MODELING SMART MARKET PRICING MECHANISM IN PACKET-SWITCHED COMMUNICATION NETWORKS

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Abstract

A considerable amount of research has been done recently into the successful QoS provisioning. One of the ways to achieve a better network service provisioning is the implementation of congestion pricing. In this paper, we are focusing on the open issues of Smart Market approach. This approach is considered to be technically inefficient and weak in the way the price is aggregated and the information about users' willingness to pay is distributed between network resources. We propose a relatively plain solution, where users express their willingness to pay for their traffic from end to end, probably differently for each Internet service they use. This information represents a bid price throughout the network, where each network resource subtracts its share of value from packet's bid price. Our conceptual study has shown that the proposed scheme is feasible and that it establishes a direct relationship between the expressed willingness to pay and the gained QoS level. To study the concept, we developed a simulation model using the systems dynamics approach, where data traffic is represented as a flow. The economic dimension of the proposed solution is represented by value flows within the simulation model. The conceptual model was then implemented in Goldsim simulation environment. The stability tests have shown that the proposed solution is feasible, and that the achieved QoS level for each network user is dependent from his willingness to pay for the traffic. The paper concludes with the discussion about implementation possibilities and future research challenges.

Keywords: Internet pricing, Congestion control, Quality of Service, Smart Market, Systems dynamics simulation, Goldsim

Presenting Author's biography

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1 Introduction

In recent years several pricing models which address the problem of Internet congestion have been studied. Today, Internet users are charged mostly on a flat-fee basis, regardless of the network load they introduce to the network, and one way to deal with congestion issues is to pay for actual traffic [1], [2]. Different QoS models have been proposed, such as price-controlled best-effort model, which introduces the general idea of usage sensitive or variable pricing - Smart Market [3]. Other possible models include per-packet or per-volume flat rate pricing, and flat rate pricing dependent from QoS class, to name just a few [4], [5]. In almost all proposed models the price is established for each network connection or a network source (e.g. routers, links). This is hard to achieve in reality for several reasons, most important one being the accounting and billing activities.

One of the open issues in the Smart Market pricing mechanism is the calculation of total costs of data transfer. These costs are basically the sum of costs incurred along the network path, or in other words, the costs for data transfer over each node-to-node connection. The Smart Market mechanism was proposed and defined for one network node only, not for the whole route the traffic is passing. One possible solution would be to include the information about the highest price the user is prepared to pay per transferred megabyte (the so called "bid price") along the whole path into his data stream. For each hop along the route, the network node which controls data entry onto the connection would subtract the connection price from the bid price indicated in the data stream. This would be repeated along the path the data stream is passing. It can happen that the bid price is too small to cover the costs of the last part of the path, so the proposition assumes at least two QoS classes - paid and unpaid traffic. The user pays according to the actual connection price, not according to his bid.

The question is, however, whether the proposed schema will work in practice. For this the stability of the system should be examined. If the basic model gives preferable estimates, the model can be used to test different pricing policies. In the article we will describe our tests of the stability and functionings of this pricing approach by using the system dynamics simulation model to test the feasibility of the pricing approach as briefly described above. Firstly, we will present a formal definition of the simulation model, which basicly includes the simulation of the computer network (connections and nodes). On the top of it, we will add the formal representation of the Smart Market pricing mechanism. Secondly, we will show how the formal definition was implemented in Goldsim, and the results we obtained from the simulation runs.

2 Simulation model development

Until recently, discrete event simulations and queuing systems were mostly used to describe and design various elements of these networks. Unfortunately, they are becoming too slow to successfully model the behavior of complex and high speed modern networks. Many proposals were given by researchers to overcome these problems. On the one hand, there are efforts to strengthen the discrete event simulations, for instance parallel simulation [6]. On the other hand, time-driven system dynamics simulation can be used to model large-scale networks. In this simulation scheme, network elements are modeled as fluid servers which process workload continuously. The data traffic is simulated as a continuous (fluid) flow in discrete time (time is partitioned into fixed-length intervals.) In the basic network model of this system's dynamics approach, physical network paths between nodes (routers) are described with matrix notation, but the actual order of network resources used was not outlined. This obstacle was corrected in [7]. Fluid models have deficiencies with the granularity of the network (in comparisson to event driven simulations), and one possible solution to this issue with the combination of fluid and discrete event sources of flows was described in [8].

