A TOOL-BOX APPROACH TO COMPUTER-AIDED GENERATION OF REDUCED-ORDER MODELS

Peter Schwarz, Jens Bastian, Christoph Clauss, Joachim Haase, Andreas Köhler, Georg Otte, Peter Schneider

Fraunhofer Institute for Integrated Circuits,
Design Automation Division,
Zeunerstrasse 38, 01069 Dresden, Germany

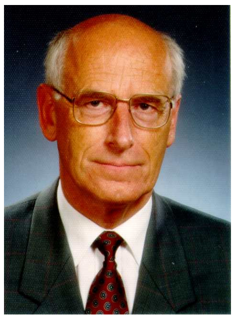
peter.schwarz@eas.iis.fraunhofer.de

Abstract

Technical systems can be characterized very often as being very complex and heterogeneous. Many simulators exist for the analysis of continuous, discrete, and hybrid systems. However, the first *modeling* steps in physical domains are not yet sufficiently assisted by CAD tools. Multi-ports and generalized KIRCHHOFFian networks proved to be powerful concepts in a physically oriented modeling methodology. If the physical system is described by partial differential equations, discretization algorithms are applied to construct models described by ordinary differential equations or equivalent lumped network models. Their parametrization can be supported by various algorithms for numerical order reduction in frequency and time domain, symbolic and semi-symbolic analysis, approximation, and optimization which are comprised in a tool box. Modeling languages as VHDL-AMS, Verilog-AMS, SystemC-AMS, and Modelica will support the practical application of these approaches.

Keywords: Model generation, Order reduction, Symbolic analysis, Modeling languages, Multi-ports, KIRCHHOFFian networks.

Presenting Author's biography



Peter Schwarz was born in Berlin, Germany. He received the diploma and the Ph.D. degree in electrical engineering from the Dresden University of Technology in 1964 and 1967, respectively. Then he worked in a CAD department of the Robotron Computer Company in Dresden. From 1982 to 1991 he was the leader of the research group "Simulation" at the Central Institute for Cybernetics and Information Processes of the Academy of Sciences in Dresden. Since 1992 he has been working at Fraunhofer IIS / EAS Dresden. He led the Modeling and Simulation department with about 30 engineers, mathematicians, and physicists before his retirement in 2006. His special interests are multi-level, mixed-signal modeling and simulation of complex heterogeneous systems (e.g., integrated circuits, MEMS, mechatronic and automation systems), methods and algorithms for model generation, web-based simulation, and knowledge transfer in life-long learning.

ISBN 978-3-901608-32-2

1 Introduction

Many technical systems like integrated circuits, micro-electro-mechanical systems (MEMS), mechatronic systems or distributed automation plants are very often complex and heterogeneous systems. They show some of the following characteristics:

- mixed-domain or "multi-physics": mechanical, electrical, thermal, fluidic, ... phenomena,
- partially close coupling between these domains, side effects, cross coupling,
- distributed and lumped (concentrated) elements,
- discrete and continuous signals and systems (in electronics: analog and digital),
- very large and stiff systems of differential equations to describe the continuous subsystems.

We will focus on continuous systems, described by partial differential equations (PDE) or ordinary differential equations (in the special forms of ODE and differential-algebraic systems, DAE). A lot of algorithms and computer programs are available for the numerical solution of such differential equations. However, there is a demand for more and better assistance in finding the equations which model the whole complex system. A powerful interdisciplinary **modeling methodology** is necessary to analyze real-world problems [1], [2], [3], [4], [5].

2 Partitioning of complex systems

Each complex system consists of subsystems and components with different functionalities. This "functional partition" is the first approach to define a complex model structure.

Fig. 1a shows the block structure of an electro-mechanical system and its corresponding model structure. If the models of the subsystems are very abstract, the simulation of the whole system with one simulator is possible. In many applications, more detailed models must be used and, therefore, the coupling of different simulators is necessary (Fig 1b).

A similar situation also exists in micro-structures. The cross-section of an electronic component, a transistor, is shown in Fig 2a in a very simplified form. Devices with such geometry can be simulated with FEM simulators. But, if hundreds or thousands of elements have to be considered together, model simplification is necessary. In Fig. 2b, the partitioning of the device (with the result of meshing in the FEM modeling) is shown. Fig. 2c shows the more abstract model composed of multi-pole elements with linear or nonlinear behavior. Unfortunately, there are no algorithms to construct such partitioned models *automatically*.

