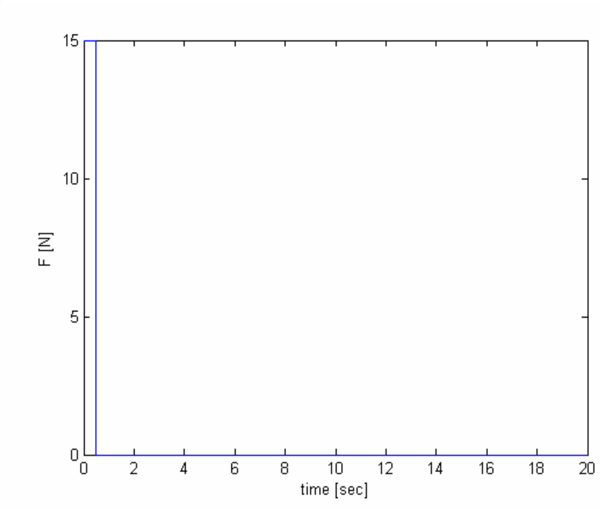


Fig. 11 Course of the impact load $F_{(t)} = F$ at the start of the mechanical system

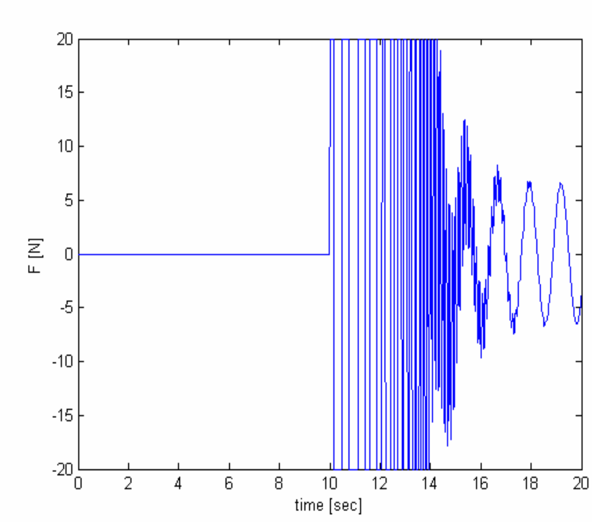


Fig. 12 Course of the feedback circuit

The actuating control signal can theoretically take an unlimited value (Fig. 11). It is necessary to respect the binding conditions of the actuator (in this case a pneumatic spring) and to insert a block into the feedback circuit, which will limit the acting control signal to a feasible value (Fig. 12).

The displacement y_2 of the absorber is shown on Fig. 6 (alt.1), Fig. 8 (alt.2) and Fig. 10 (alt.3). If we use the air-operated spring with the changed coefficient k_2 with the possibility to regulate the pressure $p_{act} = F_{act}/s$ of the air in the operated spring in dependence to the displacement y_1, it is possible to reduce this displacement y_1 in the mechanical system to a minimum.

6 References

[1] G. F. Franklin, J. D. Powell, M. L. Workman, F. L. Lewis. Control of Dynamic Systems. *Addison Wesley,* New Jersey, 2001.