In this paper, we are developing these ideas further. Our goal is to develop a relatively simple simulation approach which could be used when designing the network topologies for backbones and corporate networks, where end-to-end traffic from particular sources to destinations should be simulated, together with testing for appropriate policies for traffic shaping. The system dynamics approach is suitable for this, since its computing complexity grows linearly with the number of network nodes (netowork size). In discrete-event simulations, the computing complexity grows superlinearly with the network size [7].

In the proposed system dynamics simulation scheme, data flows between network nodes are represented in matrix notation. Before the network topology can be constructed we need to define its basic elements:

- source (s) - data traffic generator; e.g. actual use of a communication service by a user,
- connection (c) - physical or virtual connection between two nodes; e.g. optical fiber, virtual private connection,
- link (l) - path between two neighboring nodes; a part of the end-to-end network path the traffic is passing,
- node (n) - data buffer for the connections.

In the notation below we will use m, o, p and q to represent the total number of sources, connections, links and nodes, respectively, in a given network topology. Each source introduces some traffic flow onto the network. Data streams flow through the network along the path, which is represented as a series of links. Links share common connections. Each connection has its capacity (bandwidth) and price.

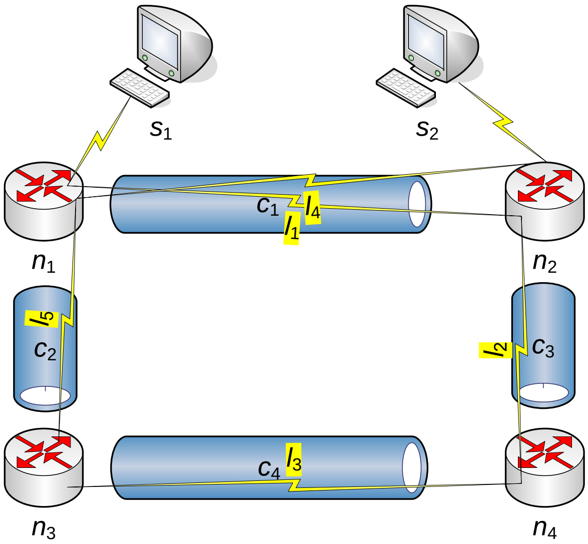

Fig. 1 Example of simple network topology with two routed data flows

An example of a simple network is represented in Fig. 1. Connections c provide means to establish data flows. The data flow from the source s_1 is routed from node n_1 to node n_3; and the route is constructed from the link l_1 from node n_1 to node n_2, the link l_2 from node n_2 to node n_4 and the link l_3 from node n_4 to node n_3. The second source s_2 uses the link l_4 from node n_2 to node n_1 and the link l_5 from node n_1 to node n_3, for the traffic to take the route from node n_2 to node n_3.

2.1 Network topology

The network topology can be provided as a set of four binary matrices. The matrix elements represent a relationship between two network elements (sources, connections, links or nodes). If elements are related, this is denoted by 1, otherwise the value of matrix element is 0. The matrices are:

- sources to links matrix, $\mathbf{S}_{sl}$,which defines the path for data flows from sources to links
- links to links adjacency matrix, $\mathbf{L}_{ll}$,which defines the path for data flows from link to link
- links to connections matrix, $\mathbf{C}_{lc}$,which defines the way connections are used by links
- links to nodes matrix, $\mathbf{N}_{ln}$,which defines the node as an origin of the link

The first and the second matrices define the basic traffic paths. Firstly, the traffic from each source is directed to one of the links (this is defined in $\mathbf{S}_{sl}$). Secondly, the traffic from each source continues its flow towards the end node according to the adjacency matrix $\mathbf{L}_{ll}$. The third matrix ($\mathbf{C}_{lc}$) represents connections sharing among links. It defines which links are occupying a particular connection. The last matrix ($\mathbf{N}_{ln}$) represents the way links use network nodes at their end-points. (This information could have been represented in another way, e.g. with the "connections to nodes" matrix.)