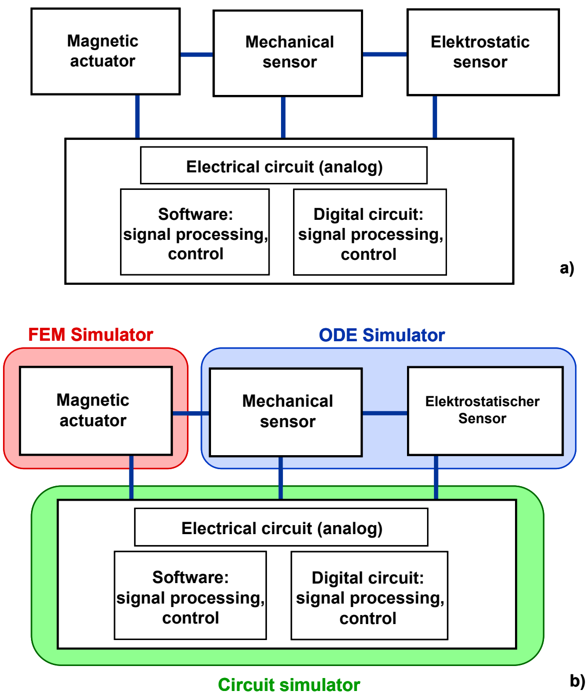


Fig 1 Partitioning of a complex system

Models have to be developed for all these subsystems. From the system simulation point-of-view, these models should be "as small as possible" to achieve a reasonable simulation time with sufficient accuracy. Detailed model description exist very often for these subsystems (e.g., transistor net lists for electronic circuits and partial differential equations for micromechanical structures) but these models are too detailed (and, therefore, too time-consuming) to be applied in <u>system</u> simulation. Models with reduced complexity

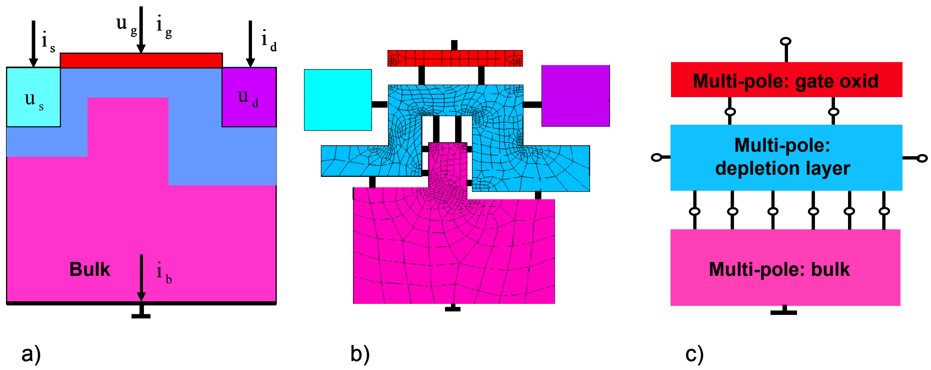


Fig. 2 Partitioning in electronic device simulation

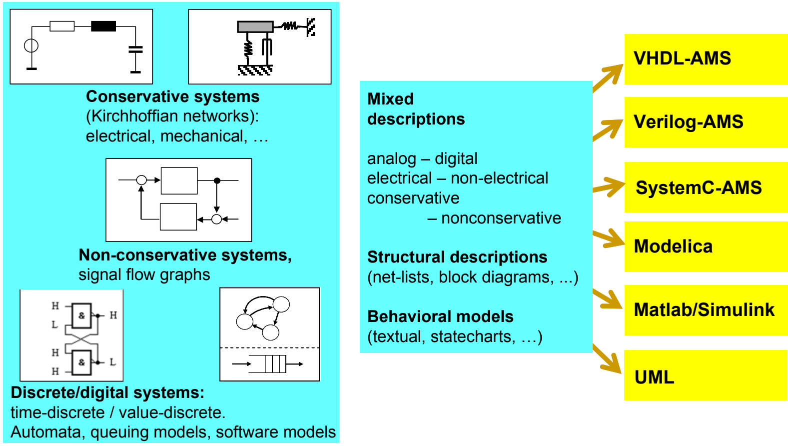


Fig. 3 Modeling languages for complex systems

and size have to be constructed. We will use the term **reduced order models** with this general meaning and will have a look also on special mathematical methods for numerical **model order reduction (MOR)**.

Another problem is the description of a mathematical model in such a way that it can be simulated within a simulator which allows embedding of user-defined behavioral models. Fig. 3 shows some "modeling languages" which can be used in the simulation of complex systems (powerful simulators support these languages!).

3 Different modeling methods

Various approaches exist for modeling complex systems. A possible taxonomy is shown in Fig. 4. The methods are divided into two main approaches: physically and mathematically oriented. The first way which is standard in many technical disciplines [6], [7], [8], [9], [10] shall be summarized only very short and the second one will be discussed more detailed in the next section.

The basic approach in physically oriented modeling is:

- hierarchical partitioning of the system into subsystems and further refinement into components,
- modeling the components behavior (if such component models are not available as simulator primitives),
- all component models are connected together to build-up the model of the complete system.

Electrical circuits are often modeled as networks. However, the network concept is not restricted to the electrical domain. It is well-known for a long time that this concept may be generalized to other physical domains like mechanics or fluidics. There are some basic requirements which have to be fulfilled by the systems to be modeled as networks:

- Networks consist of *elements* (components) and ideal *links* between them.

