MODELLING OF FACTS DEVICES FOR TRANSIENT-STABILITY ASSESSMENT USING DIRECT METHODS

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Abstract

To analyze the behaviour of electric-power systems (EPSs), the simulation on the base of the proper model is the only practical solution. Variouos models of an EPS are used for various types of the analysis. The paper deals with the phenomena of the transient stability, i.e., the stability of an EPS when subjected to large disturbance. While a typical method for transient-stability assessment is the repetition of numerical integration of EPS's nonlinear differential equations, in this paper we focus on an alternative, simplified method, i.e., the Lyapunov direct method. Using this method, an EPS should be presented with an energy function, i.e., with the set of nonlinear algebraic (non-differential) equations. The energy function for the EPS was already constructed. However, with the introduction of new devices like FACTS devices this energy function should be supplemented in order to consider the effect of these devices. The paper presents how to model FACTS devices and how to supplement the existing energy function for the case of a STATCOM with an energy function (STATCOM-ESS). An injection model of a STATCOM-ESS is constructed and according to this model the energy function for the EPS that includes a STATCOM-ESS is constructed. The energy function is developed for the EPS with preserved structure, therefore the effect of the STATCOM-ESS is described by an additional term that can be added to the existing energy function. This approach enables a simultaneous consideration of multiple FACTS devices connected at various nodes of an EPS. The adequacy of newly constructed energy funtion was proved by numerical examples of transient-stability assessment using the Lyapunov direct method. The proposed energy functions proved to be adequate and the results show an improvement of transient stability.

Keywords: Power-system control, power-system stability, FACTS devices.

Presenting Author's biography

Valentin Ažbe received his B.Sc., M.Sc. and Dr.Sc degrees from The University of Ljubljana, Slovenia, in 1996, 2003, and 2005, respectively. After receiving his diploma he worked with IBE, consulting engineers, Slovenia, as a project manager in the Department for Overhead-Lines design. In 2000 he joined the Department of Power Systems and Devices at the Faculty of Electrical Engineering, The University of Ljubljana, where he has since worked as a junior researcher. In 2005 he became a Teaching Assistant. His areas of interest include system analysis, FACTS devices, power-system protection and DC power-system analysis.

1 Introduction

The vast electric-power systems (EPSs) may be considered as the largest and most expensive of man-made systems. In recent years the market liberalization affect drastically the operation of EPSs, which under economical pressure and increasing amount of transactions are being operated much closer to their limits than previously. Due to utilization of transmission networks for more flexible interchange transactions, the need for EPS's dynamic analysis has grown significantly.

To analyze the dynamics of an EPS, simulation is practically the only way to do it. Various transients can be simulated according to the speed of these transients: a) ultrafast transients caused by atmospheric discharges on the exposed transmission lines, b) medium-fast transients caused by short-circuit phenomena and c) slow transients – electromechanical oscillations of synchronous machine rotors, denoted as a transient stability. According to the type of transients, various models of an EPS should be developed.

This paper deals with analyzing of the slowest, but most important type of transients, i.e., transient stability. From a physical viewpoint, transient stability may be defined as the ability of an EPS to maintain machines' synchronous operation when subjected to large disturbance. From the system theory viewpoint, transient stability is a strongly nonlinear, high-dimensional problem.

Typical method for transient-stability assessment is the repetition of numerical integration of EPS's nonlinear differential equations for various disturbances (e.g. for various times of fault clearing). Application of computers and special software tools enables a detailed modelling of an EPS and gives precise and reliable results. Without special software tools this method is practically useless. Therefore, simplified methods were searched in the past (before the computer era) and the only one applicable for the transient stability assessment of EPS proved to be the Lyapunov direct method. In recent years, the Lyapunov direct method became interesting again because of its simplicity, which might enable its application in on-line transient stability assessment. For this task a repetition of numerical integration of EPS's nonlinear differential equations—usually denoted as "simulation method"—still seems to be too slow for practical application.

This paper is focused on the Lyapunov direct method. The main task for application of this method is the construction of proper energy function that represents an EPS with the set of algebraic nonlinear equations. This energy function can be found only with the intuition. For the EPS without FACTS devices various energy functions are already developed. In the case of FACTS devices included in an EPS, those energy functions should be supplemented.

The paper presents how to model FACTS devices and how to supplement the existing energy function for an EPS. Fore some of FACTS devices a supplememnted EPS's energy functions were already constructed: for a static var compensator (SVC) in [1]-[2], for an SSSC in [3], for an UPFC in [4] and for phase-shifting transformers in [5]. In this paper we summarize the procedures and construct a supplemented energy function for the case of the EPS that include a STATCOM with an energy-storage system (STATCOM-ESS).