For instance, the network topology for the traffic from the first source s_1 to the node n_3 in Fig. 1 can be represented by the following matrix notation:

$$\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$
$$\mathbf{L} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\mathbf{N} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

The traffic utilizes three links, three connections and four nodes. We can see that the traffic from the source s_1 is directed to the first link (as defined in **S**), and it continues its way by using the second and the third link (as defined in **L**). The links l_1, l_2 and l_3 are occupying connections c_1, c_3 and c_4, respectively (as defined in **C**). Similarly, the three links are connected together and are utilizing the nodes n_1, n_2 and n_4 (as defined in **N**). Normally, connections are fixed whereas links can be changed dynamically to represent dynamic routing activities in the network.

Three characteristics of the data traffic are important for our purpose:

- the nature of the network load the sources are introducing to the network,
- capacities of connections (bandwidth), and
- capacities of nodes (buffer size).

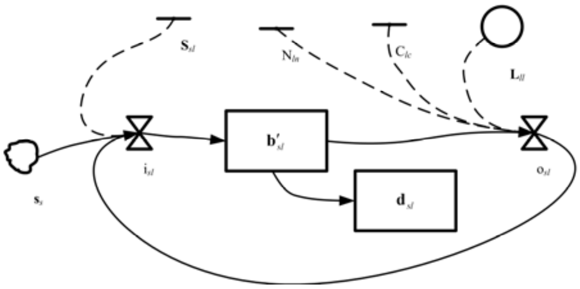

Fig. 2 Conceptual representation of the network model

A conceptual diagram of the proposed computer network model is shown in Fig. 2, where $\mathbf{i}_{sl}$ and $\mathbf{o}_{sl}$ represent input and output flows from routers, and $\mathbf{b'}_{sl}$ is the matrix with buffer occupancy data, for each source - link pair. $\mathbf{S}_{sl}$, $\mathbf{C}_{lc}$ and $\mathbf{N}_{ln}$ are sources to links matrix, links to connections matrix and links to nodes matrix, respectively. $\mathbf{L}_{ll}$ (links to links matrix) represents the routing behavior of the network. In our formal model, it is represented as a constant, but the model can be further adapted to include dynamic routing activities, simply by changing $\mathbf{L}_{ll}$. Diagram also shows $\mathbf{d}_{sl}$ as a collector of dropped packets.

The above general schema can be used in various ways. When designing complex networks one can easily simulate network behavior under different circumstances and different setups, even to include different levels of computer networks.

Optimization techniques can be applied, together with stochastic (Monte Carlo) approaches, for instance solving the problems of optimal routing flows with known end-to-end traffic demand, design problems with multi-layer networks, restoration designs to recover from failures, etc. Stochastic processess are normaly used to generate data traffic on its sources. They can be used also for other purposes, such as random failures on parts of the network. In the rest of the paper, we are adapting the general network model to test the behavior of a price-controlled end-to-end QoS provision in computer networks.

2.2 Value flows

Value flows (v) represent the "other side" of the data traffic. This is the value the users are willing to pay to send or receive data by the network. The willingness to pay for the traffic which enters the network in time t for all sources can be represented as a vector:

$$\mathbf{w}_s(t) = \begin{bmatrix} w_1(t) & w_2(t) & \cdots & w_m(t) \end{bmatrix}^{\mathrm{T}} \qquad (2)$$

Value flows travel along the network links in a similar fashion as data traffic flows, except that at each node the corresponding value is subtracted from the value flow, for each source separately. This value is the cost of data transfer along the next connection. The cost of data transfer is essentially the product of the amount of data and current price of data transfer for this connection. If there is not enough value, the data traffic is considered to be free and it doesn't receive priority service.

Similarly as we did for data traffic, each value flow which comes to a node can be described as a sum of direct and indirect flows:

$$\mathbf{i}_{sl}^{v}(t) = \mathbf{w}_s(t)\mathbf{S}_{sl} + \mathbf{o}_{sl}^{v}(t-z) \qquad (3)$$

Since in each moment data buffers on nodes contain some data packets, we should collect information about the value associated with these data:

$$\mathbf{b}_{sl}^{v}(t) = \\ = \int_{t_0}^{t} \left[\mathbf{i}_{sl}^{v}(s) - \mathbf{o}_{sl}^{v,(n)}(s) - \mathbf{o}_{sl}^{v,(s)}(s) - \mathbf{d}_{sl}^{v}(s) \right] ds + \mathbf{b}_{sl}^{v}(t_0) \qquad (4)$$

where value in buffers is being reduced by outgoing data packets which have been buffered, by outgoing data packets which are coming through the incoming ports and by dropped packets, all multiplied by users' willingness to pay denoted in that packets:

$$\begin{aligned} \mathbf{o}_{sl}^{v,(n)}(t) &= \mathbf{o}_{sl}^{d,(n,P)}(t) \otimes \mathbf{w}_{sl}^{(n)}(t) \\ \mathbf{o}_{sl}^{v,(s)}(t) &= \mathbf{o}_{sl}^{d,(s,P)}(t) \otimes \mathbf{w}_{sl}^{(s)}(t) \\ \mathbf{d}_{sl}^{v}(t) &= \mathbf{d}_{sl}(t) \otimes \mathbf{w}_{sl}^{(n)}(t) \end{aligned} \qquad (5)$$

Here, operator ⊗ represents matrix multiplication on term-by-term basis, such that:

$$\mathbf{A} \otimes \mathbf{B} = (a_{\iota\lambda})(b_{\iota\lambda}) = (a_{\iota\lambda} b_{\iota\lambda}) \qquad (6)$$

where

$$\iota = 1,\ldots,\mu; \lambda = 1,\ldots,\nu \qquad (7)$$

and μ, ν represent the number of rows and the number of columns, respectively. Similarly, operator ∅ will represent matrix division on term-by-term basis in the rest of the paper:

$$\mathbf{A} \varnothing \mathbf{B} = (a_{\iota\lambda})/(b_{\iota\lambda}) = (a_{\iota\lambda} / b_{\iota\lambda}) \qquad (8)$$

Value can leave a node in two ways. Firstly, it can be (partly) "consumed" by a node which represents the payment for data traffic along the next network link. Secondly, the remaining value of a data flow should follow the data flow.

The value which remains on nodes (payments) is current connection price multiplied by the amount of paid traffic which leaves the network node through that connection (output data flows):

$$\mathbf{m}_{sl}(t) = \mathbf{o}_{sl}^{d,(P)}(t) \otimes \mathbf{p}_{s\bullet l}(t) \qquad (9)$$

After the payment has been collected the remaining value flows together with the data flow. The best way that we can calculate this remaining value is to split it to the value which originates from data in buffers (n) and the value originating from inflows (s):

$$\mathbf{o}_{sl}^{v}(t) = \mathbf{o}_{sl}^{v,(n)}(t) + \mathbf{o}_{sl}^{v,(s)}(t) \qquad (10)$$

For the first we have:

$$o_{sl}^{v,(n)}(t)=\begin{cases} o_{sl}^{d,(n,P)}(t)\left[w_{sl}^{(n)}(t-u)-p_{s\cdot l}(t)\right], \\ \quad \text{if } w_{sl}^{(n)}(t-u)>p_{s\cdot l}(t) \\ 0, \text{otherwise} \end{cases} \tag{11}$$

and for the second:

$$o_{sl}^{v,(s)}(t)=\begin{cases} o_{sl}^{d,(s,P)}(t)\left[w_{sl}^{(s)}(t-u)-p_{s\cdot l}(t)\right], \\ \quad \text{if } w_{sl}^{(s)}(t-u)>p_{s\cdot l}(t) \\ 0, \text{otherwise} \end{cases} \tag{12}$$

both calculated for each source-link pair:

$$\begin{aligned} s&=1,\ldots,m \\ l&=1,\ldots,p \end{aligned} \tag{13}$$

Output value flows are then routed through the network according to:

$$\mathbf{o}_{sl}^{v}(t)=\mathbf{o}_{sl}^{\prime v}(t)\mathbf{L}_{ll} \tag{14}$$

Finally, for the data flow originating from buffers (*n*) we can estimate users' willingness to pay for each source-link pair as:

$$\mathbf{w}_{sl}^{(n)}(t)=\mathbf{b}_{sl}^{v}(t)\oslash\mathbf{b}_{sl}^{d}(t) \tag{15}$$

The corresponding willingness to pay for inflows (*s*) is:

$$\mathbf{w}_{sl}^{(s)}(t)=u\mathbf{i}_{sl}^{v}(t)\oslash u\mathbf{i}_{sl}^{d}(t) \tag{16}$$

2.3 Measurements

Since dynamic pricing mechanism depends from traffic measurements, this can be established in our simulation model relatively easy. The dynamic price mechanism should be sensitive to the connections occupancy, so we measure average connection occupancy of paid and free traffic:

$$\bar{\mathbf{o}}_{c}^{d}(t-x,t)=\bar{\mathbf{o}}_{c}^{d,(P)}(t-x,t)+\bar{\mathbf{o}}_{c}^{d,(N)}(t-x,t) \tag{17}$$

the average values being calculated for the time interval [*t*-*x*, *t*].