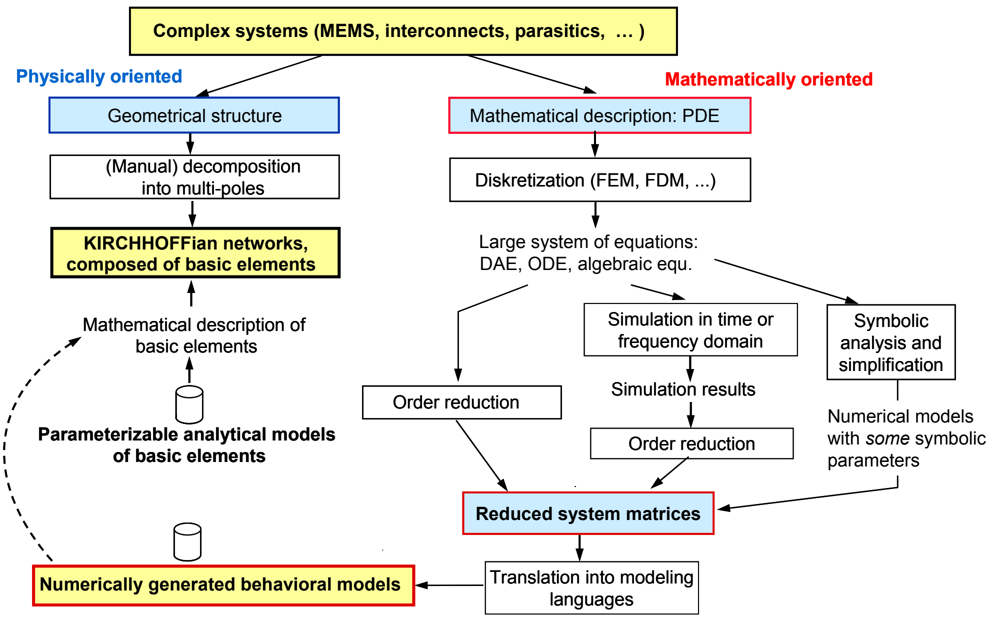


Fig.4 Modeling methods

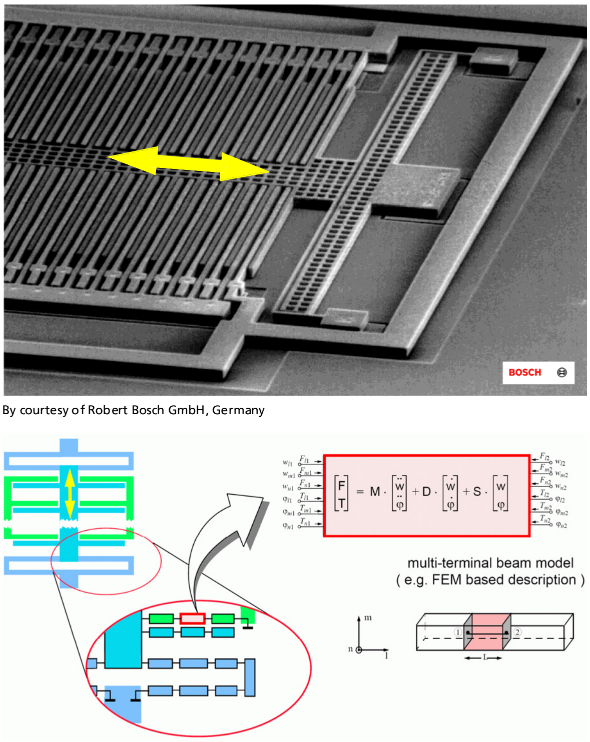


Fig. 5 Network modeling of a mechanical sensor

- There are two kinds of quantities (variables) in various physical domains:
  flow (or *through)* quantities, e.g., electrical currents or mechanical forces;
  difference (or *across*) quantities, e.g., electrical voltages or fluidic pressures.
- The elements are usually described as *two-poles* and *multi-poles*.
- The flow and difference quantities fulfill two *conservation laws*:
  the sum of all *difference* quantities between nodes along each closed cycle is zero;
  the sum of all *flow* quantities into each node is zero.

The relation between flow and difference quantities is mostly given as implicit (or explicit) equations or differential equations, solely depending on the terminal quantities and, possibly, internal states. The conservation laws are known in the electrical domain as KIRCHHOFF's voltage and current law. Therefore, the general (electrical as well as non-electrical) networks are sometimes named "generalized KIRCHHOFFian networks" [5].

This approach is illustrated in Fig. 5. A micromechanical acceleration sensor [11] is modeled by the interconnection of very small cantilever elements for which a behavioral model exists: a linear six-dimensional, second-order ODE system for translational and rotational quantities in all of the three directions in space.

A more mathematically oriented modeling method starts with the differential equations describing the system (or the subsystem / component) to be modeled. In our context we will focus on ordinary differential equations. Therefore, handling of partial differential equations (discretization in space, geometrical meshing of 2D or 3D problems) has to be carried out with other tools. Fortunately, many FEM simulators allow the export of the internally generated ODEs which can be processed in the next modeling steps.