The paper is organized as follows. In section 2 the Lyapunov direct method and the energy function for an EPS is described. In section 3 an energy function for an EPS including a FACTS device—i.e., STATCOM-ESS—is constructed after the model of this device is presented. Section 4 presents numerical examples of transient-stabilitay assessment using Lyapunov direct method that prove the correctness of newly constructed energy function. Section 5 draws conslusions.

2 Lyapunov direct method

The principle of the Lyapunov direct method was defined by a Russian mathematician and physicist A.M. Lyapunov at the end of 19^{th} century and is sometimes denoted as Lyapunov second method. A method was constructed for the mechanical model and can be evidently presented on the case of a ball rolling on the three-dimensional bowl-shaped surface presented in Fig. 1. The main idea about stability assessment using the direct method is—instead of observing the path of rolling ball (which is determined with the set of second-order differential equations, similarly as swing equations for an EPS)—to determine the kinetic energy of a ball and compare it to the potential energy of the bowl-shaped surface. The advantage of this method is that previously mentioned kinetic and potential energy is defined with the set of algebraic (non-differential) equations.

Besides its application in mechanical systems the Lyapunov direct method can be applied for other systems like the EPS. Crucial for this method is to define the system with the proper set of algebraic equations—it is the most pretentious part of the Lyapunov direct method. As the most suitable functions that define the system proved to be the one based on physical laws, like e.g., energy functions.

The accuracy of the Lyapunov direct method used for transient-stability assessment of an EPS primarily depends on the accuracy of the energy function, which further depends on the level of simplification of an EPS. In General, energy functions are divided to those that preserve the structure of an EPS and to those that need its reduction. The developement of the energy functions for EPSs reached its zenith in 1980's and today it still continue, mainly in correlation with the request of transmission-system operators for on-line transient stability assessment.

Considering FACTS devices to be included in EPS, energy functions should be supplemented in order to consider the effect of these devices on transient stability. In other case, the results might be far from the reality.

2.1 Construction of energy function for an EPS

A precondition for application of direct method is the construction of an adequate function that describes an EPS with the set of algebraic non-differential equations. Various functions were created, but the only useful one proved to be a function that represents an EPS as the sum of kinetic and potential energy after the fault clearing [6] and was obtained as the first integral of non-linear differential equations (swing equations) of the EPS.

In [7] and [8] the construction procedure of energy function for an EPS with preserved structure is presented. The result of these procedures is the energy function for an EPS with N-bus and m-generators:

$$V\left(\tilde{\omega},\tilde{\phi},U\right)=V_{\mathrm{k}}\left(\tilde{\omega}\right)+V_{\mathrm{p1}}\left(\tilde{\phi},U\right)+V_{\mathrm{p2}}\left(\tilde{\phi}\right)+K \quad (1)$$

where $\tilde{\phi}=\left[\tilde{\delta}^{\mathrm{T}},\tilde{\theta}^{\mathrm{T}}\right]^{\mathrm{T}}$,

δ is a vector consisting of m rotor angles,
θ is a vector consisting of N bus voltage angles,
ω is a vector consisting of m rotor velocities and
U is a vector consisting of N bus voltage magnitudes

The tilde "~" denotes the values in the center-of-angle system. K is an arbitrary constant, usually chosen so that it places the origin of (1) at zero. V_{k} is the kinetic energy. The rest of the total system energy (1) is the potential energy, where V_{p2} represents the potential energy of the active part of the loads, and V_{p1} stands for:

$$\begin{aligned}V_{\mathrm{p1}}\left(\tilde{\phi},U\right)=&-\sum_{i=1}^{m}P_{\mathrm{m}i}\tilde{\phi}_i+\sum_{i=m+1}^{m+N}\int\frac{Q(U_i)}{U_i}\mathrm{d}U_i\\&-\frac{1}{2}\sum_{i=1}^{m+N}\sum_{j=1}^{m+N}B_{ij}U_iU_j\cos\tilde{\phi}_{ij}\end{aligned} \quad (2)$$

where $P_{\mathrm{m}i}$ is the i-th machine mechanical input, $Q(U_i)$ is the voltage-dependent reactive part of the load at bus i, and B_{ij} represents the susceptance in an augmented admittance matrix. The potential energy of the system loads V_{p2} can be analytically solved only in the case of constant active loads at each of N buses:

$$V_{\mathrm{p2}}=\sum_{i=m+1}^{m+N}P_i\cdot\tilde{\phi}_i \quad (3)$$

If the reactive part of the load $Q(U_i)$ in (2) at each of N buses is defined as a constant susceptance B_{load}, integral in (2) can be analytically solved:

$$\begin{aligned}V_{\mathrm{p1}}\left(\tilde{\phi},U\right)=&-\sum_{i=1}^{m}P_{\mathrm{m}i}\tilde{\phi}_i+\frac{1}{2}\sum_{i=m+1}^{m+n}B_{\mathrm{load}}U_i^2\\&-\frac{1}{2}\sum_{i=1}^{m+N}\sum_{j=1}^{m+N}B_{ij}U_iU_j\cos\tilde{\phi}_{ij}\end{aligned} \quad (4)$$

The kinetic energy V_{k} is the energy accumulated in generators' rotors due to their acceleration, while the sum of the potential energy V_{p1} and V_{p2} stand for the ability of the system to receive the energy accumulated in rotors, i.e., angles of rotors according to the center-of-angle of an EPS.

2.2 The use of the energy function

The application of the energy function in the direct method can be illustrated with a mechanical analogy as a ball rolling on a potential energy surface, as in [9]. This visualization is presented in Fig. 1. The potential energy V_{p} of a given post-fault system depends on the machine angles. In the case of a three-machine system the potential energy V_{p} can be presented as a surface on a three-dimensional chart, where the horizontal axes represent the angle of two machines according to the third machine (i.e., δ_1 and δ_2 according to Fig. 1). The potential-energy surface has a local minimum at the stable equilibrium point, which corresponds to the machine angles during the post-fault steady-state operation. Around this stable equilibrium point the potential-energy surface forms a bowl-shaped area, which is the area of stable system operation.

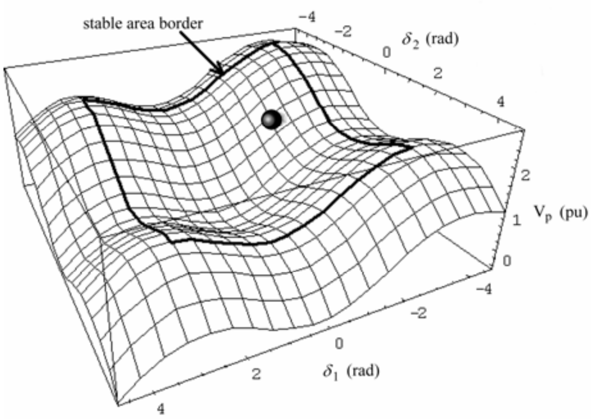

Fig. 1. A ball on a potential-energy surface.

The position of the ball presents an operating point of an EPS. The kinetic energy of the system is equated to the kinetic energy of a ball that rolls along the potential-energy surface according to the generator swing trajectory. In steady-state operation the ball stands still at the stable equilibrium point. However, when the fault occurs, the ball is pushed toward the edge of the bowl-shaped area of the potential-energy surface until the fault is cleared. Depending on the total of the kinetic and potential energies of the ball at the time of fault clearing, the ball can either escape from the bowl over the saddle (i.e., an unstable case) or it can continue to oscillate within the bowl (i.e., a stable case). To assess the stability of the system the kinetic energy is compared with the potential energy at the border of the stable area. To set the point at this border—i.e., a critical value V_{cr}—that the sum of kinetic and potential energy of a ball should not exceed in order to maintain stability, is after the construction of the energy function (1) the next crucial step in application of the direct method. The exact point where the ball crosses the border analytically can not be found and is usually estimated using various methods [10] that in general can be divided into three groups:

a) Basicly the Lyapunov direct method set the critical value V_{cr} as the value of the energy function at the lowest saddle. In this case we are at the "safe side", but the results are unfortunately too pessimistic and values of all saddle points should be calculated.

b) The next option for determing of V_{cr} is to set it as the value of the energy function at the saddle point that is the nearest to the system's trajectory. This method requires comprehensive calculations. However, numerical simulations can be applied for this task that lead to hybrid methods that combine numerical simulations and direct methods.

c) One of the easiest methods is the one that determs V_{cr} as the value of the energy function (1) at the point where the ball crosses the stable-area border in the case that the fault is not eliminated. This method is denoted as a potential energy boundary surface (PEBS) method and gives good results if the trajectory of the system after the fault clearing matches well to the trajectory of faulted system.