Different policy scenarios lead to different measurement strategies. In this paper we are limiting the structure of measurements to the dimensions of paid/free traffic and to inflow and buffer traffic:

$$\begin{aligned} \bar{\mathbf{o}}_{c}^{d,(P)}(t-x,t)&=\bar{\mathbf{o}}_{c}^{d,(n,P)}(t-x,t)+\bar{\mathbf{o}}_{c}^{d,(s,P)}(t-x,t) \\ \bar{\mathbf{o}}_{c}^{d,(N)}(t-x,t)&=\bar{\mathbf{o}}_{c}^{d,(n,N)}(t-x,t)+\bar{\mathbf{o}}_{c}^{d,(s,N)}(t-x,t) \end{aligned} \tag{18}$$

Average values are obtained by the values of $\mathbf{o}_{sl}^{d,(n,P)}(t)$, $\mathbf{o}_{sl}^{d,(s,P)}(t)$, $\mathbf{o}_{sl}^{d,(n,N)}(t)$ and $\mathbf{o}_{sl}^{d,(s,N)}(t)$.

2.4 Pricing

Price of the traffic via each network connection is being dynamically recalculated as:

$$\begin{aligned} &\mathbf{p}_c(t)=\mathbf{p}_c(t-u)\otimes \\ &\otimes\left\{\mathbf{e}_c\left[\mathbf{c}_c^{(P)}(t)+\bar{\mathbf{o}}_c^{d}(t)\right]+\mathbf{c}_c^{(P)}(t)-\bar{\mathbf{o}}_c^{d}(t)\right\}\oslash \\ &\oslash\left\{\mathbf{e}_c\left[\mathbf{c}_c^{(P)}(t)+\bar{\mathbf{o}}_c^{d}(t)\right]-\mathbf{c}_c^{(P)}(t)+\bar{\mathbf{o}}_c^{d}(t)\right\} \end{aligned} \tag{19}$$

where $\mathbf{e}_c$ is a vector of price elasticities for each network connection and $\mathbf{c}_c^{(P)}(t)$ is the connection capacity available for paid traffic. The elasticity can be estimated by employing the methodology further developed in [2].

3 Simulation model implementation

As a simulation tool we used GoldSim [9], which implements a systems dynamics approach, and the models can be enriched with discrete events.

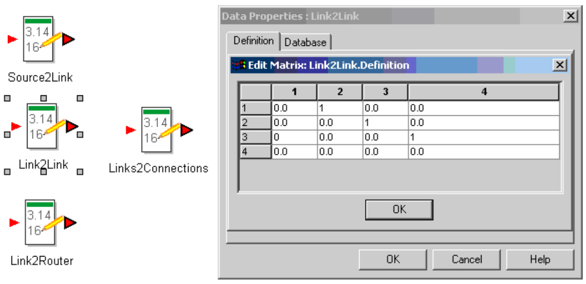

Fig. 3 The network topology is represented in matrix notation

In our model, the network topology is represented by matrices, as we have shown in previous chapter. For instance, the matrix Link2Link in Goldsim implementation (please see Fig. 3) shows that the first link is connected to the second, the second is connected to the third, and the third is connected to the fourth link.

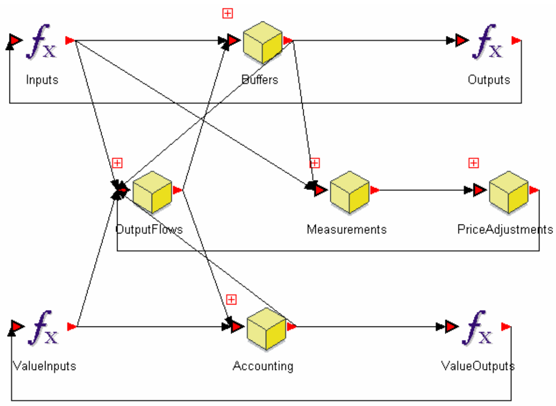

Fig. 4 The main structure of the network model

The model elements are grouped in four modules (containers), where the most complex one is the container which represents the communication network (other containers represent such parts as source behavior and parameters of the model, e.g. the above mentioned topology). The structure of the network model can be seen on Fig. 4.