4. Mathematical Order Reduction

Three approaches to the reduction of the order (= the state space dimension) of the equations describing the system have been under investigation for several years now:

a) Linear systems of differential equations (of first or second order) were numerically reduced by projection methods, especially with KRYLOV subspace algorithms [12], [13], [14], [15]. The systems of equations have to be externally available, which can not be guaranteed if commercial simulators are used.

b) Based on the result of FEM *simulation* in the time or frequency domain, order reduction is possible with methods which have their origins in model identification for control system design [12], [13].

c) Symbolic analysis methods calculate analytical formulas instead of numbers to describe the system behavior [16], [17], [18]. For the analysis of *large* systems the analysis algorithms are always combined with simplification algorithms (otherwise the resulting equations would be so large that they could not be handled). This is a very special way of "model reduction" which can be combined with other approaches.

4.1 Projection methods based on moment matching

In many cases, very large systems of differential equations (the dimension may be 10^4 or 10^5 or larger!) result from FEM or FDM modeling and discretization in space of the PDE. Furthermore, the analysis of electrical interconnect systems [19], [20], [21] or modeling of micro-mechanical systems [22], [23] with

Description of the original system with linear differential equations:

1st order systems

$$\left.\begin{aligned} \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{G}\,\mathbf{x} &= \mathbf{B}\,\mathbf{u} \\ \mathbf{y} &= \mathbf{L}\,\mathbf{x} \end{aligned}\right\}$$

$$\mathbf{C}, \mathbf{G} \in \mathbb{R}^{N\times N};\ \mathbf{B} \in \mathbb{R}^{N\times p},\ \mathbf{L} \in \mathbb{R}^{m\times N}$$

2nd order systems

$$\left.\begin{aligned} \mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} &= \mathbf{B}^{\text{in}}\mathbf{u} \\ \mathbf{y} &= \mathbf{B}_1^{\text{out}}\mathbf{x} + \mathbf{B}_2^{\text{out}}\dot{\mathbf{x}} \end{aligned}\right\}$$

$$\mathbf{M}, \mathbf{D}, \mathbf{K} \in \mathbb{R}^{N\times N},\ \mathbf{B}^{\text{in}} \in \mathbb{R}^{N\times m};\ \mathbf{B}_1^{\text{out}}, \mathbf{B}_2^{\text{out}} \in \mathbb{R}^{p\times N}$$

Fig. 6 Typical structure of systems of differential equations for order reduction

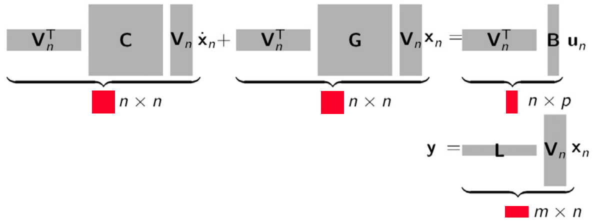


Fig. 7 Reduction of order by multiplication with projection matrices

KIRCHHOFFian networks may result in extremely large systems of equations, too. In Fig. 6, two widely used structures of differential equations are summarized as examples for electrical RC systems (**C** and **G** correspond to electrical capacitances and resistances, respectively; **M**, **D** and **K** are related to mechanical masses, damping effects, and stiffness). But the same structures occur in modeling thermal systems and electrical RLC circuits, respectively.

The reduction of the dimensions, also w.r.t to the output vector **y**, which corresponds to the interface signal to the environment of the modeled subsystem, is illustrated in Fig. 7.

Based on the passivity preserving ENOR (Efficient Nodal Order Reduction) algorithm for linear second-order systems [21], a modified variant has been developed and proven in many applications [23]. This algorithm has been further improved with special attention to proper deflation of basis vectors and numerical stability through selective re-orthogonalization [24]. By choosing certain expansion points, the approximation of the original transfer function can be improved in desired regions of the frequency domain. For multiple-input, multiple-output (MIMO) systems, the algorithm allows to simultaneously match a desired amount of block moments corresponding to different expansion points.

This method has been applied to the system simulation of an electronically controlled yaw rate sensor [22], [25] (Fig.8). The mechanical subsystem has been modeled with about 45000 variables in the original FEM model. To design the electronic subsystems, a reduced order model with 40 variables was generated.

4.2 Simulation-based order reduction

Simulation-based order reduction has some specific advantages. It is not necessary to export the system matrices (this export feature is not supported by all FEM simulators!). Additionally, the system to be modeled can consist of different subsystems (which are, e.g., modeled with transcendental functions which arise in the context of electrical transmission lines). In the simplest case it is only necessary to determine the impulse response of the system.

We restrict us to linear time-discrete systems which are described at equidistant time-points t_k by

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k + Du_k$$

x_k is the N-dimensional state vector, u_k is the m-dimensional input vector, and y_k is the q-dimensional output vector. The matrix A is $N \times N$, B is $N \times m$, C is $q \times N$, and D is $q \times m$. These matrices and the order N are assumed to be *not explicitly known*.