3 Modelling of FACTS devices – the case of a STATCOM -ESS

The construction procedure of the energy function for an EPS that includes FACTS devices is presented on the case of a STATCOM with an energy-storage system (STATCOM-ESS). An injection model of a FACTS device should be constructed firstly. Then, according to this injection model, the supplement to the existing energy function for an EPS without FACTS devices is constructed and this supplement is denoted as the energy function for a chosen FACTS device. In following paragraphs the injection model and the energy function for a STATCOM-ESS is presented.

3.1 Injection Model

A STATCOM-ESS is a parallel current source. Generally, it is composed of a parallel transformer, a current source converter and a DC energy-storage system. A STATCOM-ESS can inject active and reactive power, and the angle between the injected current and the bus-voltage can take any value and is not limited to the angles 90° and –90°, as it is in the case of a STATCOM without an energy-storage system (neglecting losses). Fig. 2 presents the scheme, the phasor diagram and the injection model of a STATCOM-ESS.

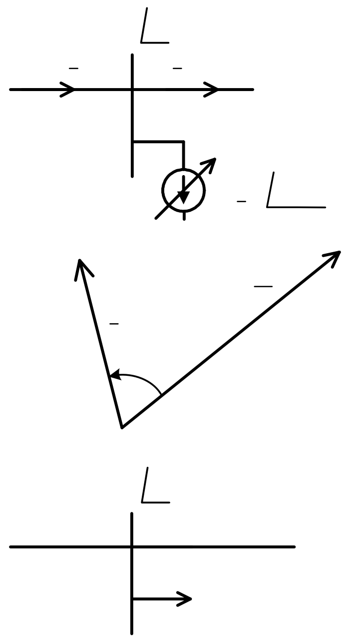

Fig. 2 STATCOM-ESS; a) scheme; b) phasor diagram; c) injection model.

Power injections, which are the basis for the construction of an energy function, are the product of the bus voltage and the injected current:

$$P_{si} = I_{\mathrm{P}} \cdot U_i \cdot \cos(\beta) \tag{5}$$

$$Q_{si} = I_{\mathrm{P}} \cdot U_i \cdot \sin(\beta) \tag{6}$$

3.2 Construction of an Energy Function

In [7] the structure-preserving energy function is constructed for the EPS without FACTS devices and we denote it as $V_{\text{without FACTS}}$. In this paper we construct the energy function for a STATCOM-ESS that can be added as an additional term to the $V_{\text{without FACTS}}$. Following the construction procedure presented in [7], the active-power injection P_{si} is multiplied by the time derivative of the voltage angle, and the reactive-power injection Q_{si} is divided by the voltage magnitude and multiplied by the time derivative of the voltage magnitude. Next, both terms are summed together and the following expression is obtained:

$$I_{\mathrm{P}} \cdot U_i \cdot \cos(\beta) \cdot \dot{\theta}_i + I_{\mathrm{P}} \cdot \sin(\beta) \cdot \dot{U}_i \tag{7}$$

If the control parameters I_{P} and β are constant, the integral of (7) can be rewritten as:

$$\begin{aligned} &\int \left[I_{\mathrm{P}} \cdot U_i \cdot \cos(\beta) \cdot \dot{\theta}_i + I_{\mathrm{P}} \cdot \sin(\beta) \cdot \dot{U}_i \right] dt \\ &= I_{\mathrm{P}} \cdot \sin(\beta) \cdot U_i + \int I_{\mathrm{P}} \cdot \cos(\beta) \cdot U_i \; d\theta_i \\ &= Q_{si} + \int P_{si} \; d\theta_i \end{aligned} \tag{8}$$

Equation (8) is the energy function for a STATCOM-ESS with constant control parameters I_{P} and β:

$$V_{\text{STATCOM-ESS}} = Q_{si} + \int P_{si}\ d\theta_i \quad (9)$$

The first part of (9) is the contribution of the reactive-power injection; therefore, it can be considered as an energy function for a STATCOM without an energy-storage system. The second part of (9) is the contribution of the active-power injection and it is in the form of an integral. This integral cannot be analytically solved because the dependence of P_{si} on θ_i isnot explicitly determined, but it is implicitly described with the set of the EPS's non-linear algebraic equations. However, (9) can be applied in a transient-stability assessment in a numerical way, as is presented in section 4.