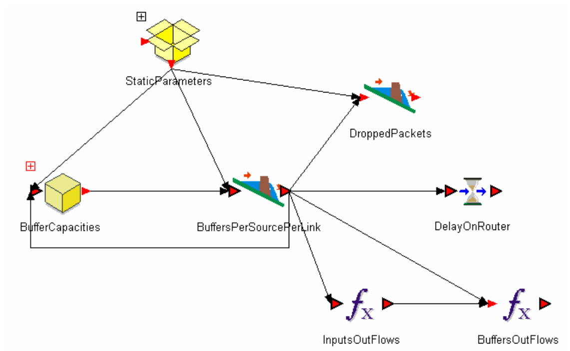

Fig. 5 The details of network node model

The top part of the model represents the data flows, while the bottom part models the value flows. (The traffic introduced into the network by sources is modeled as being stochastic.) The middle part on Fig. 4 includes the calculation of output flows (both data and value flows), together with the traffic measurements and price adjustments in the case of variable pricing. Fig. 5 shows the details of network node model, which includes such estimations and calculations as buffer capacities, dropped packets, the delay on network node, etc. The model tracks data and value flows for each source, network node and connection in time.

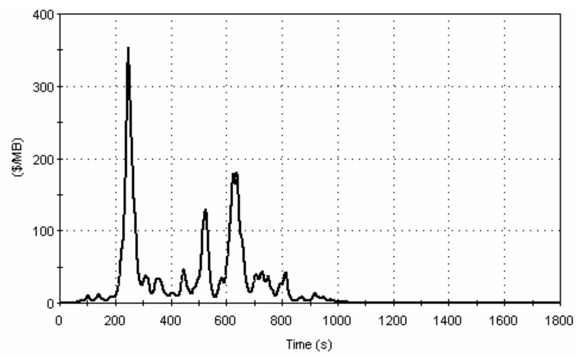

Fig. 6 The connection price

Figures 6, 7, 8 and 9 give the results from running one realization, where a single connection (and link) is shared among 20 sources. Each source introduces 2 Mbps of unpaid traffic on average. The connection capacity is 200 Mbps, so it is on its limits (the network is in congestion state). The first user (source) decides to pay for his traffic, and the price he is willing to pay is at most 80 \$ per MB. We can see from figure 4 that the connection price is sometimes above his bid. (The connection price is variable; it is recalculated each second, which introduces relatively quick and drastic changes to its value.) Figure 7 represents the data stream from the first user for which he is prepared to pay for. We can see that in time intervals where the connection price is higher than his bid, the data stream changes to another (unpaid) QoS class.

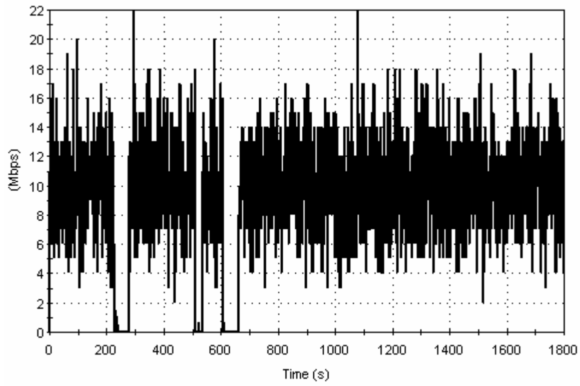

Fig. 7 Paid network traffic for the first user

Figure 8 shows the data traffic overflow rate for the first user. Data traffic overflow rate represents dropped packets, and is dependent from traffic state (congestion), the buffers capacity on network nodes, and QoS class. When user pays for his traffic, the dropping of packets doesn't occur (paid traffic has a higher priority than unpaid one). Figure 9 shows data traffic overflow rate for his fellow user, who is not prepared to pay for his traffic, and is othewise behaving in similar fashion as the first user.