The Hankel matrix

$$\mathsf{H}_p = \begin{pmatrix} g_1 & g_2 & \cdots & g_p \\ g_2 & g_3 & \cdots & g_{p+1} \\ \vdots & \vdots & \ddots & \vdots \\ g_p & g_{p+1} & \cdots & g_{2p} \end{pmatrix}$$

can be constructed from the impulse responses of this system calculated at the first *2p* time-points. Any full-rank decomposition of H_p in a $pq \times \tilde{N}$ Matrix $\tilde{\mathsf{O}}_p$ and a $\tilde{N} \times pm$ Matrix $\tilde{\mathsf{R}}_p$ with $\tilde{N} \le p$ is qualified to determine the observability and the controllability matrices of a system $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ of order $\tilde{N}$ which approximates the given system (A, B, C, D) of order $N > \tilde{N}$. A reliable and numerically stable method of full-rank decomposition is the Singular Value Decomposition (SVD). A procedure using these relations is a simplified version of the well-known class of N4SID algorithms (Numerical Subspace-based State-

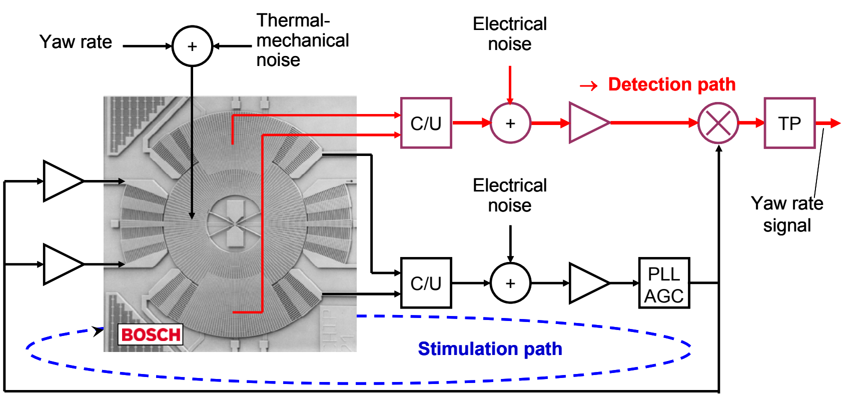


Fig. 8 Yaw rate sensor and its signal-processing electronic circuitry

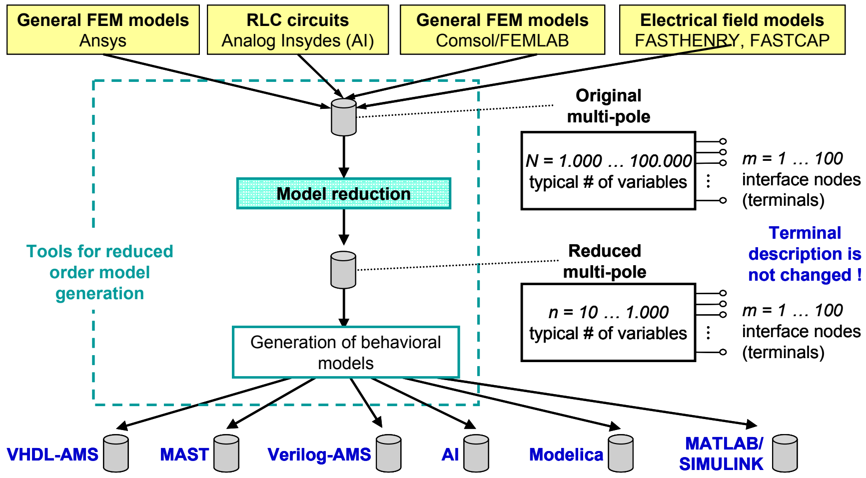


Fig. 9 Tool-suite for the generation of reduced-order models

Space System Identification) [26], [27], [28]. To guarantee a high accuracy of the approximated system matrices, the system $(\tilde{A},\tilde{B},\tilde{C},\tilde{D})$ should be calculated in a first step with a "high" dimension, e.g. $\tilde{N} = 10^2 \ldots 10^3$. In a second step, these “medium sized” matrices can be reduced by matrix-based order reduction, e.g. by the above mentioned projection method, to achieve a system of low order $\tilde{\tilde{N}} < \tilde{N}$ [29].

All the described methods have been implemented in form of a tool-suite (Fig. 9) with interfaces to widely-used FEM simulators (and to Analog Insydes, see Sec. 4.4). The most important modeling languages are supported. The system is open to include new or improved MOR algorithms.