Based on the treatment of the active loads in [7], where it is shown that a true Lyapunov energy function without an integral part can be obtained only for constant active-power loads, the energy function for a STATCOM-ESS was searched, with the angle β controlled in such a way that P_{si} is constant. Assuming that $\underline{I}_{\text{P}}$ and P_{si} are constant, the integral of (7) can be rewritten as:

$$\int I_{\text{P}} \cdot \cos(\beta) \cdot U_i\ d\theta_i + \int I_{\text{P}} \cdot \sin(\beta)\ dU_i \quad (10)$$

The first part of (10) is the integral of the active-power injection over the angle θ_i. Because the active-power injection is constant, the first integral in (10) can be easily solved. In the second integral of (10) the angle β is not constant, but the integral can be transformed as:

$$\begin{aligned} &\int I_{\text{P}} \cdot \sin(\beta)\ dU_i \\ &= \int \Big[I_{\text{P}} \cdot \sin(\beta) \cdot \dot{U}_i + I_{\text{P}} \cdot U_i \cdot \cos(\beta) \cdot \dot{\beta} \\ &\quad - I_{\text{P}} \cdot U_i \cdot \cos(\beta) \cdot \dot{\beta} \Big]\ dt \end{aligned} \quad (11)$$

The first two terms in square brackets can be denoted as the time derivative of the injected reactive power Q_{si}, while the third part of (11) inherits a term for injected active power that is constant. Therefore, (11) can be rewritten as:

$$\begin{aligned} &\int \left[\frac{d}{dt} \left[I_{\text{P}} \cdot \sin(\beta) \cdot U_i \right] \right] dt\ -\ \int P_{si}\ d\beta \\ &= I_{\text{P}} \cdot \sin(\beta) \cdot U_i - P_{si} \cdot \beta \\ &= Q_{si} - P_{si} \cdot \beta \end{aligned} \quad (12)$$

Now (12) can be inserted into (10) and the energy function for a STATCOM-ESS with a constant current magnitude, I_{P}, and constant injected active power, P_{si}, is obtained:

$$V_{\text{STATCOM-ESS}} = Q_{si} + P_{si} \cdot (\theta_i - \beta) \quad (13)$$

The energy function (1) for the EPS without FACTS devices (denoted as $V_{\text{without FACTS}}$) can now be upgraded to represent the energy function for the EPS with a STATCOM-ESS:

$$V_{\text{with STATCOM-ESS}} = V_{\text{without FACTS}} + V_{\text{STATCOM-ESS}} \quad (14)$$

where $V_{\text{STATCOM-ESS}}$ stands for the energy function (9) or (13).

4 Numerical examples

The proof of correctness and a demonstration of the application of the newly constructed energy function (14) for an EPS comprising STATCOM-ESS were carried out on an example of transient-stability assessment. In order to obtain the critical energy of the system V_{cr} a potential-energy boundary-surface (PEBS) method [8] was used, in which the critical clearing time (CCT) is the time instant when the sum of the kinetic and potential energy of the system along the fault-on trajectory equals the maximum of the potential energy along the same fault-on trajectory.

In order to prove the correctness of the proposed energy functions we compared the results—i.e., the CCTs—obtained by the direct method with the CCTs obtained by the simulation method, i.e., by the time-domain step-by-step simulation. In a single-machine infinite-bus (SMIB) test system the trajectory of the system is uniformly given and therefore the CCTs obtained should be equal, regardless of the method applied [8]. However, exactly the same system model has to be used in both the simulation and the direct method. For this reason we applied the *Mathematica* computer program. We are aware that a mathematical tool is not the most appropriate for a simulation of electric-power system dynamics, and the use of, e.g., *PSCAD, EMTP* or the *Netomac* program, would have been much easier to implement. However, in order to ensure that the system modeling is absolutely identical in the simulation and when applying direct methods we chose *Mathematica*.

The SMIB test system with a STATCOM-ESS is presented in Fig. 3. The data for this system can be found in [4]. The generator is presented as a classical model with the initial voltage at BUS1 set to 1 p.u. at 30°. The disturbance is a three-phase short-circuit near BUS1, according to Fig. 3, and it is assumed to be eventually cleared, i.e., the system's post-fault configuration is identical to the pre-fault one. The pre-fault and fault-on values of the STATCOM-ESS's controllable parameters are set to 0. The CCTs were obtained using a time-domain simulation, and directly with the use of the energy function (14).