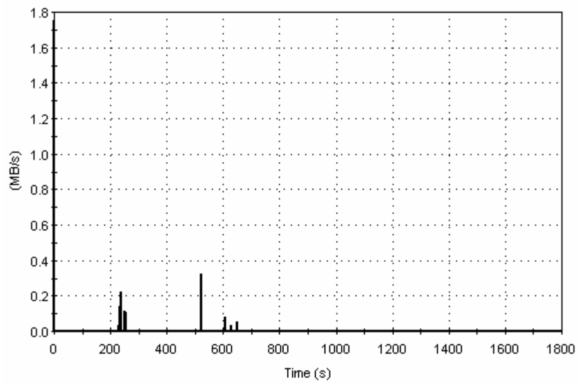

Fig. 8 Traffic overflow rate (dropped packets) of the first user (mostly paid traffic)

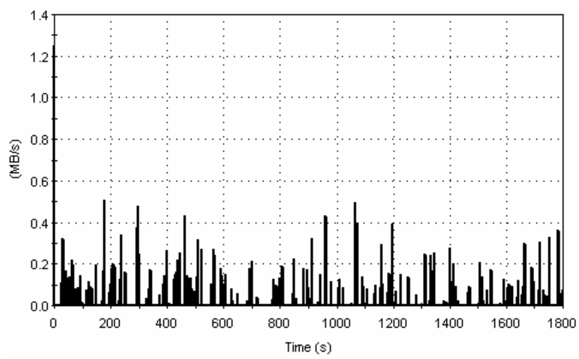

Fig. 9 Traffic overflow rate (dropped packets) of the second user (unpaid traffic)

Figure 10 shows the average values of the percentage of dropped packets for the traffic allong the route compared to the expressed willingness to pay (bold lines, denoted by fixed in legend). It can be seen that the expressed willingness to pay greatly influences the established QoS level, measured by the percentage of dropped packets. Different calculation intervals were tested (the intervals when the network nodes calculate a new price), with different policies regarding the ratio of bandwidth available for paid and unpaid traffic (fixed or adaptive according to the traffic conditions). There are no significant differences in the effectiveness of the pricing mechanism due to different price calculation intervals.

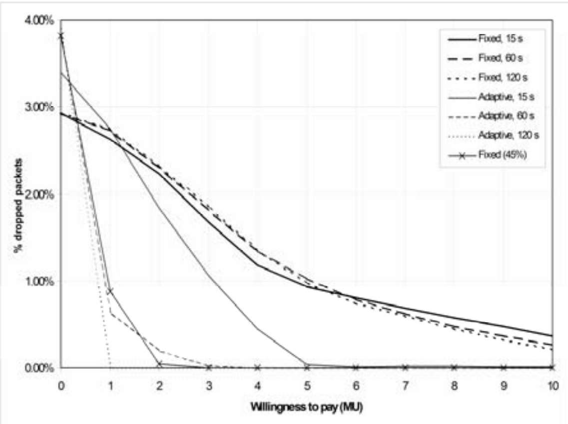

Fig. 10 Average percentage of dropped packets compared to expressed willingness to pay under different policies

4 Conclusion

The proposed pricing mechanism has some interesting properties. Firstly, it doesn't exclude the best-effort network traffic. Routers, servers and client applications with new protocols could be gradually installed as needed, if a new practice would prove to be useful (to users and network operators). It is not necessary that the transition is instantaneous. Secondly, network operator must decide just on three parameters, the amount of connection capacities reserved for paid traffic (if that kind of policy is chosen), the price calculation interval and the averaging period of measured traffic.

Besides exploring the feasibility of the proposed pricing mechanism, we have shown that the communication networks can be successfully modeled with system dynamics approach. In our model, matrix notation is used to represent the topology of the network, together with routing characteristics of the network, and this is the main advantage of our approach. Besides this, the modelling of systems where traffic characteristics are important (like priority class) is possible, as we have shown in example, where we tested the behavior of a price-controlled QoS end-to-end provision in computer networks.

The development of a new simulation model for decision making purposes (e.g. when deciding about investments into a particular network topology) with this kind of approach is very straightforward and simple process, since only the topology of the network and the routing behavior should be described by defining matrices **L**, **C** and **N**. It can be used when designing complex networks, together with optimization techniques and stochastic (Monte Carlo) modelling. However, if the model is considered to be used in analyses where detailed data packet attributes should be tracked (for each data packet), the model should be enriched with discrete-event servers. There is also a possibility to combine a system dynamics approach and agent-based simulations, where detailed characteristics of network elements are described as properties of object instances, and different network element types are defined as object classes.

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