4.3 An outlook to nonlinear MOR

Many systems can not be considered as linear. Obviously, model generation, simplification, and order reduction is much more complicated in the nonlinear case. First research results are encouraging but many problems remain to be solved [12], [31], [32], [33]. An approach to reducing *nonlinear* systems is the Trajectory Piecewise Linear (TPWL) method [34], [35]. Given a simulated trajectory called training trajectory, we choose some linearization points (Fig. 10). Now, the nonlinear behavior of the system can be approximated in a neighborhood of the training trajectory by a weighted combination of multiple linear

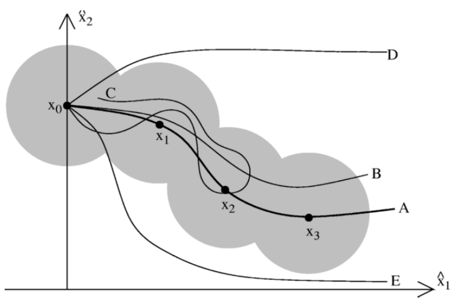


Fig. 10 Nonlinear order reduction with linearization along trajectories

models. These linear models are obtained by Proper Orthogonal Decomposition (POD) [12]. In a certain sense, POD is an extension of Singular Value Decomposition to integral equations by utilizing system identification methods.

In some special applications, nonlinear MOR can be realized by smart partitioning of a complicated nonlinear system into coupled subsystems which can be modeled with different methods. This may be useful not only at system level but also at component level to handle strongly coupled effects. The following example [36] illustrates the problem and a solution approach. A micromechanical device consisting of a cantilever is stimulated by electrostatic forces which result from the applied electrical voltage. The mechanical and the electrical subsystems are characterized by different mathematical formalisms. The partitioning approach is:

- decoupling into a mechanical cantilever model which can be *linear* (due to the very small dimensions of the device), and the electro-static field model,
- the *nonlinear* electro-static subsystem has been manually discretized and is modeled by n nonlinear transducer elements (force sources and capacitances),
- both parts are modeled as multi-poles and their coupling is realized by connecting their terminals.

The whole system is highly nonlinear (e.g., with hysteresis), but the partial models are relatively simple: a mechanical subsystem generated automatically via order reduction and a physically oriented modeled electrostatic subsystem containing transducers with nonlinear behavior. The capacitances are nonlinear functions of the distances x_i between a piece of the cantilever and the substrate. In Fig. 11 this approach is illustrated.

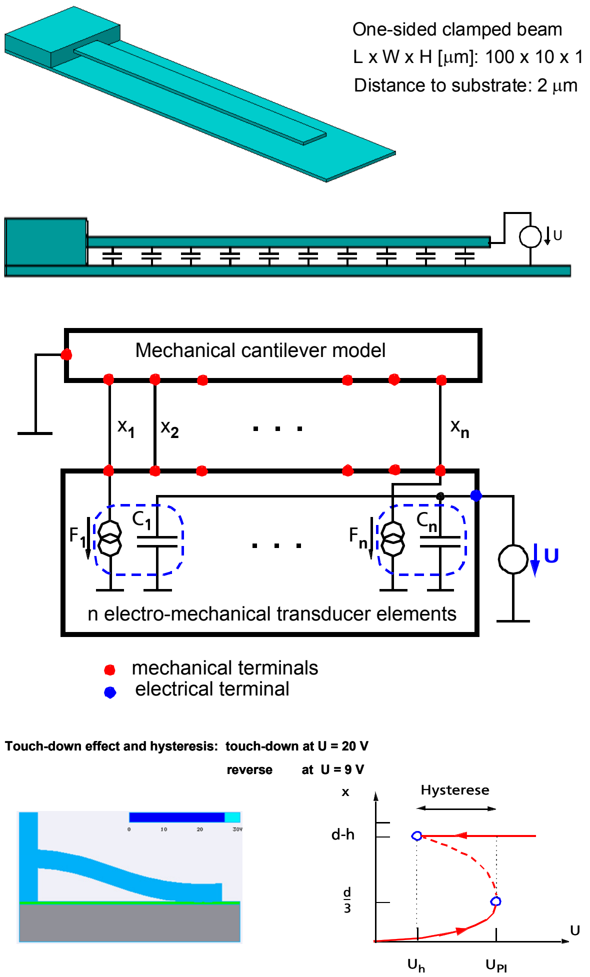


Fig. 11 Nonlinear electro-mechanical microsystem: mechanical structure, multipole model, touchdown effect and hysteresis

Another field of current research is *parametric model order reduction*, also called symbolic or multidimensional MOR. Consider a linear time invariant system with system matrices dependent on certain parameters:

$$C(\lambda)\dot{x} + G(\lambda)x = Bu$$
$$y = B^T x + Du$$

with $\lambda = (\lambda_1, \ldots, \lambda_k) \in \mathbf{R}^k$.

The aim is to find a reduced system with matrices still depending on these parameters. Using a multidimensional TAYLOR series expansion of C and G, one can extend existing projection schemes to parametric systems [19], [20]. Unfortunately, it is not always possible to obtain such a parametric systems in explicit form, e.g. when using black-box tools. A heuristic approach would be to generate several snapshots of the system matrices by varying parameters around some nominal values. These snapshots are combined with a multidimensional regression method, which leads to the desired linear system with parameterized system matrices.

4.4 Symbolic Analysis

Symbolic analysis is applied in the context of computer-aided model generation [17], [18], [37], [38] and can be used for MOR, too. The commercial tool Analog Insydes [39] is mainly used in modeling analog electronic circuits. But the internal algorithms handle the simplification of systems of nonlinear differential equations in general and not only for electronics. This simplification is coupled with simulation-based error control and, therefore, the generated models are very reliable. For the application in multi-physics modeling, a library of basic mechanical elements has been developed [40]. Fig. 12 shows its application to a micromechanical sensor (similar to the system shown in Fig. 5).