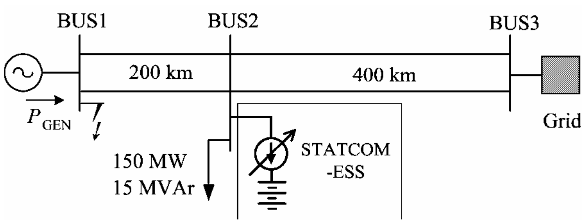

Fig. 3. SMIB test system with STATCOM-ESS

Two energy functions—i.e., (9) considering constant I_P and β and (13) considering constant control parameters I_P and P_{si}—were developed for a STATCOM-ESS to be included in the energy function for the EPS (14).

The resulting CCTs considering (9) are presented in Tab. 1 for various I_P limits and for angles β that give a maximum transient-stability improvement.

Tab. 1. CCTs obtained in a SMIB test system including STATCOM-ESS with constant I_P and β.

		Simulation method	Direct method
I_P [pu]	β [º]	CCT [ms]	CCT [ms]
0	~	106	106
0.1	135	114	114
0.2	135	120	120
0.3	120	126	126

The resulting CCTs considering (13) are presented in Tab. 2 for various I_P limits and various active-power injections P_{si}. The voltage magnitude U_i at BUS2 during the first-swing angles' propagation considerably decreases, and consequently the constant active power P_{si} injected within this period is limited to small values. The P_{si} presented in Tab. 2 are the maximum possible that at the same time give maximum CCTs. The negative sign of P_{si} means that the active power flows from the system to the STATCOM-ESS.

Tab. 2. CCTs obtained in a SMIB test system including STATCOM-ESS with constant I_P and P_{si}.

		Simulation method	Direct method
I_P [pu]	P_{si} [pu]	CCT [ms]	CCT [ms]
0	0	106	106
0.1	0	113	113
0.1	–0.01	114	114
0.2	0	119	119
0.2	–0.022	120	120
0.3	0	125	125
0.3	–0.033	126	126

Comparing the CCTs from Tab. 2 at P_{si} = 0 to the CCTs from Tab. 1, it is clear that for the employed test system the effect of the active-power injection of a STATCOM-ESS to the transient-stability improvement is relatively small according to the effect of a reactive-power injection.

5 Conclusions

Simulation is the only possible way for studying of an EPS. The simulation of its dynamic behaviour is traditionally performed by munerical "time-domain" integration of non-linear differential equations. In this paper an alternative to this method, i.e., the Lyapunov direct method, is described from the viewpoint of including FACTS devices into the EPS. The way how to model FACTS devices and how to construct their energy functions as a supplement to the existing energy function for an EPS is presented on the case of a STATCOM with energy-storage system. To prove the adequacy of the newly constructed energy functions, we apply them in numerical examples of transient-stability assessment using the Lyapunov direc method. The proposed energy functions proved to be adequate and the results show an improvement of transient stability. Further work will be focused on the control strategies based on the energy functions.

6 References

[1] K. R. Padiyar, V. Immanuel. Modelling of SVC for stability evaluation using structure preserving energy function. *Electric Power & Energy Systems*, 16:339-348, 1994.

[2] I. A. Hiskens, D. J. Hill. Incorporation of SVCs into energy function methods. *IEEE Trans. Power Systems,* 7: 133-140, 1992.

[3] R. Mihalic and U. Gabrijel, A structure-preserving energy function for a Static Series Synchronous Compensator. *IEEE Trans. Power Systems*, 19: 1501-1507, 2004.

[4] V. Azbe, U. Gabrijel, R. Mihalic and D. Povh. The Energy Function of a General Multimachine System with a Unified Power Flow Controller. *IEEE Trans. Power Systems,* 20:1478-1485, 2005.

[5] A. Gabrijel and B. Mihalic. Phase-shifting transformers in a structure-preserving energy function. *Electric Power Systems Research*, 74: 323-330, 2005.

[6] P. Sauer, M. A. Pai. Power System Dynamics and Stability. Prentice Hall. 1998.

[7] T. Van Cutsem, M. Ribbens-Pavella. Structure preserving direct methods for transient stability analysis of power systems. *24th Conference on Decision and Control*, pp. 70-77, 1985.

[8] M. A. Pai. Energy Function Analysis For Power System Stability. Kluwer Academic Publishers. 1989.

[9] T. Athay, R. Podmore,S. Virmani. A practical method for the direct analysis of transient stability. *IEEE Trans. Power Apparatus and Systems*, 98:573-582, 1979.

[10]M. Pavella, P.G. Murthy. Transient stability of power systems. John Wiley & Sons, 1994.