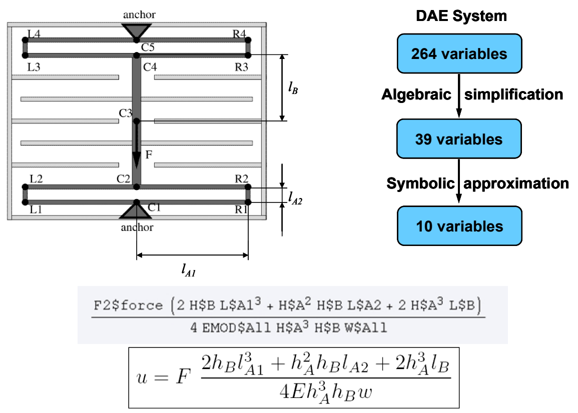


Fig 12 MOR with symbolic analysis for a micromechanical device

5 Summary and Conclusion

Many modeling methods have been developed in the last decades to handle multi-physics systems. The most promising approaches are:

- modeling with (generalized) KIRCHHOFFian networks
- model order reduction of very large systems of linear ordinary differential equations
- symbolic analysis (combined with simplification) of linear and nonlinear ordinary differential equations
- application of optimization programs [41] for model parameterization and of approximation programs to fit model characteristics to measured or simulated data (e.g. in piecewise-linear or table-based models)

Progress in the development of modeling languages (Modelica, VHDL-AMS, etc.) and their support by powerful system simulators is another basis of successful modeling. These languages support in a nearly perfect way

- hierarchical partitioning of complex systems into simpler subsystems,

- formulation of different models with appropriate mathematical formalisms (e.g., linear and nonlinear DAE as well as state-space descriptions, discrete-time models and difference equations, automata, discrete event-queue organized models) and its common computation in *one* model,
- clear distinction between the interface of the subsystem (or component) and its internal description as behavioral and/or structural models,
- mechanisms for the exchange various internal model descriptions (but with the same interface),
- signal exchange with other models only via the terminals.

Therefore, the *combination* of different modeling approaches is supported by these languages.

Tool-boxes, consisting of

- various model generation algorithms,
- optimization and approximation algorithms for model parameterization,
- multi-physics model libraries,
- interface modules to widely-used FEM simulators,
- translation modules to transform mathematical model descriptions into all widely-used modeling languages,

can support the modeling of very large and complex systems. These systems have to be *manually* partitioned in an appropriate manner to apply the available algorithms to the subsystems.

Due to the intensive research activities, further progress in nonlinear order reduction and coupling with other methods (e.g., symbolic analysis) can be expected. But in certain situations, current state-of-the-art tools are already powerful enough for the analysis of complex heterogeneous systems.

References

[1] B.F. Romanowicz. Methodology for the Modeling and Simulation of Microsystems. Kluwer, Dordrecht, 1998.

[2] S.D. Senturia. Microsystem Design. Kluwer, Dordrecht, 2000.

[3] P. Schwarz. Physically oriented modeling of heterogeneous systems. 3. IMACS Symp. MATHMOD, Wien, 2000, pp. 309-318

[4] P. Schwarz, P. Schneider. Model Library and Tool Support for MEMS Simulation. SPIE's Conf. Microelectronic and MEMS Technology, Edinburgh, 2001.

[5] G. Wachutka. Tailored modeling: a way to the 'virtual micro transducer fab' ? Sensor and Actuators A 46-47 (1995), pp. 603-612.

[6] H.E. Koenig, W.A. Blackwell. Electromechanical System Theory. McGraw-Hill, New York 1961.

[7] P.E. Wellstead. Introduction to Physical System Modelling. Academic Press, London 1979.

[8] E. Tonti. The reason for analogies between physical theories. Appl. Math. Modelling 1(1976), 37-50.

[9] P. Voigt, G. Wachutka. Electro-fluidic microsystem modeling based on Kirchhoffian network theory. Sensor and Actuators A 66 (1998)1-3, 6-14.

[10] G.K. Fedder, Q. Jing. A hierarchical circuit-level design methodology for micromechanical systems. IEEE Trans. CAS-II, 46(1999)10), pp. 1309-1315.

[11] R. Neul et al. A modeling approach to include mechanical microsystem components into system simulation. Proc. Design, Automation & Test Conf. (DATE'98), Paris, 1998, pp. 510-517.

[12] A.C. Antoulas et al. A survey of model reduction methods for large-scale systems. Contemporary Mathematics, 280 (2001), pp. 193-219.

[13] C. Antoulas, D.C. Sorensen. Approximation of large-scale dynamical systems: An overview. International J. of Applied Mathematics and Computational Science 11 (2001), pp. 1093-1121.

[14] R.W. Freund. Model Reduction Methods Based on Krylov Subspaces. Numerical Analysis Manuscript No. 03-4-01, Bell Laboratories, 2003.

[15] R.-C. Li, Z. Bai. Structure-preserving model reduction using a Krylov subspace projection formation. Comm. Math. Sciences, 3(2005) pp. 179-199.

[16] G. Gielen, W. Sansen. Symbolic Analysis for Automated Design of Analog Integrated Circuits, Kluwer Academic Publishers, Boston (USA), 1991

[17] E. Hennig, J.M. Tweer, R. Sommer. Enhanced symbolic matrix approximation techniques. Proc. SMACD'98, Kaiserslautern (Germany), Oct. 1998, pp. 199–206

[18] T. Wichmann, Symbolische Reduktionsverfahren für nichtlineare DAE-Systeme. Ph.D. dissertation, Shaker Verlag, Aachen, 2004.

[19] P. Gunupudi, M. Nakhla. Multi-Dimensional Model Reduction of VLSI Interconnects. Proc. CICC 2000, pp. 499-502.

[20] X. Li, P. Li, L.T. Pileggi. Parameterized interconnect order reduction with explicit-and-implicit multi-parameter moment matching for inter/intra-die variations. Proc. ICCAD 2005, pp. 806-812.

[21] B.N. Sheehan. ENOR: Model order reduction of RLC circuits using nodal equations for efficient factorization. Proc. 36th Design Automation Conference, 1999.

[22] S. Reitz, J. Bastian, J. Haase, P. Schneider, P. Schwarz. System level modeling of microsystems using order reduction methods. Symp. Proc. Design, Test, Integration and Packaging of MEMS/MOEMS (DTIP), Cannes, France 2002, 365-373.

[23] J. Bastian, J. Haase. Order reduction for second order systems. 4th MATHMOD, Vienna, 2003, pp. 418-424.

[24] A. Köhler. Modellreduktion von linearen Deskriptorsystemen erster und zweiter Ordnung mit Hilfe von Block-Krylov-Unterraumverfahren. Diploma thesis, TU Bergakademie Freiberg, 2006.

[25] S. Reitz et al. System Level Modeling of the Relevant Physical Effects of Inertial Sensors using Order Reduction Methods. Proc. DTIP 2004, Montreux (Switzerland), 2004, pp. 383-387.

[26] P. van Overschee, B. de Moor. Subspace algorithms for the identification of combined deterministic-stochastic systems. Automatica 30(1994)1, pp. 75-93.

[27] P. van Overschee, B. de Moor. A unifying theorem for three subspace system identification algorithms. Automatica 31(1995)12, pp. 1853-1864.

[28] M. Viberg. Subspace-based methods for the Iidentifikation of linear time-invariant systems. Automatica 31(1995)12, pp.1835-1851

[29] G. Otte, S. Reitz, J. Haase. Generation of linear models using simulation results. 4. IMACS-Symp. MATHMOD, Vienna, 2003, pp. 436-443.

[31] S. Senturia, N.R. Aluru, J. White. Simulating the behavior of MEMS devices: computational methods and needs. IEEE Trans. Computational Science & Engineering, January 1997, pp. 30-54.

[32] L.D. Gabbay et al. Computer-aided generation of nonlinear reduced-order dynamic macromodels. J. Micromechanical Systems 9(2000)2, pp. 262-278.

[33] J.R. Phillips. Projection-based approaches for model reduction of weakly nonlinear, time-varying systems. IEEE Trans. CAD 22(2003)2, pp. 171-187.

[34] M.J. Rewieński. A Trajectory Piecewise-Linear Approach to Model Order Reduction of Nonlinear Dynamical Systems. PhD Thesis, Massachusetts Institute of Technology, 2003.

[35] A. Rewienski, J. White. Trajectory piecewise-linear approach to model order reduction and fast simulation of nonlinear circuits and micro-machined devices. IEEE Trans. CAD 22(2003)2, pp. 155-170.

[36] J. Haase, S. Reitz, P. Schwarz. Ermittlung mehrdeutiger Kennlinien elektrisch-mechanischer Systeme mit VHDL-AMS-Simulatoren. Workshop "Multi-Nature-Systems 2001", Hamburg, 21.02.01, S.25-32.

[37] R. Sommer, J. Broz, T. Halfmann. Automated behavioral modeling and analytical model-order reduction by application of symbolic circuit analysis for multi-physical systems. This Conference.

[38] T. Halfmann, T. Wichmann. Symbolic Methods in Industrial Analog Circuit Design, Scientific Computing in Electrical Engineering, Springer, 2006.

[39] J. Broz et al. Analog Insydes 2.1 Manual, Fraunhofer ITWM, Kaiserslautern, 2005.

[40] J. Broz et al. Automated symbolic modelling approach for the design of mechatronical systems. IEEE Int. Conf. Computer Aided Control Systems Design, Munic, 2006.

[41] P. Schneider et al. A modular approach for simulation based optimization of micro systems. Intern. Symp. Microelectronics and Assembly, Singapore, 2000, pp. 71-82.

